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Stiffness and Damping Parameters for Dynamic Analysis of Retaining Walls

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ABSTRACT

The displacements of retaining walls developed during earthquakes have been recognized for many years. Mathematical models have been developed to simulate the bahavior of retaining walls during earthquakes. The determination of the soil parameters needed for solution of the displacements has neither been adequately described nor used in a realistic analysis.

In this paper theories describing spring and damping constants of rigid retaining walls, for both the base-soil and the back-fill, are discussed along with the recommended solutions.

INTRODUCTION.

Dynamic behavior of rigid retaining walls has been of concern in seismically active regions. Mathematical models and computational procedures have been developed to simulate the dynamic behavior of retaining walls in order to determine the displacements of the walls resulting from earthquakes. However, satisfactory methods of determining stiffness and damping parameters for both foundation soils and backfill materials have not yet been developed. Both stiffness and damping depend upon strain and this results in non-linear behavior of soils, which must be considered in any realistic analysis.

Stiffness and damping values have been evaluated for footings of various shapes for machine foundations. Barkan (1962) developed a method of estimating the dynamic response of machine foundations by using elastic springs to represent the foundation soil. His approach does not consider the effect of the geometry of the foundation, but only its area, regardless of shape. Neither geometrical or material damping effects are considered. Solutions for circular footings only, in different modes of vibrations, have been developed by Bycroft (1956), Lysmer and Richart (1966), and Hall (1967). To apply these solutions to foundations of shapes other than circular, an equivalent radius needs to be calculated. Solutions for dynamic response of arbitrary shaped foundations have been developed by Dobry and Gazetas (1986), and Gazetas and Tassoulas (1987a,b). Their theory is valid for circular, rectangular, and strip foundations, including solutions for vertical, horizontal, rocking and torsion modes of motion. For the horizontal and rocking modes of motion, solutions are available in both the short and longitudional directions of the foundation. In the case of strip foundations, the stiffness and damping values become insignificant for the horizontal and rocking mode of motion in the longitudional direction. The solutions for torsional mode of motion do not exist.

The analysis and solution of the dynamic response of retaining walls during earthquakes are significantly different than those of machine foundations. Because the retaining walls are embedded on one side, the displacements observed may be significant away from the fill, but small, if any, towards the fill. This is due to the spring and damping constants representing the backfill material being different for the active and passive conditions. Further, the forces from the backfill always act in the same direction, i.e. away from the backfill. It should also be noted that strains developed in the base soil are not necessarily the same as those developed in the backfill material.

Damping of soils is an important factor in the dynamic analysis of foundations. There are two types of damping to be considered: (1) Radiation or geometrical damping, which is a function of decrease in density energy due to wave propagation in an elastic medium. (2) Material damping, which is due to hysteretic behavior of soils. Material damping can be observed from the dynamic hysteretic stress-strain relationship developed during dynamic loading, and it is a function of the strain level developed in the soil mass.

In this paper, known theoretical solutions of stiffness and damping are discussed which can be applied to evaluate the dynamic response of retaining walls during earthquakes. These solutions have been adopted in the solutions of response of rigid retaining walls subjected to earthquake conditions (Rafnsson,1991).

STIFFNESS AND DAMPING PARAMETERS.

Retaining walls are subjected to soil reactions and damping at both the wall-base and wall-face. Realisticaly, motions of rigid retaining walls will consist of simultaneous sliding and rocking. Therefore, the following parameters need to be determined at the base: (a) Soil stiffness in sliding. (b) Soil stiffness in rocking. (c) Geometrical damping in sliding. (d) Geometrical damping in rocking. (e) Material damping in sliding. (f) Material damping in rocking. There will be six corresponding quantities for the back-fill.

The solutions for stiffness and geometrical damping for foundation bases were developed for machine foundations.

These solutions have not been applied to other engineering problems although they may be valid for structures such as retaining walls.

Both geometrical and material damping are expressed as viscous damping for ease in analytical solutions.

Each of the parameters, mentioned above, will be discussed in the following sections.

BASE SOIL.

The displacements of retaining walls developed during an earthquake are mostly due to horizontal translation and rocking of wall. Also, the torsional motion is not important for retaining walls. Therefore, sliding and rocking stiffness and damping are discussed only. However, for the sliding mode of motion, the vertical geometrical damping relationship is applied for the back-fill.

Barkan's Theory

Barkan (1962, see Prakash, 1981a) studied the dynamic behavior of foundations and base soils and developed a theory of elastic (static) springs for computation of the dynamic response of machine foundations. The base soil parameter in his theory is the coefficient of uniform compression (C_u), expressed as:

$$C_u = c_s \frac{E}{1-\nu^2} \frac{1}{\sqrt{A}} \tag{1}$$

Where c_s is a coefficient dependent upon the length-to-width ratio of foundation.
ν is Poisson's ratio.
E is the modulus of elasticity.
A is the foundation area.

