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SYNOPSIS This paper deals with the theoretical study concerning soil-pile interaction in vertical vibration for both the floating pile and the pile group. The analysis is made by applying the elastic wave theory to the viscoelastic layer overlying on the rigid bedrock. Further, the displacement responses of the pile and the complex stiffness at the pile head subjected to the harmonic excitation at the top of the pile are obtained for various parameters.

INTRODUCTION

The soil-pile interaction problem should be clarified in order to understand the dynamic behavior of the structures on the soft ground subjected to earthquakes or wind. Especially, the vertical vibration of piles is one of the most important problems in the case of floating piles or rocking of pile groups. Recently, the theoretical studies on soil-pile interaction in vertical vibration have been reported by Nogami et al. (1976) for the case of the end bearing pile, by Novak et al. (1977), by Gyoten et al. (1978),(1980) and by Wolf et al. (1978) for the case of floating pile and pile groups.

It is extremely difficult to obtain the rigorous analytical solution, because this includes the displacement and the resistance force from the soil layer at the pile tip for the floating pile and the vibration of the soil layer with the cavities of the pile group. Therefore, this paper adopts the analytical model of the soil rod connected with the floating pile tip and the systematic analytical solution is obtained by neglecting the influence of the cavities of all other piles except the origin pile in the pile group.

In this analysis, the following assumptions are made:
(1) The soil overlying on the rigid bedrock is a visco-elastic, homogeneous and isotropic layer with the hysteretic damping.
(2) The horizontal displacement is negligibly small.
(3) There are no normal stress on the free surface of the soil layer and no displacement at the bottom of the soil layer.
(4) The pile and the soil rod contact with the surrounding soil perfectly.
(5) The pile is perfectly elastic and the soil rod connected with the pile tip is made of the same material as that of the surrounding soil. These are vertical and have an equal circular cross-section.

DYNAMIC RESISTANCE FACTOR OF A SOIL LAYER

To obtain the dynamic resistance factors influencing floating piles and pile groups by the soil layer and the other piles through the soil medium, the behavior of the vibrating soil layer must be first analyzed. The visco-elastic soil layer overlying on the rigid bedrock is assumed to have a cavity of the origin pile at r=0, as shown in Fig. 1 where the coordinate system is also indicated.

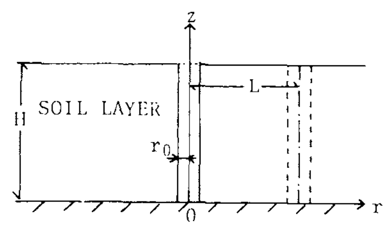

Fig. 1. Model of Soil Layer

By neglecting the horizontal displacement, the equation of vertical motion, $w(r,z)e^{i\omega t}$, of the visco-elastic medium can be written as

$$[(\lambda+2\mu)+i(\lambda'+2\mu')]\frac{\partial^2}{\partial z^2}(w\ e^{i\omega t})$$
$$+(\mu+i\mu')(\frac{\partial}{r\partial r}+\frac{\partial^2}{\partial r^2})(w\ e^{i\omega t})=\rho\frac{\partial^2}{\partial t^2}(w\ e^{i\omega t}) \qquad (1)$$

where λ,μ=the real part of Lamé's constants, λ', μ'=the viscosity coefficients associated with λ and μ respectively, ρ=soil density, ω=frequency of excitation, t=time and $i=\sqrt{-1}$. As the solution for Eq. (1) under the boundary conditions; w=0 at $r\to\infty$, $\sigma_z=0$ at z=H, w=0 at z=0, the amplitudes of the vertical displacement and the resistance force which occur on the circumference of the origin pile are obtained as

$$w(r_0,z)=\sum_{n=1}^{\infty} w_n\sin(h_n z)\ ,\ w_n=A_nK_0(q_nr_0) \qquad (2)$$

$$P_f(z)=2\pi r_0\tau_{rz}(r_0,z)=-\sum_{n=1}^{\infty}\alpha_n w_n \sin(h_n z) \qquad (3)$$

where w_n is defined as the amplitude of the displacement in the n-th wave mode, A_n is the integration constant and τ_{rz} is the shear stress amplitude at $r=r_0$. The resistance factor of the soil layer, α_n, is defined as

