29 Mar 2001, 4:00 pm - 6:00 pm

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CRITICAL ACCELERATION AND SEISMIC DISPLACEMENT OF VERTICAL GRAVITY WALLS BY A TWO BODY MODEL

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Paper No. 7.02

ABSTRACT

Under the assumption that as a result of earthquake loading the backfill behind a gravity wall reaches an active state, and witl further increase in the earthquake acceleration the wall slides outwards, the soil-wall system consists of two bodies, each slidinı along a different inclination: (a) the active soil wedge that slides with the inclination of least resistance in the backfill, and (b) the wall that slides along the soil-wall boundary at the base. This paper first gives the equation of motion of the 2-block sliding systen described above that models the seismic response of vertical gravity walls retaining dry sand. Then, using the principle of limi equilibrium it gives analytical expressions giving (a) the angle of the prism of the active soil wedge, and (b) the corresponding value of the critical acceleration. Finally, differences between the predicted displacement by the new model and those of Newmark's sliding-block model are detected and discussed.

INTRODUCTION

Shaking table tests have illustrated that in gravity walls retaining dry sand, as the horizontal earthquake force increases, the shear force in the backfill behind the wall increases, until an active soil wedge is formed behind the wall. With further increase in the horizontal force, the wall and the active soil wedge move outwards. Displacements accumulate each time that the applied horizontal acceleration is larger that the critical. Fig.1a gives typical mode of failure measured in shaking-table tests by Nishimura et al (1995).

Sliding-block analysis has been proposed to model this response. The commonly used solution is given by Richards and Elms (1979): the force acting on the wall is estimated by the Mononobe-Okabe (M-O) equation (Okabe, 1926 and Mononobe and Matsuo, 1929), and the weight of the wall to prevent motion is estimated by considering the inertial force of the wall. The M-O force acting on the wall boundary corresponds to the soil prism that produces maximum force on the wall, when an inertia force is applied in the backfill. Further, the displacement caused by this horizontal force is estimated by Newmark's (Newmark, 1965) block-on-an-inclined-plane model.

However, under the assumptions that a wedge in the backfill behind the wall reaches an active state, and that the wall and the backfill slide outward, the soil-wall system consists really of two bodies: (a) the active soil wedge that slides with the inclination of least resistance in the backfill, and (b) the wal that slides along the soil-wall boundary in the foundation During relative movement, the force on the wall-backfil boundary does not equal to the force given by the M-C equation, as it depends not only on the forces that act on the prism formed in the backfill, but also on the forces that act or the wall. The relative velocity of the wall can be related to the relative velocity of the soil wedge by the restriction o compatibility of velocities.

This paper first gives the equation of motion of the 2-block sliding system described above, when an horizonta earthquake is applied. The geometry considered is that of a vertical wall retaining dry backfill, not necessarily havinį horizontal ground surface. Then, using the principle of limi equilibrium the paper derives analytical expressions givinį (a) the angle of the prism of the active soil wedge, and (b) the corresponding value of the critical acceleration. Finally differences between the predicted displacement by the nev model and those of Newmark's sliding-block model are detected and discussed.

EQUATIONS OF MOTION

The sliding system consists of two bodies shown in Fig.2a the wall (body 1) and the active soil wedge (body 2). The wal slides with inclination α_1 and the soil wedge with inclinatior

α'_2. The distance moved by each body in its direction of sliding is denoted as u_i. At the soil-wall interface between the bodies both normal and shear force components exist. Their sum produces a single force, P_a, which acts at an angle φ_3 to the interface. Figure 2b gives all the forces acting on the two bodies. For each body i, (i=1 for the wall, i=2 for the active soil wedge), these forces are: (a) the weight of the mass W_i, (b) the horizontal seismic force $k(t)W_i$, (c) the force from the other mass in contact P_a, (d) the normal force N_i between the body and the slip surface and (e) the shear force resisting relative movement $N_i \cdot \tan\varphi_i$.

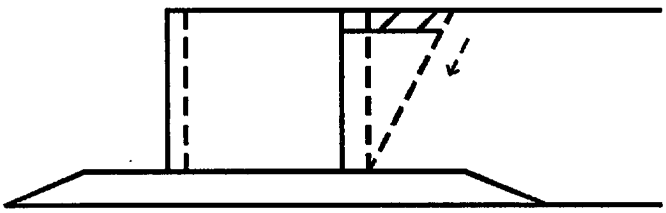

Fig. 1 Typical mode of failure of gravity walls observed in shaking-table tests (Nishimura et al, 1995).