This relationship is limited to foundation areas equal to or less than 10m². The relationships between C_u and the coefficient of elastic non-uniform compression (C_ϕ) for rocking, and coefficient of uniform shear (C_τ) for sliding, are recommended as follows:

$$C_u = 2\,C_\tau \tag{2a}$$

$$C_\phi = 2\,C_u \tag{2b}$$

The corresponding spring constants in vertical (k_z), sliding (k_x), and rocking (k_ϕ) modes of motion are given as:

$$k_z = C_u\,A \tag{3a}$$

$$k_x = C_\tau\,A \tag{3b}$$

$$k_\phi = C_\phi\,I \tag{3c}$$

Where I is the moment of inertia of the contact area about the rotational axis. Further, the critical damping values of the base soil in different modes of vibrations may be determined for vertical, horizontal, and rocking vibrations, respectively, as:

$$c_{cz} = 2\sqrt{m\,k_z} = 2\sqrt{m\,C_u\,A} \tag{4a}$$

$$c_{cx} = 2\sqrt{m\,k_x} = 2\sqrt{m\,C_\tau\,A} \tag{4b}$$

$$c_{c\phi} = 2\sqrt{M_{mo}\,k_\phi} = 2\sqrt{M_{mo}\,C_\phi\,I} \tag{4c}$$

Where m is the mass of the foundation.
M_{mo} is the mass moment of inertia about the axis of rotation.

Barkan does not consider geometrical or material damping in his analysis; however, the expressions of critical damping shown above may be used to estimate material damping. A material damping factor (ξ) is determined as a function of strain as discussed further in this paper. Then, material damping (c_m) is given by:

$$c_m = \xi\,c_c \tag{5}$$

Where c_c is the critical damping in a particular mode of vibrations.

Hall's Analog

Hall (1967) developed an analog for sliding and rocking vibrations of foundations resting on an elastic-half-space. The expression of (static) sliding spring constant and (static) sliding geometrical damping constant for circular footings are:

$$k_x = \frac{32(1-\nu)}{7-8\nu}\,G\,r_o \tag{6}$$

$$c_x = \frac{18.4(1-\nu)}{7-8\nu}\,r_o^2\sqrt{\rho\,G} \tag{7}$$

Where G is the shear modulus.
ρ is the soil density.
r_o is the equivalent radius of the foundation.

The relationships for the sliding spring and damping constants are valid for certain ranges of geometry of the problem only (Lysmer and Richart, 1966, and Prakash, 1981a). If the footing is not circular, the equivalent radius (r_o) can be determined as:

$$r_o = \sqrt{\frac{(2L)\,(2B)}{\pi}} \tag{8}$$

Where L and B are half the length and half width of the foundation, respectively.

The solutions for (static) spring and (static) geometrical damping constants in rocking vibrations of circular footings are as:

$$k_\phi = \frac{8\,G\,r_o^3}{3(1-\nu)} \tag{9}$$

And:

$$c_\phi = \frac{0.80\,r_o^4\sqrt{G\,\rho}}{(1-\nu)(1+B_\phi)} \tag{10}$$

The dimensionless inertia ratio in rocking (B_ϕ) has been defined as:

$$B_\phi = \frac{3(1-\nu)}{8}\;\frac{M_{mo}}{\rho\,r_o^5} \tag{11}$$

It should be noted that these solutions are valid for certain ranges of B_ϕ (0.5 - 6.0) and dimensionless frequency ratio (a_o, see Eq.14) (0 - 1.5) only. The equivalent radius for rectangular foundations in rocking vibrations is:

$$r_o = \sqrt[4]{\frac{(2L)\,(\,2B\,)^3}{3\,\pi}} \tag{12}$$

Rectangular and Strip Foundations

Gazetas (1983) discussed the state of the art of analyses of the dynamic response of machine foundations resting on an elastic-half-space. More comprehensive work on the parameters representing the dynamic response of foundations was carried out by Dobry and Gazetas (1986). In this study, solutions for the spring and geometrical damping constants were developed for: (1) Arbitrarily shaped foundations, (2) rectangular foundations, and (3) rigid strip foundations. However, the areas of the arbitrarily shaped foundations are characterized by the circumscribed rectangle of the dimensions 2L by 2B. In strip foundations ($2L \rightarrow \infty$) the solutions of stiffness and damping must be independent of the length of the foundations.

Although, the solutions are developed for static conditions, the dynamic spring constant ($\bar{k}$) is related to the static spring constant (k) through a dimensionless stiffness ratio ($\tilde{k}$) as follows:

$$\bar{k} = \tilde{k}\, k \qquad (13)$$

The ratio $\tilde{k}$ is highly dependent on Poisson's ratio (ν), geometry (L/B), and the dimensionless frequency factor (a_o), which is defined as:

$$a_o = \frac{\omega\, r_o}{V_s} = \omega\, r_o \sqrt{\frac{\rho}{G}} \qquad (14)$$

Where ω is the frequency of excitation.
r_o is equal to B in rectangular and strip footings.

A typical relationship of $\tilde{k}$ and the dimensionless frequency factor (a_o) for rectangular and strip footings is shown for horizontal vibrations ($\tilde{k}_x$) by Fig.1, and for vertical vibrations ($\tilde{k}_z$) by Fig.2. The dimensionless stiffness ratio is primarily dependent on the L/B ratio and a_o for the horizontal and the vertical vibrations. The corresponding constant ($\tilde{k}_\phi$) for rocking vibrations (Fig.3) depends on a_o only.