$$\alpha_n=2\pi r_0\mu(1+iDs)q_n\frac{K_1(q_n r_0)}{K_0(q_n r_0)} \qquad (4)$$

In the above equations, $K_0(x)$, $K_1(x)$ are the modified Bessel functions of order zero and the first order of the second kind respectively and the parameters h_n, q_n are expressed as

$$h_n=(\pi/2H)(2n-1)$$
$$q_n{}^2=\frac{\{\eta^2+i[Dv(\eta^2-2)+2Ds]\}h_n{}^2-(\omega/Vs)^2}{1+iDs} \qquad (5)$$

where $\eta=Vl/Vs=\sqrt{[(\lambda+2\mu)/\mu]}$, Vl and Vs are the longitudinal and the shear wave velocities in the elastic medium and $Dv=\lambda'/\lambda$ and $Ds=\mu'/\mu$ are the hysteretic damping ratios.

Further, in the case of the pile away from the origin by the distance L, as shown in Fig. 1, the influence of the cavity of its pile is neglected. The pile is assumed to be the one-dimensional soil rod made of the surrounding soil. The distributions of the displacement and of the resistance force vary along the circumference of the soil rod. However, these values can be estimated at its center, when the distance L is relatively longer than the pile diameter. The soil rod vibrates due to the resistance force, $\bar{P}_f$, from the soil layer, then the equation of vertical motion, $w_s e^{i\omega t}$, can be written as

$$\bar{P}_f(z)e^{i\omega t}=\pi r_0{}^2\rho\frac{\partial^2}{\partial t^2}(w_s\ e^{i\omega t})$$
$$-\pi r_0{}^2\mu\{\eta^2+i[Dv(\eta^2-2)+2Ds]\}\frac{\partial^2}{\partial z^2}(w_s\ e^{i\omega t}) \qquad (6)$$

where w_s is equal to the displacement of the soil layer at r=L and similar to Eq. (2):

$$w_s=\sum_{n=1}^{\infty}\bar{w}_n\sin(h_n z)\ ,\ \bar{w}_n=A_n K_0(q_n L) \qquad (7)$$

Substituting Eq. (7) into Eq. (6) and considering Eq. (5) yield

$$\bar{P}_f(z)=\sum_{n=1}^{\infty}\beta_n\ \bar{w}_n\sin(h_n z) \qquad (8)$$

where $\bar{w}_n$ is the amplitude of the displacement of the soil rod and the resistance factor, β_n, is defined as

$$\beta_n=\pi r_0{}^2\mu(1+iDs)q_n{}^2 \qquad (9)$$

The displacement w_n including the integration constant may be determined by the boundary condition at $r=r_0$.

VIBRATION OF A FLOATING PILE

The floating elastic pile which perfectly connects with the one-dimensional visco-elastic soil rod at the pile tip is shown in Fig. 2.

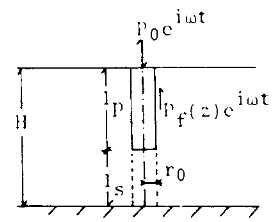

Fig. 2. Model of Soil-Floating Pile System

The pile-soil rod is assumed to sustain the vertical harmonic excitation, $P_0e^{i\omega t}$, at the pile head and the resistance force, $P_f(z)e^{i\omega t}$, from the surrounding soil. Then the equation of vertical motion, $w\ e^{i\omega t}$, of the pile-soil rod is written as

$$M_p\frac{\partial^2}{\partial t^2}(w_p e^{i\omega t})-E_pS\frac{\partial^2}{\partial z^2}(w_p e^{i\omega t})=P_f(z)e^{i\omega t}$$
$$M_s\frac{\partial^2}{\partial t^2}(w_s e^{i\omega t})-E_sS\frac{\partial^2}{\partial z^2}(w_s e^{i\omega t})=P_f(z)e^{i\omega t} \qquad (10)$$

where the subscripts p and s denote the pile and the soil rod respectively, M=mass per unit length, E=Young's modulus and S=cross-sectional area of the pile. E_s means the modulus of visco-elasticity, in the state of plane strain, like as in the surrounding soil and is defined as

$$E_s=[(\lambda+2\mu)+i(\lambda'+2\mu')]\pi r_0{}^2/S \qquad (11)$$

The resistance force, $P_f(z)$, is expressed similar to Eq. (3) and rewritten as

$$P_f(z)=-\sum_{m=1}^{\infty}\alpha_m w_m\sin(h_m z)\ ,\ h_m=\frac{\pi}{2H}(2m-1) \qquad (12)$$