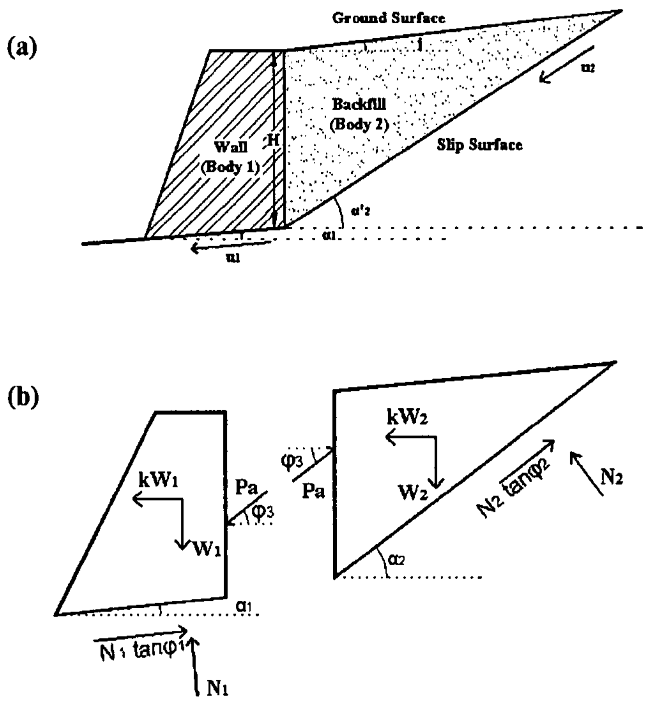

Fig. 2 (a) General geometry of soil - wall system considered in the present analysis. (b) Forces acting on the two bodies.

Application of equilibrium for each body separately gives:

$$m_1\ddot{u}_1 = \frac{1}{\cos\varphi_1}\left[m_1 g\sin(\alpha_1-\varphi_1)+k(t)m_1 g\cos(\alpha_1-\varphi_1)+ +P_a\cos(\varphi_3-\alpha_1+\varphi_1)\right] \tag{1a}$$

and

$$m_2\ddot{u}_2 = \frac{1}{\cos\varphi_2}\left[m_2 g\sin(\alpha'_2-\varphi_2)+k(t)m_2 g\cos(\alpha'_2-\varphi_2)- -P_a\cos(\varphi_3-\alpha'_2+\varphi_2)\right] \tag{1b}$$

where the dot indicates differentiation with respect to time, $k(t)g$ is the applied acceleration record and g is the acceleration of gravity.

It is assumed that total contact exists on the shearing surface between the wall and the backfill. Thus, the component of the movement perpendicular to this surface should be the same for both moving bodies. This gives:

$$\frac{du_1}{du_2} = \frac{u_1}{u_2} = \frac{\sin(90^\circ+\alpha'_2)}{\sin(90^\circ+\alpha_1)} = \frac{\cos\alpha'_2}{\cos\alpha_1} = \lambda \tag{2}$$

Equations (1a), (1b) and (2) give that the governing equation describing the motion of the whole system is:

$$\ddot{u}_2 = Z_2\cdot(k(t)-k'_c)\cdot g \tag{3}$$

where:

$$Z_2 = \frac{m_1\cos(\alpha_1-\varphi_1)\cos(\varphi_3-\alpha'_2+\varphi_2)+m_2\cos(\alpha'_2-\varphi_2)\cos(\varphi_3-\alpha_1+\varphi_1)}{\lambda m_1\cos\varphi_1\cos(\varphi_3-\alpha'_2+\varphi_2)+m_2\cos\varphi_2\cos(\varphi_3-\alpha_1+\varphi_1)} \tag{4}$$

and the critical acceleration coefficient required for motion is:

$$k'_c = \frac{m_1\sin(\varphi_1-\alpha_1)\cos(\varphi_3-\alpha'_2+\varphi_2)+m_2\sin(\varphi_2-\alpha'_2)\cos(\varphi_3-\alpha_1+\varphi_1)}{m_1\cos(\alpha_1-\varphi_1)\cos(\varphi_3-\alpha'_2+\varphi_2)+m_2\cos(\alpha'_2-\varphi_2)\cos(\varphi_3-\alpha_1+\varphi_1)} \tag{5}$$