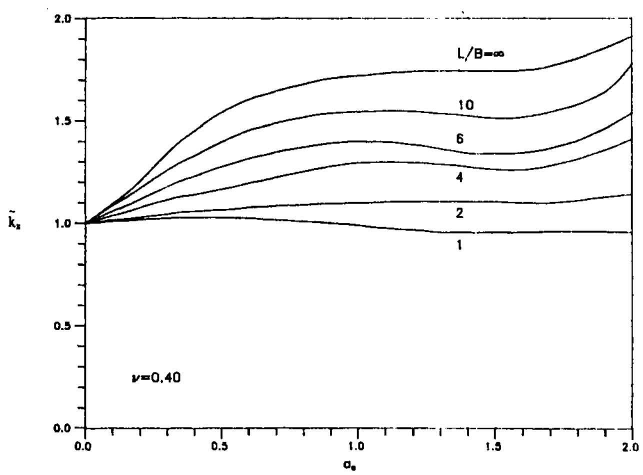

Figure 1. The horizontal dimensionless stiffness parameter (After Gazetas and Tassoulas, 1987a).

The vertical static stiffness per unit length of rectangular foundation (k_z) is evaluated according to the following expression (Dobry and Gazetas, 1986):

$$k_z = S_z \frac{G}{1 - \nu} \qquad (15)$$

Where the dimensionless static stiffness parameter (S_z) is for rectangular foundations:

$$S_z = 0.8 \qquad \text{for } \frac{B}{L} < 0.02 \qquad (16a)$$

$$S_z = 0.73 + 1.54\left(\frac{B}{L}\right)^{0.75} \qquad \text{for } \frac{B}{L} > 0.02 \qquad (16b)$$

In the case of strip foundations Eq.(16a) is valid. The static horizontal stiffness per unit length of foundation (k_x) in the short direction is determined as (Dobry and Gazetas,1986):

$$k_x = S_x \frac{G}{2 - \nu} \qquad (17)$$

In which the dimensionless static stiffness parameter (S_x) is given by Eq.(18) in the case of rectangular foundations of area 2L by 2B (Gazetas and Tassoulas, 1987a):

$$S_x = 2 + 2.50\left[\frac{B}{L}\right]^{0.85} \qquad (18)$$

For strip footings the value of S_x becomes equal to 2.

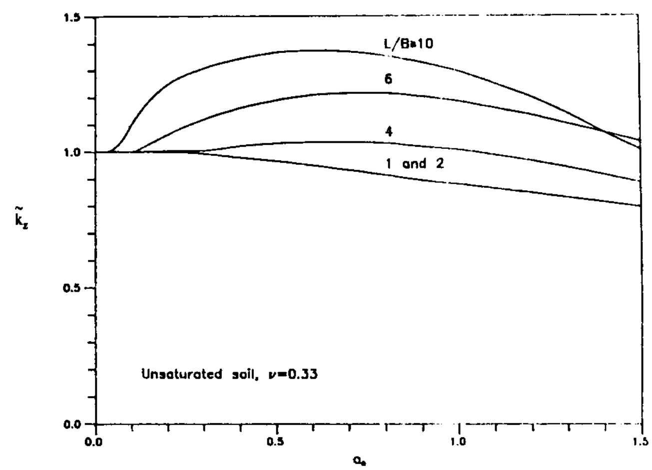

Figure 2. The vertical dimensionless stiffness parameter (After Dobry and Gazetas, 1986).

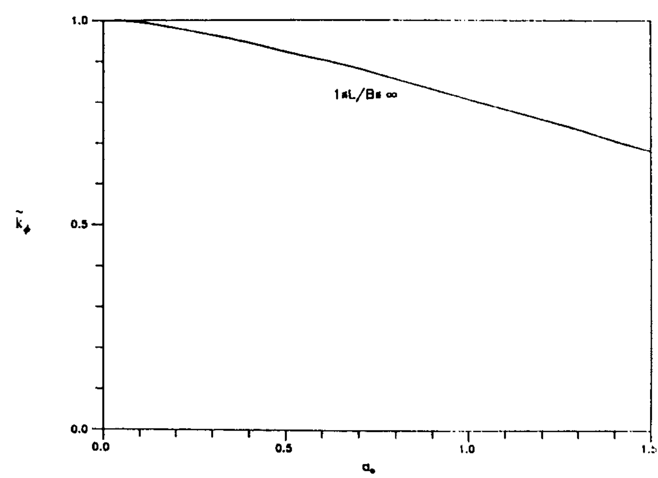

Figure 3. The rocking dimensionless stiffness parameter (After Dobry and Gazetas, 1986).