Each solution of Eq. (10) is obtained as a sum of the homogeneous solution and the non-homogeneous solution; hence

$$w_p(z)=A_p\cos(\kappa_p z)+B_p\sin(\kappa_p z)+\sum_{m=1}^{\infty}C_p\sin(h_m z)$$
$$w_s(z)=A_s\cos(\kappa_s z)+B_s\sin(\kappa_s z)+\sum_{m=1}^{\infty}C_s\sin(h_m z) \qquad (13)$$

where A_p, B_p, A_s and B_s are the integration constants, and κ_p, κ_s, C_p and C_s are defined as

$$\kappa_p{}^2=\frac{M_p\omega^2}{E_pS}\ ,\qquad C_p=\frac{1}{E_pS}\ \frac{\alpha_m w_m}{\kappa_p{}^2-h_m{}^2}$$
$$\kappa_s{}^2=\frac{M_s\omega^2}{E_sS}\ ,\qquad C_s=\frac{1}{E_sS}\ \frac{\alpha_m w_m}{\kappa_s{}^2-h_m{}^2} \qquad (14)$$

The four integration constants can be determined under the boundary conditions; $w_s=0$ at $z=0$, $w_s=w_p$ and $E_sS\frac{dw_s}{dz}=E_pS\frac{dw_p}{dz}$ at $z=l_s$, $E_pS\frac{dw_p}{dz}=P_0$ at $z=H$, and become

$$A_p=[\frac{P_0}{E_p\kappa_p}J+\cos(\kappa_p H)\sum_{m=1}^{\infty}C_m\alpha_m w_m]/(\Delta S)$$

$$B_p=[\frac{P_0}{E_p\kappa_p}I+\sin(\kappa_p H)\sum_{m=1}^{\infty}C_m\alpha_m w_m]/(\Delta S)$$

$$A_s=0 \tag{15}$$

$$B_s=[P_0+\sum_{m=1}^{\infty}D_m\alpha_m w_m]/(\Delta S)$$

where

$$\Delta=I\cos(\kappa_p H)-J\sin(\kappa_p H)$$

$$I=E_p\kappa_p\sin(\kappa_p l_s)\sin(\kappa_s l_s)+E_s\kappa_s\cos(\kappa_p l_s)\cos(\kappa_s l_s)$$

$$J=E_p\kappa_p\cos(\kappa_p l_s)\sin(\kappa_s l_s)-E_s\kappa_s\sin(\kappa_p l_s)\cos(\kappa_s l_s) \tag{16}$$

$$C_m=\kappa_s(\frac{1}{\kappa_s^2-h_m^2}-\frac{E_s}{E_p}\frac{1}{\kappa_p^2-h_m^2})\cos(\kappa_s l_s)\sin(h_m l_s)-h_m(\frac{1}{\kappa_s^2-h_m^2}-\frac{1}{\kappa_p^2-h_m^2})\sin(\kappa_s l_s)\cos(h_m l_s)$$

$$D_m=\kappa_p(\frac{E_p}{E_s}\frac{1}{\kappa_s^2-h_m^2}-\frac{1}{\kappa_p^2-h_m^2})\sin(\kappa_p l_p)\sin(h_m l_s)-h_m(\frac{1}{\kappa_s^2-h_m^2}-\frac{1}{\kappa_p^2-h_m^2})\cos(\kappa_p l_p)\cos(h_m l_s)$$

Then the displacement of the soil on the circumference of the pile-soil rod is expressed by Eq. (2). Since the pile-soil rod is assumed to contact with the soil layer, the displacements of the soil layer, Eq. (2), and of the pile-soil rod, Eq. (13), must be identical. Expanding Eq. (13) into the Fourier sine series of argument $h_n z$, the identical equation is written as

$$\sum_{n=1}^{\infty}w_n\sin(h_n z)=\sum_{n=1}^{\infty}[B_sF_s+A_pF_{p1}+B_pF_{p2}+\sum_{m=1}^{\infty}(C_sG_s+C_pG_p)]\sin(h_n z) \tag{17}$$

where

$$F_s=\frac{2}{H}\int_0^{l_s}\sin(\kappa_s z)\sin(h_n z)dz$$

$$G_s=\frac{2}{H}\int_0^{l_s}\sin(h_m z)\sin(h_n z)dz$$

$$F_{p1}=\frac{2}{H}\int_{l_s}^{H}\cos(\kappa_p z)\sin(h_n z)dz \tag{18}$$

$$F_{p2}=\frac{2}{H}\int_{l_s}^{H}\sin(\kappa_p z)\sin(h_n z)dz$$

$$G_p=\frac{2}{H}\int_{l_s}^{H}\sin(h_m z)\sin(h_n z)dz$$

Eq. (17) can be solved for w_n which is determined from the following complex simultaneous equation.