Also, the interface force P_a equals to:

$$P_a = m_1 m_2\frac{P_1}{P_2}g \tag{6}$$

where:

$$P_1 = \sec\varphi_2\cdot[k(t)\cos(\alpha'_2-\varphi_2)+\sin(\alpha'_2-\varphi_2)]- \lambda\sec\varphi_1\cdot[k(t)\cos(\alpha_1-\varphi_1)+\sin(\alpha'_1-\varphi_1)]$$

$$P_2 = m_1\lambda\cos(\varphi_3-\alpha_1+\varphi_1)\sec\varphi_1+m_2\cos(\varphi_3-\alpha'_2+\varphi_2)\sec\varphi_2$$

Using equation (2), Eq.(3) can be expresssed in terms of the distance moved u_1 as:

$$\ddot{u}_1 = Z_1\cdot(k(t)-k'_c)\cdot g \tag{7}$$

where:

$$Z_1 = \lambda \cdot Z_2 \qquad (8)$$

A similar (but more general) slope has been considered by Sarma and Chlimitzas (2000). The governing equation of motion and the critical acceleration are equivalent to those given by equations (2), (3), (4) and (5) for the particular geometry of Fig. 2a.

CRITICAL ANGLE α_2 AND VALUE OF k_c.

Analytical Solution

According to the limiting equilibrium method in the backfill - soil system, the angle α_2 corresponds to the soil wedge angle formed in the backfill, α'_2, that produces instability with the minimum possible applied acceleration. According to Eq.(3), this value of α'_2 can be obtained by the minimization of k'_c. By invoking simple trigonometric rules, equation (5) is written as:

$$k'_c = AA/BB \qquad (9)$$

with:

$$AA = m_1 \cdot \{\tan(\varphi_1 - \alpha_1) \cdot [1 + \tan\varphi_2 \tan\alpha'_2] - [\tan\varphi_2 - \tan\alpha'_2]\tan\varphi_3\} + m_2 \cdot [\tan\varphi_2 - \tan\alpha'_2] \cdot [1 - \tan\varphi_3]\tan(\varphi_1 - \alpha_1) \qquad (10)$$

and

$$BB = m_1 \cdot \{[1 + \tan\varphi_2 \tan\alpha'_2] - \tan\varphi_3[\tan\varphi_2 - \tan\alpha'_2]\} + m_2 \cdot [1 + \tan\varphi_2 \tan\alpha'_2] \cdot [1 - \tan(\varphi_1 - \alpha_1)\tan\varphi_3] \qquad (11)$$

The mass m_2 of the soil wedge behind the wall can be expressed in terms of the angle α'_2, the inclination of the backfill i, the height of the wall H and the unit weight of the soil γ, as:

$$m_2 = 0.5\gamma H^2 / [(\tan\alpha'_2 - \tan i) \cdot g] \qquad (12)$$

Substitution of Eqs. (10) - (12) into Eq. (9) gives a second order equation of $\tan\alpha'_2$ that includes the critical acceleration k'_c as a variable:

$$A_1 \tan^2\alpha'_2 + A_2 \tan\alpha'_2 + A_3 = 0 \qquad (13)$$

where:

$$A_1 = x \cdot (\beta + f) \cdot (\varepsilon - k'_c) \qquad (14)$$

$$A_2 = (1 - \varepsilon \cdot f) \cdot (1 + \beta \cdot k'_c) + [f \cdot \kappa - 1 + \beta \cdot (f + \kappa)] \cdot (k'_c - \varepsilon)x \qquad (15)$$

$$A_3 = (1 - \varepsilon f) \cdot (k'_c - \beta) + \kappa \cdot x \cdot (1 - \beta f) \cdot (\varepsilon - k'_c) \qquad (16)$$

and the dimensionless quantities β, ε, κ, f, x are:

$$\beta = \tan\varphi_2, \varepsilon = \tan(\varphi_1 - \alpha_1), \kappa = \tan i, f = \tan\varphi_3, x = \frac{2m_1 g}{\gamma H^2} \qquad (17)$$

