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SYNOPSIS With dynamic stiffness of elastic half-space, the Rayleigh wave dispersion in transversely isotropic soil is analysed by Finite-layer and Semi-infinite layer method. Only is matrix eigenvalue involved, avoiding the calculation procedure encounted in analytical method. Two examples prove the deduction correctly, and show that soil anisotropy influences dispersion dramaticaly. It is possible to study soil anisotropy and characteristics of its dynamical responces from its surface wave dispersion.

INTRODUCTION

Dispersion characters of surface waves including Rayleigh wave and Love wave are widely applied to engineering test. In earthquake engineering, site responces and foundation vibrations are closely linked with surface wave dispersion. The classical method for Rayleigh wave dispersion analysis, Haskell (1953) , was improved by Dunkin(1965) greatly, with which Crampin (1970) studied the layered transversely isotropic deposit . So far, the classical methods are still faced with the problems of high-frequency overflow and lost of accuracy.
Lysmer and Waas (1970)(1972) put forward Finite-layer method to dispersion analyses, developed by kausel (1986) for anisotropic solid in the case of rock base (Fig.1). Hossain (1984) applied FEM and Infinite element to calculate the surface wave dispersion of the model as shown in Fig.2. Xia(1992) et al improved semi-infinite layer in isotropic deposit layered system. It is possible to modify Lysmer's method for layered anisotropic system, so that transersely isotropic model is suggested. In fact, for semi-infinite layer, it is just a problem of
stiffness matrix of surface wave in transversely isotropic half-space. This problem is solved by Kausel (1991) in form of vibration. The authors solve it combining with Finite-layer method in form of surface wave.

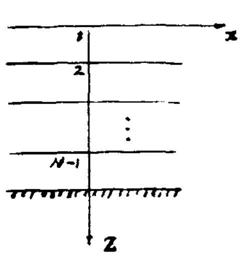

Fig.1 System Model With Rock Base

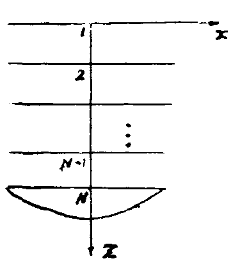

Fig.2 System Model With Elastic Half-space

PLANE WAVES IN TRANSVERSELY ISOTROPIC DEPOSIT SYSTEM

The matrix of the elastic constants for medium with transversely isotropic symmetry is

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & & & \\ c_{12} & c_{11} & c_{13} & & & \\ c_{13} & c_{13} & c_{33} & & & \\ & & & c_{44} & & \\ & & & & c_{44} & \\ & & & & & (c_{11}-c_{12})/2 \end{bmatrix} \quad (1)$$

For isotropic, there are:

$$c_{11}=c_{33}=\lambda+2\mu,\quad (c_{11}-c_{12})/2=c_{44}=\mu,\ c_{13}=\lambda$$

For plane waves propagated in a direction specified by direction cosines(n_1, n_2, n_3), let

$$(u,\ v,\ w)=(U,\ V,\ W)\, e^{i\omega t}\cdot \exp[-ik(n_1x+n_2y+n_3z)]$$

Substitute it into equations of motion, by setting the determinant of coefficients to zero, one can obtain velocity eqnation. Two special cases may be considered immediately:
(a) For transmission along the unique axis, $n_3=1$ and $n_1=n_2=0$, $C^2=c_{33}/\rho$ and $c^2=c_{44}/\rho$ are solutions. The first term corresponds to a vertically travelling pure compression Wave(PV) and the second is a double root corresponding to a vertically travelling shear wave with horizontal particle motion. The degeneracy is caused by the SV and SH waves becoming indistinguishable.
(b) For transmission along the direction perpendicular to the z axis, that is in x -y plane, $n_3=0$, the solutions are as following:

$c^2=c_{11}/\rho$	compression	PH
$C^2=C_{44}/\rho$	shear	SV
$c^2=\dfrac{c_{11}-c_{12}}{2\rho}$	shear	SH

The anisotropic factors are introduced for convience:

$\phi = c_{11}/c_{33}, \quad \eta = (c_{11}-2c_{44})/c_{13}$

$\phi = \eta = 1$ for isotropic case.