$$w_n-\sum_{m=1}^{\infty}\alpha_m w_m\{\frac{D_mF_s+C_m[F_{p1}\cos(\kappa_p H)+F_{p2}\sin(\kappa_p H)]}{\Delta S}+\frac{1}{E_sS}\frac{G_s}{\kappa_s^2-h_m^2}+\frac{1}{E_pS}\frac{G_p}{\kappa_p^2-h_m^2}\} \tag{19}$$
$$=P_0[F_s+\frac{1}{E_p\kappa_p}(J\,F_{p1}+I\,F_{p2})]/(\Delta S)$$

Finally, the displacement of the pile can be evaluated by Eq. (2) and the complex stiffness of the soil-pile system at the pile head is obtained as

$$K=\frac{P_0}{w_p(H)} \tag{20}$$

The particular cases; $\kappa_p=h_m$, $\kappa_s=h_m$, $\omega=0$, may be obtained from the above equations as the limit cases for κ_p, $\kappa_s\to h_m$ or $\omega\to 0$. However, the relationships in the explicit forms have been derived by modifying the above procedure.

VIBRATION OF A PILE GROUP

The identical piles forced to vibrate vertically due to the harmonic excitation, $P_j e^{i(\omega t-\phi_j)}$, applied at each pile head are shown in Fig. 3.

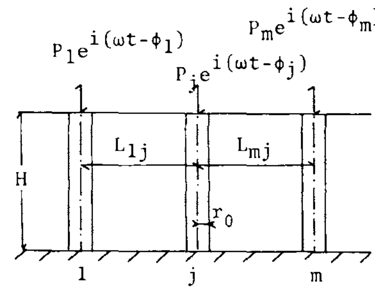

Fig. 3. Model of Soil-Pile Group System

As the resistance forces generated in one pile by the vibrations of the other piles have been obtained, the equation of motion for each pile can be expressed as the similar equation to the case of the single pile. Thus, for the origin pile j,

$$M_p\frac{\partial^2}{\partial t^2}[w_{pj}e^{i(\omega t-\phi_j)}]-E_pS\frac{\partial^2}{\partial z^2}[w_{pj}e^{i(\omega t-\phi_j)}]=P_{fj}(z)e^{i(\omega t-\phi_j)} \tag{21}$$

where the symbols for the pile are the same as for the floating pile and ϕ is the phase lag of the excitation at each pile. The total resistance force, $P_{fj}(z)$, from the soil layer on the circumference of the pile j is a sum of ${}_jP_{fj}$ occurring due to the vibration of the pile j itself and ${}_kP_{fj}$ generated by the vibration of the other piles; hence

$$P_{fj}=\sum_{k=1}^{m}{}_kP_{fj} \tag{22}$$

where m is the number of piles. ${}_jP_{fj}$ is expressed similar to Eq. (3) and rewritten as

$${}_jP_{fj}=-\sum_{n=1}^{\infty}\alpha_n w_{nj}\sin(h_n z) \tag{23}$$

while ${}_kP_{fj}$ is given by Eq. (8) and rewritten as

$${}_kP_{fj}=\sum_{n=1}^{\infty}\beta_n\,{}_kw_{nj}\sin(h_n z) \tag{24}$$

where w_{nj} in Eq. (23) and $_kw_{nj}$ in Eq. (24) are the unknown amplitudes of the displacements of the soil on the circumference of the pile j and w_{nj} is the displacement due to the vibration of the pile j itself and $_kw_{nj}$ is the displacement generated by the vibration of the other piles. In the same way as the floating pile, the solution of Eq. (21) for the vertical displacement of the pile j, w_{pj}, under the boundary conditions; $w_{pj}=0$ at z=0, $E_pS\frac{dw_p}{dz}=P_j$ at z=H, is reduced to

$$w_{pj}=\sum_{n=1}^{\infty}\frac{1}{E_pS(h_n^2-\kappa_p^2)}\{\frac{2}{H}P_j(-1)^{n-1}-\alpha_nw_{nj}+\sum_{k=1}^{m}\beta_n\ {}_kw_{nj}\}\sin(h_nz)\ ;k\neq j \quad (25)$$