The static rocking stiffness per unit length of rectangular foundations is determined as (Dobry and Gazetas, 1986):

$$k_\phi = S_{rx} \frac{G}{1-\nu} (I)^{0.75} \quad (19)$$

Where the dimensionless static stiffness parameter (S_{rx}) is (Dobry and Gazetas, 1986):

$$S_{rx} = 2.54 \qquad \text{for } \frac{B}{L} < 0.4 \quad (20a)$$

$$S_{rx} = 3.2\left(\frac{B}{L}\right)^{0.25} \qquad \text{for } \frac{B}{L} > 0.4 \quad (20b)$$

The static rocking stiffness per unit length of foundation in the case of strip foundations is not valid for S_{rx} equal to 2.54 as whould be expected, but instead the following equation is recommended (Dobry and Gazetas, 1986):

$$k_\phi = \frac{\pi G B^2}{2(1-\nu)} \left(1 + \left[\frac{\ln(3-4\nu)}{\pi}\right]^2\right) \quad (21)$$

It should be noted that for B/L ratio below 0.4 Eq.(19) gives lower values than for the rocking spring constant than Eq.(21).

The geometrical damping coefficients are determined as well by Dobry and Gasetaz (1986). The dynamic vertical damping ($\bar{c}_z$) is determined as follows for rectangular footings:

$$\bar{c}_z = \tilde{c}_z \frac{3.4}{\pi(1-\nu)} \rho v_s A = \tilde{c}_z \frac{3.4}{\pi(1-\nu)} A \sqrt{G\rho} \quad (22)$$

Which per unit length of both rectangular and strip foundation becomes:

$$\bar{c}_z = \tilde{c}_z \frac{3.4\,(2B)}{\pi(1-\nu)} \sqrt{G\rho} \quad (23)$$

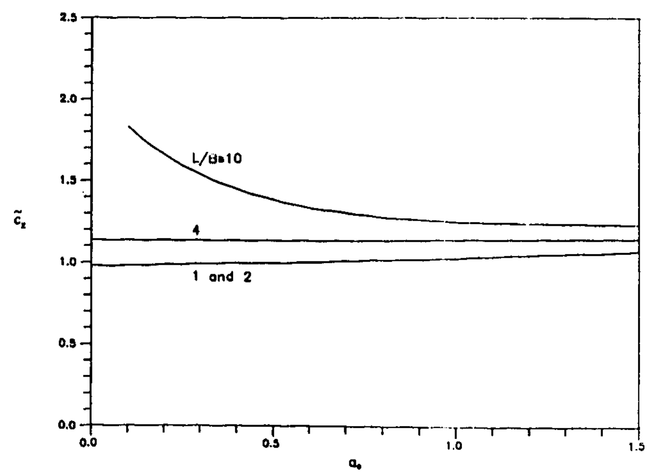

Figure 4. The vertical dimensionless damping parameter (After Dobry and Gazetas, 1986).

The vertical dimensionless damping ratio ($\tilde{c}_z$) depends on the L/B ratio and a_o (Fig.4). The dynamic horizontal damping value in the short direction ($\bar{c}_x$) is expressed by:

$$\bar{c}_x = \tilde{c}_x \rho v_s A \quad (24)$$

Which, per unit length of strip foundations and in terms of shear modulus, becomes:

$$\bar{c}_x = \tilde{c}_x 2B \sqrt{G\rho} \quad (25)$$

The dimensionless damping ratio ($\tilde{c}_x$) depends on the L/B ratio, a_o, and the Poisson's ratio, for values of a sub o below 1 (Fig.5). In the case of rigid retaining walls the value of a_o can be expected to be lower than 1 (Rafnsson, 1991). Similarly, the dynamic damping (c_ϕ) for rectangular footings in rocking in the short direction is given by (Dobry and Gazetas, 1986):

$$c_\phi = \tilde{c}_\phi \rho v_{la} I \quad (26)$$

Where $\tilde{c}_\phi$ is the dimensionless damping ratio for rocking depending on L/B ratio and a_o (Fig.6).

V_{La} is the "Lysmer's Analogue Wave Velocity" expressed as:

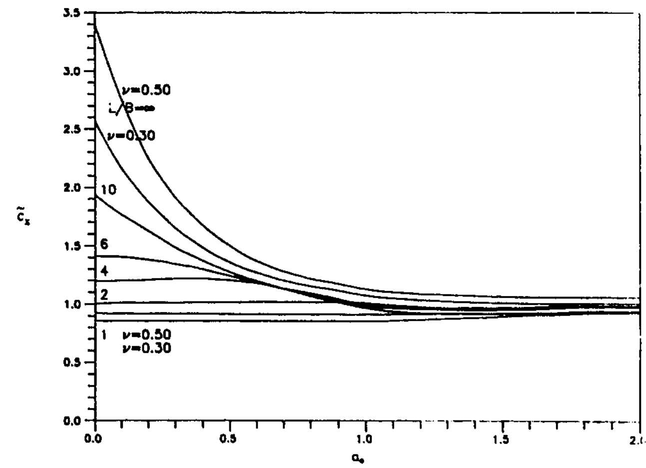

Figure 5. The horizontal dimensionless damping parameter (After Gazetas and Tassoulas, 1987b).