FINITE LAYER ANALYSIS

The displacements of Rayleigh wave, an in-plane propagation are shown:

$$\begin{bmatrix} u \\ w \end{bmatrix} = \begin{bmatrix} f(z) \\ g(z) \end{bmatrix} \exp[-ik(x-ct)] \qquad (2)$$

where k wave number, c phase velocity, $i=\sqrt{-1}$.
For isotropic solid, Lysmer and Waas gave the solution which can be modified for travsversely isotropic case. The stiffness matrix of rectangular element can be obtained according to virtual work theorem.

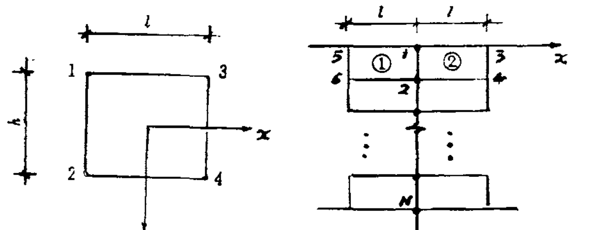

Fig. 3 Discrete Element and System

In Fig. 3, the element ① and ② both have the same stiffness matrix $[k]^1$, and $[k]^2$. Because of the displacements of element ① and ②, force acting on point 1 in x direction is

$$\begin{aligned} F_{1x} = & K^1_{11}u_{1x} + K^1_{12}u_{1z} + K^1_{13}u_{2x} + K^1_{14}u_{2z} \\ & + K^1_{15}u_{3x} + K^1_{16}u_{3x} + K^1_{17}u_{4x} + K^1_{18}u_{4z} \\ & + K^2_{51}u_{5x} + k^2_{52}u_{5z} + k^2_{53}u_{6x} + K^2_{54}u_{6x} \\ & + K^2_{55}u_{1x} + K^2_{56}u_{1z} + K^2_{57}u_{2x} + K^2_{58}u_{2x} \end{aligned} \qquad (3a)$$

where m, i, j in k^m_{ij} represent the numbers of element, row and column number. According to equation 2, the displaments of point 3, 4, 5, 6, can be expressed in form of the displacements of point 1 and 2. Equation (3a) will be condensed as following while $l \to 0$,

$$F_{1x} = k_{11}u_{1x} - k_{12}iu_{1z} + k_{13}u_{2x} - k_{14}iu_{2z} \qquad (3b)$$

Similar solutions can be obtained for F_{1z}, F_{2x}, F_{2z}, which results in following matrix:

$$\begin{bmatrix} F_{1x} \\ -iF_{1z} \\ F_{2x} \\ -iF_{2z} \end{bmatrix} = [k]_e \begin{bmatrix} u_{1x} \\ -iu_{1z} \\ u_{2x} \\ -iu_{2z} \end{bmatrix} \qquad (4a)$$

where $[k]_e =$

$$\begin{bmatrix} c_{11}k^2h/3+c_{44}/h & c_{13}k/2-c_{44}k/2 & c_{11}k^2h/6-c_{44}/h & -c_{13}k/2-c_{44}k/2 \\ & c_{33}/h+c_{44}k^2h/3 & c_{13}k/2+c_{44}k/2 & -c_{33}/h+c_{44}k^2h/6 \\ & & c_{11}k^2h/3+c_{44}/h & -c_{13}k/2+c_{44}k/2 \\ \text{symmetry} & & & c_{33}/h+c_{44}k^2h/3 \end{bmatrix}_{4\times 4} \times 1 \qquad (4b)$$

There exits a common factor "l" in matrix $[k]_e$, which will be deleted as global matrix is formed. This matrix can be degenerated to the isotropic case (Chen, 1991) as well as mass matrix.