From the assumption of the pile to contact with the soil layer, the relationship on the circumference of the pile j becomes

$$w_{pj}=\sum_{n=1}^{\infty}\{w_{nj}+\sum_{k=1}^{m}\ {}_kw_{nj}\}\sin(h_nz)\ ;k\neq j \quad (26)$$

Hereon, $_kw_{nj}$ is expressed by the displacement of the soil on the circumference of the other piles , w_{nk}, due to their own harmonic vibration, $e^{i(\omega t-\phi_k)}$ and follows by considering the phase lag $(\phi_j-\phi_k)$ of the pile k behind the pile j as

$$_kw_{nj}=\ _k\Psi_j\ w_{nk}\ ;k\neq j \quad (27)$$

where $_k\Psi_j$ is the influence factor of the soil displacement on the circumference of the pile j through the soil medium due to the vibration of the pile k. From the relationship between Eqs. (2) and (7), $_k\Psi_j$ is obtained as

$$_k\Psi_j=\frac{K_0(q_nL_{kj})}{K_0(q_nr_0)}\ e^{i(\phi_j-\phi_k)}\ ;k\neq j \quad (28)$$

This influence factor depends on the phase differences of the harmonic excitations and the distances among piles. Finally, from the identical equation (26), the following equation for the pile j must hold with respect to n. Substituting Eqs. (25) and (27) into Eq. (26) yields for k≠j

$$\{E_pS(h_n^2-\kappa_p^2)+\alpha_n\}w_{nj}+\{E_pS(h_n^2-\kappa_p^2)-\beta_n\}\sum_{k=1}^{m}\ _k\Psi_j\ w_{nk}=\frac{2}{H}P_j(-1)^{n-1} \quad (29)$$

where the unknown displacement w_{nk} (k=1,··,j,·· ,m) are included. Similar equations may be obtained for the other piles. By solving the simultaneous equation composed of m equations, w_{nk} can be determined. The displacement of the pile group can be evaluated by Eqs. (26) and (27), respectively. The complex stiffness of the soil-pile system at the pile head is defined as Eq. (20).

NUMERICAL RESULTS AND DISCUSSION

Some of the numerical results are shown in Figs. 4 to 7. In the numerical calculation, the complex responses of the displacement w' and the stiffness k' at the pile head normalized by their static values have been obtained. The displacement w' includes the amplitude and the phase lag. The complex stiffness k' represents the stiffness and the damping for the real and the imaginary parts, respectively. In these figures, ν is Poisson's ratio of the soil, $\bar{\rho}$ is the mass density ratio of the soil to the pile, D(=Dv=Ds) is the hysteretic damping factor of the soil, $\bar{V}$ is the velocity ratio of the shear wave of the soil layer to the longitudinal wave of the pile and $\omega_p=(\pi/2H)\sqrt{[E_pS/M_p]}$.