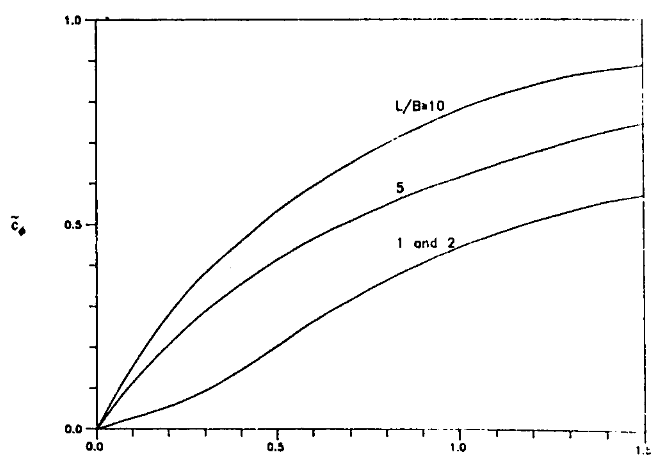

Figure 6. The rocking dimensionless damping parameter (After Dobry and Gazetas, 1986).

$$v_{La} = \frac{3.4\, v_s}{\pi(1-\nu)} = \frac{3.4}{\pi(1-\nu)}\sqrt{\frac{G}{\rho}} \qquad (27)$$

For rigid strip footings the above expression (Eq.26) becomes:

$$c_\phi = \frac{\tilde{c}_\phi\, \rho\, v_{la}\, I}{2L} = \frac{\tilde{c}_\phi\, \rho\, v_{la}\, (2B)^3}{12} \qquad (28)$$

The evaluation of the damping constants by Dobry and Gazetas (1986) and Gazetas and Tassoulas (1987b) presented in the above discussion consider the geometrical damping only. Lysmer (1980) presented a method of evaluating the total damping at any strain level. Dobry and Gazetas (1986) adopted his method which is presented by Eq. (29) for the total damping developed in the soil mass at any specified strain level.

$$c(\xi) \cong c + \frac{2\,\bar{k}}{\omega}\,\xi \qquad (29)$$

Where $\underline{c}$ is the dynamic geometrical damping.
$2\bar{k}/\omega$ is the critical damping of the soil mass.
ξ is the material damping ratio.

The computed response of rectangular and strip foundations based upon the above stiffness and damping parameters has been supported by experimental investigations by Dobry, Gazetas, and Stokoe (1986).

Nandakumaran (1973) developed force-displacement relationships for both the base soil and the backfill material (Fig.7) below and behind retaining walls. The theory for the force-displacement relationship of the base-soil depends on the following assumptions: (1) An elastic wedge (Fig.8) is formed in the soil below the wall. The wedge is determined by the base width of the wall and two sides inclined at the angle of friction of the base soil (ϕ_b) to the base. (2) On application of a lateral force to the wall, the resistance offered by the soil is equal to the passive resistance at the lowest part of the triangle (ABC). (3) The displacement required to form the resistance force is 5 to 10 % of the height of the triangle (h). The stiffness of the soil in sliding is obtained by dividing the maximum force by defined yield displacement. Geometrical damping was not included in Nandakumaran's analysis.

Discussion

The theories by Barkan (1962) and Hall (1967) are primarly developed for circular foundations and not applicable to strip foundations as in the case of retaining walls. However, the solutions by Dobry and Gazetas (1986) and Gazetas and Tassoulas (1987a,b), include solutions for rectangular and rigid strip foundations. In the proposed rigid retaining wall problem the horizontal static stiffness (k_x) is evaluated according to Eq.(17),and the static rocking stiffness is determined by Eq.(21). Then, Eq.(13) is used to determine the dynamic stiffness of the soil.

Nandakumaran procedure requires the yield displacement of the base-soil to be estimated which is rather a difficult quantity to assess. Also, the procedure underestimates the value of the horizontal spring constant.

Both material and geometrical damping of the base soil are determined as follows for the sliding and rocking modes of motion.

The dynamic material damping ($\bar{c}_{xm}$) in the case of sliding is determined as follows:

$$c_{xm} = 2\xi\sqrt{\bar{k}_x m} \qquad (30)$$

Where $\bar{k}_x$ is the dynamic stiffness in sliding.

The material damping in rocking ($\bar{c}_{\phi m}$) is found as follows:

$$c_{\phi m} = 2\xi\sqrt{\bar{k}_\phi M_{mo}} \qquad (31)$$

Where $\bar{k}_\phi$ is the dynamic stiffness in rocking.

The material damping ratio (ξ) is strain depended. The determination of damping ratio is discussed further in the paper.

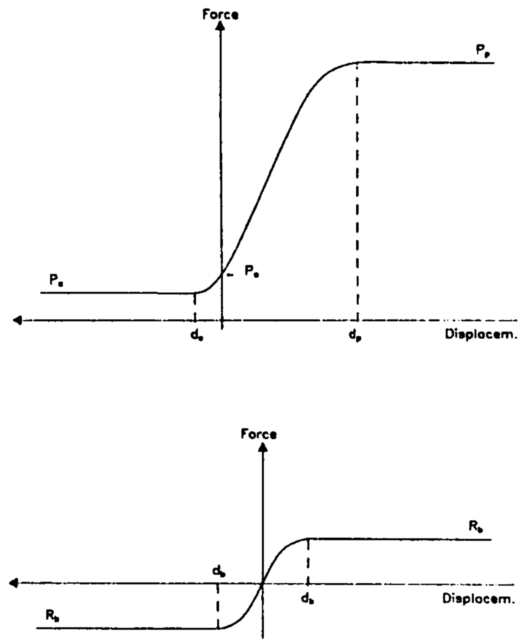

Figure 7. The Force-displacement relationship behind and below retaining walls (After Nandakumaran, 1973).