SEMI-INFINITE LAYER ANALYSIS

Biot carried out the propagator matrix for layered anisotropic system, and kausel developed it in different way and obtained the stiffness matrix of a transversely isotropic half-space subjcted to dynamic loads (Kausel, 1991). The authors deduce this matrix in form of surface wave for dispersion analysis. It is apparent that we just follow the steps of kausel with different emphasis.
The quations of In-plane motion are

$$\rho\frac{\partial^2 u}{\partial t^2} - c_{11}\frac{\partial^2 u}{\partial x^2} - c_{44}\frac{\partial^2 u}{\partial z^2} - (c_{13}+c_{44})\frac{\partial^2 w}{\partial x \partial z} = 0 \qquad (6a)$$

$$\rho\frac{\partial^2 w}{\partial t^2} - (c_{13}+c_{44})\frac{\partial^2 u}{\partial x \partial z} - c_{44}\frac{\partial^2 w}{\partial x^2} - c_{33}\frac{\partial^2 w}{\partial z^2} = 0 \qquad (6b)$$

Assuming the form of Rayleigh wave

$$\begin{bmatrix} u \\ w \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} e^{pz} e^{i(wt-kx)} \qquad (7)$$

Substitution of equation (7) into (6a), (6b) leads to an eigenvalue problem

$$\begin{bmatrix} -\rho w^2+k^2c_{11}-p^2c_{44} & ikp(c_{13}+c_{44}) \\ -ikp(c_{13}+c_{44}) & -\rho w^2+k^2c_{44}-p^2c_3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = 0 \qquad (8)$$

One can obtain the equation about p from the determinant of matrix.

$$tp^4+mp^2+n=0 \qquad (9)$$

where

$t = c_{33} \cdot c_{44}$

$m=m_1k^2,\ n=n_1k^4$

$m_1=(c_{13}+c_{44})^2-(c_{11}\cdot c_{13}+c_{44}^2)+\rho c^2(c_{44}+c_{33})$

$n_1=(c_{11}-\rho c^2)(c_{44}-\rho c^2)$

Considering radiation condition, $Re(p)<0$

$$P=\begin{bmatrix} k\cdot p_1 \\ k\cdot p_2 \end{bmatrix}=-k\cdot\frac{\sqrt{-m_1\pm\sqrt{m_1^2-4tn_1}}}{2t} \quad (10)$$

Solving next for 'b' from equation (8), with the arbitrary value, say a=1, it comes out

$$b_j=i\frac{c_{11}-\rho c^2-p_j^2c_{44}}{p_j(c_{13}+c_{44})} \qquad j=1,2 \qquad (11)$$

Following kausel's step (1991), it gives following matrix:

$$\begin{bmatrix} \tau xz \\ \sigma z \end{bmatrix}_{z=0}=[k']\begin{bmatrix} u \\ w \end{bmatrix}_{z=0} \qquad (12a)$$

where $[k']=$

$$\begin{bmatrix} c_{44}(kp_1b_2-kp_2b_1) & c_{44}[(p_2-p_1)\cdot k+ik(b_1-b_2)] \\ c_{33}b_1b_2k(p_1-p_2)+ikc_{13}(b_1-b_2) & c_{33}(b_2kp_2-b_1kp_1) \end{bmatrix}$$

(12b)

Matrix $[k']$ is just the transversely isotropic elastic half-space stiffness matrix in form of surface wave. It is proved that matrix $[k']$ is consistant with that obtained by kausel under dynamic loading. By setting its determinant tozero, one can obtain the Rayleigh equation for transversely isotrpic half-space, while kausel explained this matrix anti-symmetric, so

$$\begin{bmatrix} \tau xz \\ -i\sigma z \end{bmatrix}_{z=0}=[k']\begin{bmatrix} u \\ -iw \end{bmatrix}_{z=0} \qquad (13a)$$

where $[k']=\dfrac{k}{b_2-b_1}\times$

$$\begin{bmatrix} c_{44}(p_1b_2-p2b_1) & ic_{44}[(p_2-p_1)+i(b_1-b_2)] \\ \text{symmetry} & c_{33}(b_2p_2-b_1p_1) \end{bmatrix}$$