The critical angle of α'_2, α_2, corresponds to the double root of the trinomial (13) (Caltabiano et al., 1999) or to the value where the graph of k'_c versus α'_2 has a minimum. The necessary condition for a double root to exist, is the vanishing of the discriminant, or the condition:

$$A_2^2 - 4A_1A_3 = 0 \qquad (18)$$

Since the critical acceleration is included in the expressions for A_1 - A_3, the above relation can be written as:

$$K_1 \cdot (k'_c)^2 + K_2 \cdot k'_c + K_3 = 0 \qquad (19)$$

where:

$$K_1 = \beta^2(\varepsilon f - 1)^2 + 2(\varepsilon f - 1) \cdot [\beta + 2f + \beta f\kappa + \beta^2(f + \kappa)]x + [1 - \beta f + (\beta + f)\kappa]^2 x^2 \qquad (20)$$

$$K_2 = 2\beta(\varepsilon f - 1)^2 - 2(\varepsilon f - 1) \cdot \{1 + \beta(2\beta + f) - \kappa(\beta + f) + \varepsilon x \cdot [\beta + 2f + \beta f k + \beta^2(f + \kappa)]\} - 2\varepsilon[1 - \beta f + \kappa(\beta + f)]^2 x^2 \qquad (21)$$

$$K_3 = (\varepsilon f - 1)^2 + 2\varepsilon(\varepsilon f - 1) \cdot [1 + \beta(2\beta + f) - (\beta + f)\kappa]x + \varepsilon^2[1 - \beta f + (\beta + f)\kappa]x^2 \qquad (22)$$

From Eq. (19) it is evident that the minimum critical coefficient k'_c, denoted as k_c, is given as:

$$k_C = \frac{-K_2 + (K_2^2 - 4K_1K_3)^{0.5}}{2K_1} \qquad (23)$$

and the double root of the Eq. (13) can be obtained from k_c as:

$$\alpha_2 = \tan^{-1}(-A_2 / 2A_1) \qquad (24)$$

where the parameters A_1 and A_2 are given by equations (14) and (15) for the value of k_c given by (23).

Comparison With Previous Solutions

The Mononobe-Okabe method calculates the maximum interface force P_a on a vertical wall for the case of a backfill with inertial horizontal and vertical acceleration (kg) and $(k_v g)$ as:

$$P_a = 0.5\gamma H^2 \cdot K_A \qquad (25)$$

where (using the notation of Fig. 2):

$$K_A = \frac{\cos^2(\varphi_2 - \psi)}{\cos\psi\cos(\varphi_3+\psi)\left[1+\left(\frac{\sin(\varphi_3+\varphi_2)\sin(\varphi_2 - i - \psi)}{\cos(\varphi_3+\psi)\cos i}\right)^{0.5}\right]^2} \tag{26}$$

and

$$\psi = \tan^{-1}[k/(1-k_v)] \tag{27}$$

Zarrabi-Kashani (1979) proves that for the Mononobe-Okabe equation, the corresponding rupture angle in the backfill, α_2, equals to:

$$\alpha_2 = \varphi_2 - \psi + \tan^{-1}\{[C_1 - \tan(\varphi_2 - \psi - i)]/C_2\} \tag{28}$$

where:

$$(C_1)^2 = \tan(\varphi_2 - \psi - i)\cdot[\tan(\varphi_2 - \psi - i) + \cot(\varphi_2 - \psi)]\cdot$$
$$\cdot[1+\tan(\varphi_3 - \psi)\cdot\cot(\varphi_2 - \psi)]$$

$$C_2 = 1 + \tan(\varphi_3 - \psi)[\tan(\varphi_2 - \psi - i) + \cot(\varphi_2 - \psi)]$$

According to the approach proposed by Richards and Elms (see introduction) the critical acceleration of a soil-wall system can be obtained from the dynamic equilibrium of the wall when the M-O force acts on the wall. For the particular case where $\alpha_1 = 0°$ we have:

$$k_c = \tan\varphi_1 - P_a\cdot\frac{\cos\varphi_3 + \sin\varphi_3\tan\varphi_1}{m_1 g} \tag{29}$$

where P_a is given by equation (25) and it corresponds to the force acting on the wall when the horizontal acceleration equals to k_c. Thus, iteration is needed to obtain k_c by Eq. (29).