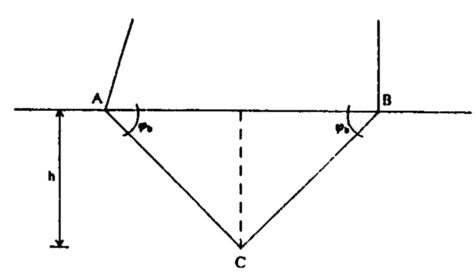

Figure 8. Formation of an elastic wedge below retaining wall (After Nandakumaran, 1973).

Hall (1967) considers circular foundations only and is therefore not as applicable to the problem discussed herein. The geometrical damping in sliding is determined according to Eq.(25). The geometrical damping for rocking is determined according to Eq.(26). Both these relationships have been developed for rectangular and strip foundations.

The total dynamic damping of the base in case of sliding is:

$$\bar{c}_{xt} = \bar{c}_x + \bar{c}_{xm} \tag{32}$$

And the total dynamic damping in the case of rocking is:

$$\bar{c}_{\phi t} = \bar{c}_\phi + \bar{c}_{\phi m} \tag{33}$$

STIFFNESS AND DAMPING OF BACKFILL MATERIAL.

Nandakumaran (1973) expressed the force-displacement relationship for the backfill as illustrated by Fig.7. Fully active pressure conditions in the backfill are assumed to develop at a displacement (d_a) of 0.5 % of the wall height, as is generally accepted. Similarly, fully passive conditions are developed at a displacement (d_p) of 5 % of the wall height, the lower end of the generally accepted range of 5 to 10 % of the wall height. The stiffness of the backfill can be obtained by simplifying Fig.7 and assuming the relationship to be bilinear (Fig.9). Since the stiffness is defined as force per unit displacement, the slope of the curves representing the backfill force-displacement is actually presenting the linear stiffness value of the back-fill.

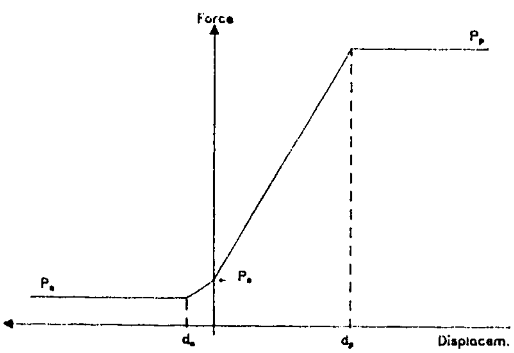

Figure 9. The force-displacement relationship of the backfill applied to determine the stiffness of the backfill (After Prakash, 1981b).

Prakash (1981b) discussed both analytical and experimental work regarding dynamic earth pressure on retaining walls, and described a simplified method to estimate the stiffness parameters of the backfill where the stiffness of the backfill is obtained by applying the above recommendations of Nandakumaran (1973). Since stiffness (spring constant) is by definition the ratio of change in force to change in displacement, the following relationships are valid for active and passive conditions, respectively:

$$k_a = \frac{P_o - P_a}{d_a} \tag{34}$$

Where P_o is the horizontal earth force at rest.
P_a is the static active horizontal earth force.

Similarily, the stiffness value that applies when the passive conditions develop in the backfill is calculated as:

$$k_p = \frac{P_p - P_o}{d_p} \tag{35}$$

Where P_p is the static passive horizontal earth force.

Discussion

Evaluation of the stiffness of the backfill material is based on Fig.9 and Eq.(34) and Eq.(35), for both the sliding and the rocking modes of motion of the retaining wall. The active and passive forces are calculated according to Coulomb's earth pressure theory. The active force is:

$$P_a = \frac{1}{2}\gamma H^2 \frac{\cos^2(\phi - \alpha)}{\cos^2\alpha \cos(\delta + \alpha)\left[1 + \sqrt{\frac{\sin(\phi + \delta)\sin(\phi - i)}{\cos(\alpha - i)\cos(\delta + \alpha)}}\right]^2} \tag{36}$$

Where H is the height of the wall.
γ is the soil unit weight.
δ is the wall-soil friction.
α is the angle of back of the wall to vertical.
ϕ is the friction of the backfill material.
i is the slope of backfill surface.

And the passive force is:

$$P_p = \frac{1}{2}\gamma H^2 \frac{\cos^2(\phi + \alpha)}{\cos^2\alpha \cos(\delta - \alpha)\left[1 - \sqrt{\frac{\sin(\phi + \delta)\sin(\phi + i)}{\cos(i - \alpha)\cos(\delta - \alpha)}}\right]^2} \tag{37}$$

The lateral force at rest is determined as:

$$P_o = \frac{1}{2}\gamma H^2 K_o \tag{38}$$

In which K_o is the coefficient of earth pressure at rest. The slope of the curve in Fig.9 from d_a to 0 and 0 to d_p represents the stiffness of the backfill for the active and passive cases, respectively, for both rocking and sliding modes of motion. The static stiffness for the active case is determined by Eq.(34). The relationships of dynamic and static stiffness given by Dobry and Gazetas (1986) and Gazetas and Tassoulas (1987a) are assumed to be valid for the backfill as well as the base soil. Therefore, the dynamic stiffness is determined as follows (Rafnsson, 1991):

$$\bar{k}_a = k_a \tilde{k}_z \tag{39}$$

Where $\tilde{k}_z$ is the dimensionless stiffness ratio (Fig.2).