(13b)

This matrix can be degenerated to isotrpic case(xia, 1992). The stress on the half-space surface is equal to the boundary stress of upper layer with opposite sign, there are

$$F_{NX}=-\frac{1}{2}\int_{-l}^{l}\tau xz\cdot dx\ ,\quad F_{NZ}=-\frac{1}{2}\int_{-l}^{l}\sigma zdx$$

(14)

on the half-space surface, the displacements

$$\begin{bmatrix} u \\ W \end{bmatrix}=\exp(-ikx)\cdot\begin{bmatrix} U_N \\ W_N \end{bmatrix} \qquad (15)$$

Substitution of equation (13), (15) into (14), as $l\rightarrow 0$, it gives

$$\begin{bmatrix} F_{NX} \\ -iF_{NZ} \end{bmatrix}=-[k']\cdot l\cdot\begin{bmatrix} U_N \\ -iW_N \end{bmatrix} \qquad (16)$$

where factor "l" in equation (16) will be deleted in global matrix.

EIGENVALUE MATRIX OF RAYLEIGH WAVE

Because of the special structure of the matrix, one can separate the matrix into quadratic equation about wave number k:

$$[A]k^2+[B]k+[E]=0 \qquad (17)$$

The wave speed is supposed to be real for realism Hossain, 1984).

For finite layer:

$[A]=$

$$
\begin{bmatrix}
\frac{c_{11}h}{3} & 0 & \frac{c_{11}h}{6} & 0 \\
 & \frac{c_{44}h}{3} & 0 & \frac{c_{44}h}{6} \\
 & & \frac{c_{11}h}{3} & 0 \\
\text{symmetry} & & & \frac{c_{44}h}{3}
\end{bmatrix} \tag{18a}
$$

$$
[B]=
\begin{bmatrix}
0 & \frac{c_{13}-c_{44}}{2} & 0 & -\frac{c_{13}+c_{44}}{2} \\
 & 0 & \frac{c_{13}+c_{44}}{2} & 0 \\
\text{symmetry} & & 0 & -\frac{c_{13}-c_{44}}{2} \\
 & & & 0
\end{bmatrix} \tag{18b}
$$

$$
[E]=
\begin{bmatrix}
\frac{c_{44}}{h} & 0 & -\frac{c_{44}}{h} & 0 \\
 & \frac{c_{33}}{h} & 0 & -\frac{c_{33}}{h} \\
\text{symmtry} & & \frac{c_{44}}{h} & 0 \\
 & & & \frac{c_{33}}{h}
\end{bmatrix} \tag{18c}
$$

Mass matrix separation can be found (Chen, 1991). The semi-infinite layer:

$$
[B]=-\frac{1}{b_2-b_1}\times
\begin{bmatrix}
c_{44}(p_1b_2-p_2b_1) & ic_{44}[(p_1-p_2)+i(b_1-b_2)] \\
\text{symmetry} & c_{33}(b_2p_2-b_1p_1)
\end{bmatrix} \tag{18d}
$$

NUMERICAL RESULTS

A two-layered road pavement structure and a three-layer soil stratum are analyzed numerically. Their characters together with subsoil are listed in table 1 and 2 as isotropic case. Fix values of c_{11} and c_{44} with variation of ϕ and η, one can obtain for anisotropy.