Numerical analyses of various cases of walls of the general geometry of Fig. 2a, showed that (a) Eq. (25) and (b) Eq. (28) produce the same value for (a) the interface force P_a and (b) the rupture angle α_2 as the equations (6) and (24) respectively, only when $k=k_c$ (and $k_v=0$). The reason is, that the previous solutions given in the literature were obtained using the equilibrium of either body or each body separately. The present solution considers the equilibrium of both bodies during motion. The two solutions do not produce the same results when relative movement occurs (i.e. when the problem is dynamic). Nevertheless, the two solutions coincide just prior to relative motion i.e. when $k=k_c$. Also, numerical analyses illustrate that Eq. (29) when iterated, produced the same value of the critical acceleration k_c as the closed - form Eq. (23).

Parametric Analyses

A computer program was written for the computation of the critical acceleration k_c and the critical angle α_2, according to the equations (23) and (24). Analyses were performed to observe the dependence of k_c and α_2 on various parameters. Results are given in Figs. 3 and 4. The following can be observed:

As anticipated, the critical acceleration increases as φ_1, φ_2, φ_3 and X increase. Specifically, the critical acceleration varies considerably with φ_1 (it exhibits a raise up to 75% from $\varphi_1 = 25°$ to $\varphi_1 = 35°$) while its variation with φ_2 is small (about 3%). The increase between $\varphi_3 = 0°$ and $\varphi_3 = 0.5\varphi_2$ is about 20%. In addition k_c decreases considerably with i.

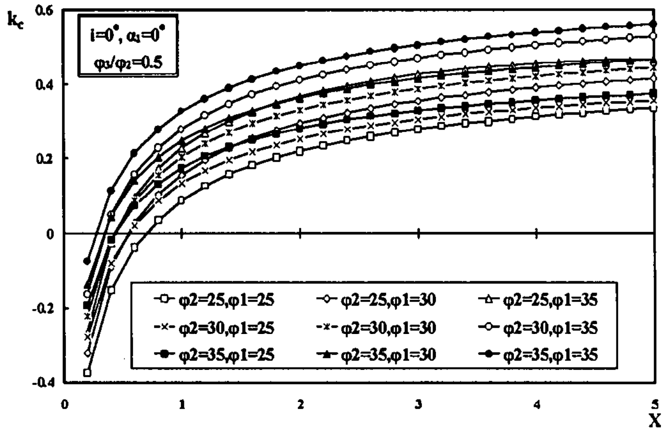

Fig. 3 Results of parametric analyses for k_c.

The critical angle α_2 increases as X increases. This means that for a wall with small weight, the slip surface is steeper. In addition α_2 depends on the soil resistance: it increases as φ_1 decreases and as the ratio φ_2/φ_1 increases. These effects are more pronounced as the factor X increases. Also, as the ratio φ_3/φ_2 and the inclination i increase, α_2 decreases.

SEISMIC DISPLACEMENTS

Small Displacements

In Newmark's sliding-block model a block slides on an inclined plane. Under horizontal excitation $k(t)g$, the governing equation of motion is:

$$\ddot{u}_o = Z_o\cdot(k(t) - k_c)\cdot g \tag{30}$$

where u_o is the displacement along the inclined plane, k_c is the critical acceleration required for motion and the factor Z_o (e.g. Sarma and Chlimitzas 2000) is given as:

$$Z_o = \frac{\cos(\varphi - \alpha)}{\cos\varphi} \tag{31}$$

where α is the inclination of the plane and φ is the friction angle acting between the block and the plane. However, because the factor Z_o, for typical values of φ and α for earth slides is close to 1, usually equation:

$$\ddot{u}_o = (k(t) - k_c) \cdot g \tag{32}$$

is solved to predict seismic ground displacements.