Similarly, the static stiffness for the passive case is determined by Eq.(35) and the dynamic stiffness for the passive case is as follows (Rafnsson, 1991):

$$\bar{k}_p = \frac{k_p}{\tilde{k}_z} \tag{40}$$

The static and dynamic rocking stiffness parameters for active and passive case are, respectively:

$$k_{a\phi} = k_a h \quad (41a)$$

$$k_{p\phi} = k_p h \quad (41b)$$

And:

$$\bar{k}_{a\phi} = k_{a\phi}\tilde{k}_{\phi} \quad (42a)$$

$$\bar{k}_{p\phi} = k_{p\phi}\tilde{k}_{\phi} \quad (42b)$$

Where h is the moment arm from the base to the point of application of the dynamic force acting on the back of the retaining wall, taken as 0.5H (Rafnsson, 1991).

$\tilde{k}_{\phi}$ is the dimensionless stiffness ratio (Fig.3).

Material and geometrical damping of backfill material are determined as follows for the sliding and rocking motions.

The dynamic sliding material damping of the backfill is determined as follows for the active case:

$$\bar{c}_a = 2\xi\sqrt{\bar{k}_a m} \quad (43)$$

Similarly, the dynamic passive material damping for the sliding is determined by the same procedure as follows:

$$\bar{c}_p = 2\xi\sqrt{\bar{k}_p m} \quad (44)$$

The dynamic rocking material damping becomes similar for both the active and passive cases, respectively, as:

$$\bar{c}_{a\phi} = 2\xi\sqrt{\bar{k}_{a\phi} M_{mo}} \quad (45)$$

And:

$$c_{p\phi} = 2\xi\sqrt{\bar{k}_{p\phi} M_{mo}} \quad (46)$$

The geometrical damping per unit length due to sliding of the retaining wall is obtained by applying Eq.(23), However, the relationship is reduced by factor K as follows:

$$\bar{c}_h = \tilde{c}_z K\left[\frac{3.4H\sqrt{G\rho}}{\pi(1-\nu)}\right] \quad (47)$$

Where K is a constant to account for the "partial" elastic half space since the backfill can be assumed to behave like an elastic half space where half of the soil medium below the top of the wall is missing. Therefore, it is reasonable to assume K to be 0.5 or less.

Total sliding damping per unit length of wall is found as follows for the active and passive conditions, respectively:

$$\bar{c}_{at} = \bar{c}_h + \bar{c}_a \quad (48)$$

And

$$\bar{c}_{pt} = \bar{c}_h + \bar{c}_p \quad (49)$$

The geometrical damping per unit length in the rocking mode of motion is determined from Eq.(28):

$$\bar{c}_{\phi} = K\left[\frac{\tilde{c}_{\phi}\,\rho\, v_{La}\,(2B)^3}{12}\right] \quad (50)$$

Where v_{La} is obtained by Eq.(27).

K has the same meaning as above.

Total rocking damping is as follows for the active and passive conditions, respectively:

$$\bar{c}_{\phi at} = \bar{c}_{\phi} + \bar{c}_{a\phi} \quad (51)$$

And

$$\bar{c}_{\phi pt} = \bar{c}_{\phi} + \bar{c}_{p\phi} \quad (52)$$

NON-LINEAR BEHAVIOR OF SOILS.

The stiffness and damping values of both the base soil and the backfill are related to the shear modulus of the soil, which is strain dependent. The non-linearity of soil can be accounted for by applying to the solutions of stiffness and damping parameters the shear modulus-shear strain relationship.

The relationship of shear strain and the ratio of shear modulus over maximum shear modulus has been studied for years. The first work was performed by Seed and Idriss (1970), followed by Seed, Wong, Idriss, and Tokimatsu (1986). The main conclusions of their research were the relationships of the shear moduli and the shear strains of clay, sand, and well-graded gravel, and the damping ratios to shear strain relationship as well. Drnevich (1985) discussed testing of soils in order to obtain the shear modulus and damping ratio of soils as a function of shear strain. The latest contribution to this topic is by Vucetic and Dobry (1991). The main difference of their study as compared to Seed and Idriss (1970) is that the curves expressing the shear modulus of clay are above the one for sand soils but not below. However, the relationship for sands are practically the same. As may be seen from Fig.10 and Fig.11, does the shear modulus reache its minimum value at shear strain of 0.01, and the damping ratio reaches its maximum at shear strain of approximately 0.1. Continuous equations have been developed for these relationships. For shear modulus and shear strain, the following relationship is valid:

$$G/G_{max} = \frac{1}{A + B(\gamma)^C} - D\gamma \quad (53)$$

Where G is the shear modulus at any specified strain levels, G_{max} is the maximum shear modulus for shear strain $\leq 1 \times 10^{-6}$, γ is the shear strain, and A, B, C, and D, are constants that depend upon the soil type (Rafnsson, 1991). Similarly, the relationship of damping ratio and shear strain is presented as follows:

$$\xi = \left[\frac{E + F\gamma^G - 1}{E + F\gamma^G}\right] I + H\gamma \quad (54)$$

Where ξ is the damping ratio. The constants E, F, G, H, and I depend upon the soil type as before (Rafnsson, 1991). By applying Eq.(53) and Eq.(54) to the solution of the displacement analysis of the retaining wall, the non-linear behavior of both the shear modulus and the damping ratio can be accounted for.