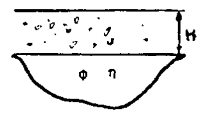

Fig. 4 Two-layer Road Pavement system

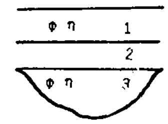

Fig. 5 Three-layer Soil Stratum

(a) Two -layer Road Pavement System

table 1

layer NO	C_{11}/C^1_{44}	C_{13}/C^1_{44}	C_{33}/C^1_{44}	C_{44}/C^1_{44}	ρ/ρ^1	H/H^1
1	2.4964	0.4964	2.4964	1.0	1.0	1.0
2	0.4389	0.0877	0.4389	0.1756	1.0	∞

The superscript is reffered to as number of layer, and $\beta^1=\sqrt{C^1_{11}/\rho^1}$.

Knoppof (Xia, 1992) calculated its isotropic case analyticaly. Figs. 6 and 7 show the dispersion curves fit the analytical solution well as ϕ=1 and η=1. Under the condition of anisotropic subsoil, at lower frequency range, the factor ϕ and η both predominate in dispersion analysis, while at higher frequency range , only factor ϕ does.

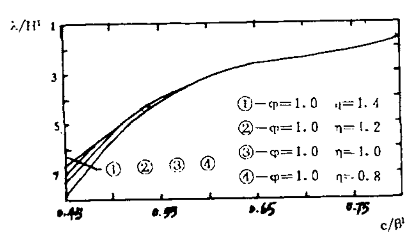

Fig. 6 Parameter Study With Variation of Factor η

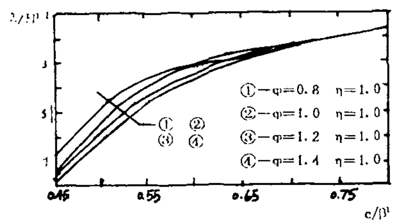

Fig. 7 Parameter Study With Variation of Factor ϕ

(b) Three-layer soil stratum, Fig. 5 .

Table 2

layer NO	c_{11}/c_{44}^1	c_{13}/c_{44}^1	c_{33}/c_{44}^1	c_{44}/c_{44}^1	ρ/ρ^1	H/H^1
1	2.71	0.71	2.71	1.0	1.0	1.0
2	3.92	1.32	3.92	1.30	1.076	0.5
3	7.29	3.11	7.29	2.09	1.283	∞

Table 3 shows anisotropy layers.

Table 3

Layer NO.	Curve ①	Curve ②	Curve ③
1	ϕ = 0.9 η = 1.0	ϕ = 1.0 η = 1.0	ϕ = 0.9 η = 1.0
2	ϕ = 1.0 η = 1.0	ϕ = 1.0 η = 1.0	ϕ = 1.0 η = 1.0
3	ϕ = 1.1 η = 1.0	ϕ = 1.0 η = 1.0	ϕ = 1.1 η = 1.0

Simmilar conclusion for ϕ and η can be seen from Fig. 8, Anisotropy is important in Rayleigh wave dispersion analysis in soil deposit even though the deposit is normally consolidated.

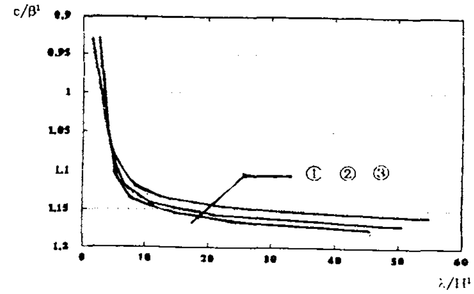

Fig. 8 Parameter Study with Variation of Factors ϕ and η

CONCLUSIONS

1. The authors provide a reliable method for Rayleigh wave dispersion analysis for transversely isotropic deposit, The deduction is fit for Love wave analysis too.
2. The Semi-infinite layer analysis is just the problem of dynamic stiffness matrix in surface wave form. So this deduction is suitable for other anisotropic model.
3. The anisotropy influnces Rayleigh wave dispersion dramatically, it is neccessary to consider anisotropy for SASW method in order to get shear wave velocity of stratum.

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