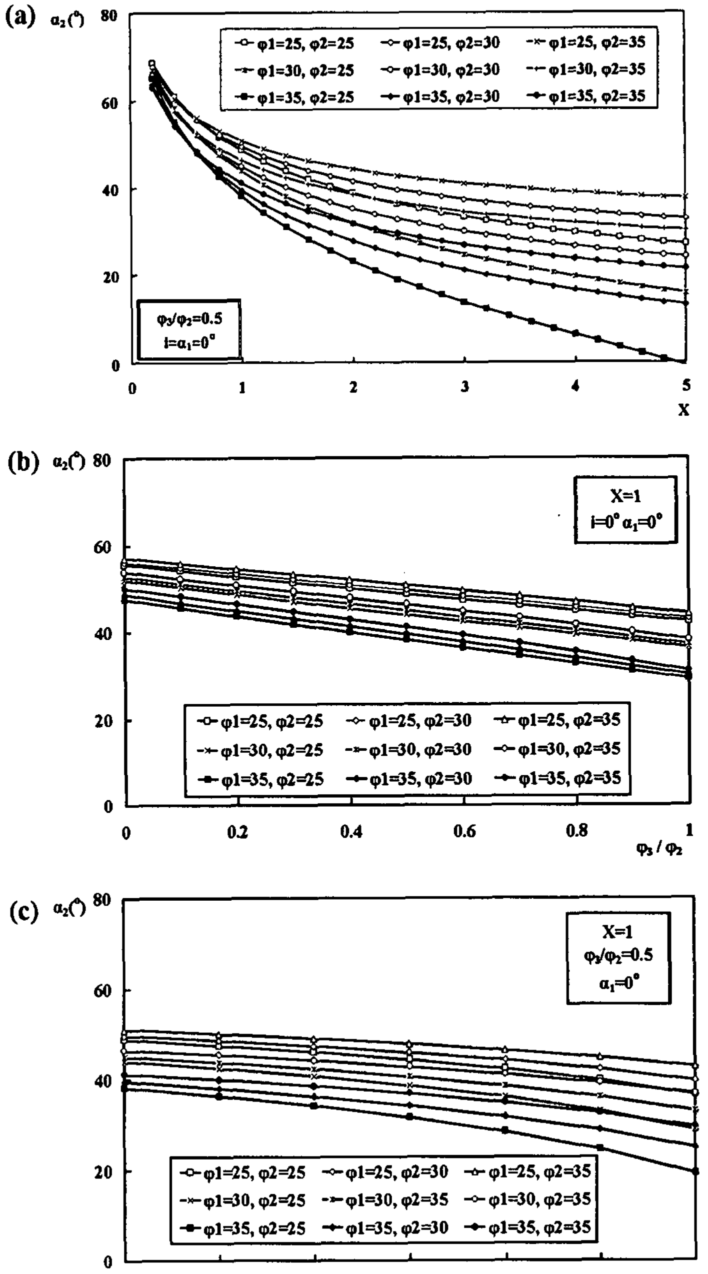

Fig. 4 Results of parametric analyses for the critical angle α_2. Variation with: (a) X, φ_1, φ_2, (b) the ratio φ_3 / φ_2, (c) the inclination i.

Many solutions of the last equation exist in the bibliography. For example Ambraseys and Srbulov (1995) analyze a large data set of earthquakes and predict the displacement u_o in terms of: (a) the ratio k_c/k_m where k_m is the maximum applied acceleration factor, (b) the value of k_m and (c) the seismological parameters of the earthquake magnitude and the epicentral distance.

From equations (3), (7) and (32) it can be inferred that the prediction of the displacements u_i (i=1 for the wall, i=2 for the soil wedge) of the new model can be related to those predicted by the sliding-block model u_o (eq.32), that can be obtained by the solution described above, as:

$$u_i = Z_i \cdot u_o \tag{33}$$

where the factors Z_i are given by equations (4) and (8) with the mass m_2 given by equation (12) and the angle α'_2 given by Equation (24). Parametric analyses were performed to investigate the factors that affect the parameters Z_i. Results of such analyses are given in Fig. 5.