It should be noted that in Fig.10 and Fig.11, the solid curves are according to Seed and Idriss (1970) except the one for clay having PI equal to 30 is from Vucetic and Dobry (1991). The dashed curves are the curves computed by Eq.(53) and Eq.(54), respectively. These curves show that the fitted curves are very close to the experimental curves.

Typical displacement values using the stiffness and damping described above and the procedure of computation described elsewhere (Rafnsson, 1991) are plotted in Fig.12 for different

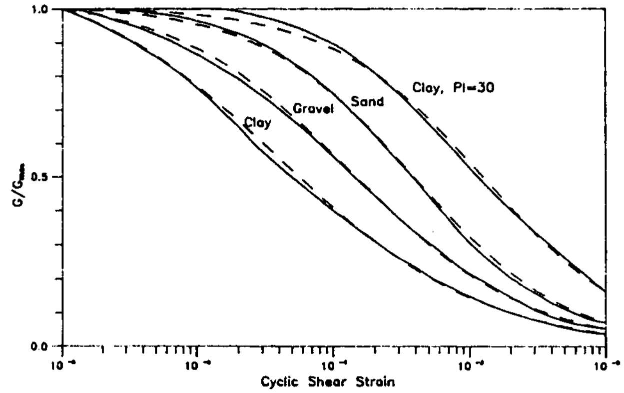

Figure 10. The observed and predicted relationship of non-dimensional shear modulus ratio and the shear strain for gravel, sand, and clay.

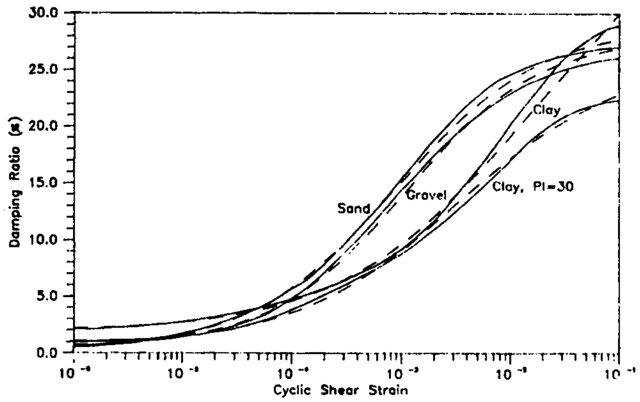

Figure 11. The observed and predicted relationship of the damping ratio and the shear strain for gravel, sand, and clay.

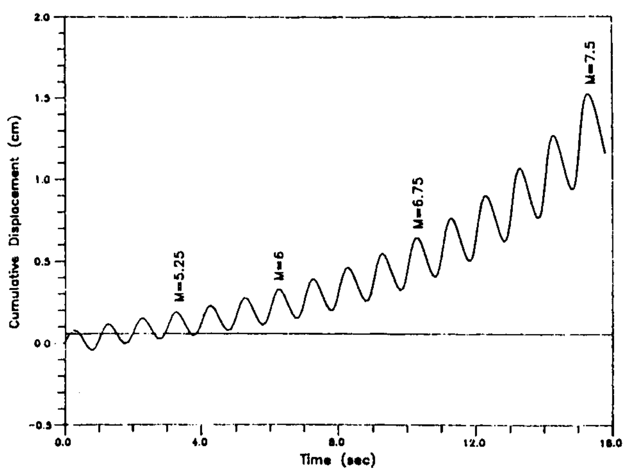

Figure 12. Observed displacements of a Retaining wall during an Earthquake.

cycles of ground motion. The earthquake input was an equivalent sinusoidal ground motion with horizontal acceleration coefficient as 0.15, and frequency of excitation 1 Hz. The retaining wall has the following dimensions, height 5 m, top width 0.5 m, and base width 1.72 m. The factor of safety against sliding is 1.5 for static conditions, and the reaction force is within 2/3 of the base width from the heel providing satisfactory factor of safety against rocking. It should be noted how the magnitude of the earthquake affects the displacement of the retaining wall. However, for the conditions used for this particular wall and exciting force, the displacement of the wall whould not be of any significance during the earthquake.

CONCLUSION

Several methods that may be applied to determine the spring constants and damping values of base soil and back fill below and behind retaining walls have been discussed. By applying any of the theories, the non-linear behavior of the soil will be accounted for since the shear modulus or the modulus of elasticity would change as the cyclic shear strain amplitude increases. The only wall-soil interaction that still is assumed to be only bilinear is the wall-backfill interaction. The application of these theories is fairly simple and does not require any complicated mathematical models to be solved. Consequently, the modeling and solution of dynamic behavior of rigid retaining walls may be solved in rather simple manner.

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