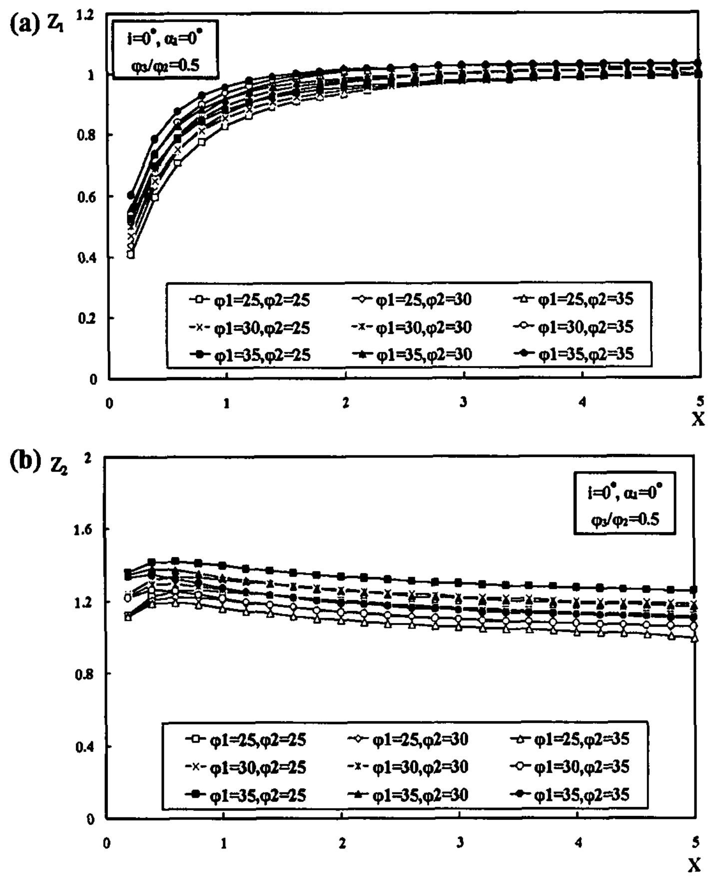

Fig. 5 Results of parametric analyses: (a) coefficient Z_1, (b) coefficient Z_2.

First, we observe that the factors Z_i are not very different than unity. The factor Z_1 increases with X, and tends towards 1 when X becomes very large. The factor Z_2 exhibits a peak at about X=0.5 and is then reduced to reach the limiting value of unity at very large values of X. The results can be compared to those of the sliding-block model: At very low values of X, the mass of the wall is unimportant compared to body two and the coefficient Z_2 equals to the factor Z_o of Newmark's sliding-block model (Eq. 31) with $\varphi = \varphi_2$ and $\alpha = \alpha_2$. When X is very large (very large wall), Z_1 is unity as expected from relation (Eq.31) of Newmark's sliding-block model, when $\alpha = \alpha_1 = 0$.

Large displacements

When large displacements develop, considerable internal mass exchange between body 2 (the soil prism) and body 1 (the mass sliding at the gentler inclination) takes place. The masses m_1 and m_2 and the lengths b_1 and b_2 change with the distance moved. At each time step, iteration is needed to change the masses and lengths of the two bodies in terms of the distance moved. In addition, the angle α_2 changes in each time step, as it depends on the relative masses m_1 and m_2. Estimation of displacements for this case is beyond the scope of the present work.

CONCLUSIONS

A 2-block sliding system models the seismic response of vertical gravity walls retaining dry sand that slide as a result of earthquake loading. Using the principle of limit equilibrium the paper gives analytical expressions giving (a) the angle of the prism of the active soil wedge in the backfill, and (b) the corresponding value of the critical acceleration. The critical acceleration required for motion is compared to that estimated (by using iterations) by the Richards-Elms solution, and it is found to be the same. Differences between the predicted displacement by the new model and those of Newmark's sliding - block model (for similar critical and applied acceleration) are detected and discussed.

ACKNOWLEDGMENT

This work was supported by the European Commission, DG XII for Science, Research and Development, Climate and Natural risks (project ENV4-CT97-0392).

NOTATION (PARTIAL LIST)

i: the inclination of the ground surface of the backfill
k'_c: the critical acceleration of the two block sliding system
k_c: the minimum value of k_c of the soil-wall system
$k(t)g$: applied horizontal acceleration record
m_1: the wall mass
m_2: the mass of the wedge formed in the backfill
u_1: the distance moved by the wall on the direction of its base
u_2: the distance moved by the backfill on the direction of its slip surface
α_1 : the inclination of the wall slip surface
α_2 : the inclination of the backfill slip surface
α'_2 : the inclination of the slip surface of the second body of the two body system
φ_1 : the friction angle on the wall base
φ_2 : the friction angle on the backfill slip surface
φ_3 : the friction angle on the wall - soil interface

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