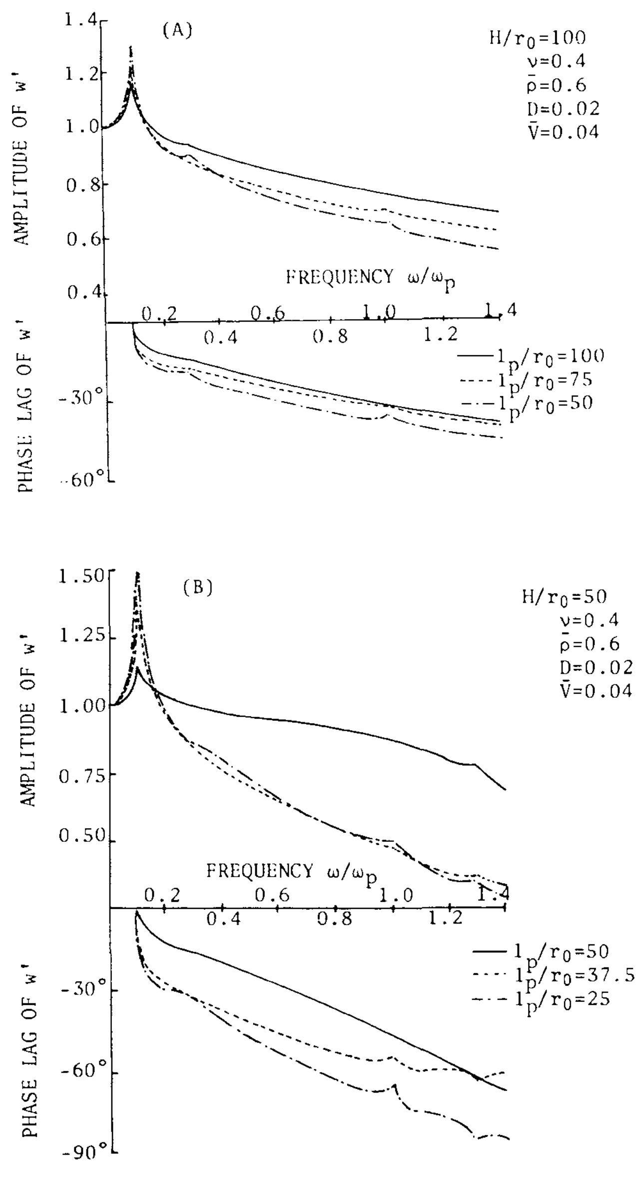

Fig. 4. Variations of Frequency Response of Amplitude and Phase Lag of Displacement with l_p/r_0

Discussion on the case of the floating pile:

The variations of w' and k' with l_p/r_0 are shown in Figs. 4 and 5, respectively. From these figures, it is mentioned that the responses of the floating pile differ from those of the end bearing pile and that the shorter the pile length is, the more the displacement and the complex stiffness at the pile head are affected by the excitation frequency, ω/ω_p and that as the wave velocity ratio $\bar{V}$ or the thickness of the soil layer H is smaller, this tendency is remarkable.

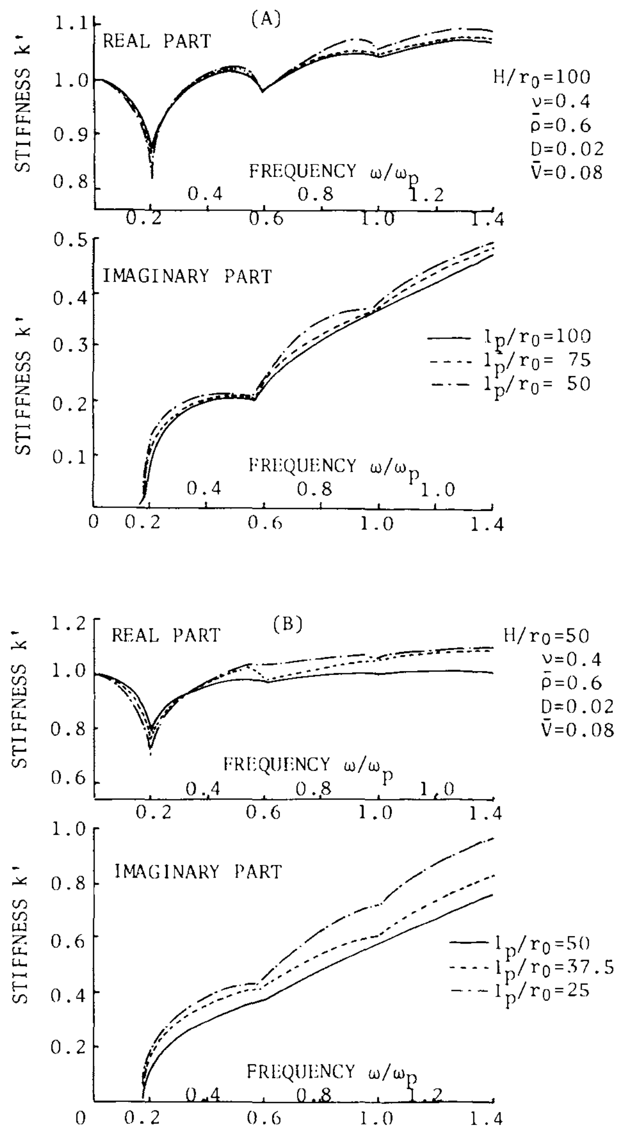

Fig. 5. Variations of Pile Stiffness vs. Frequency with l_p/r_0

Discussion on the case of two piles in phase on behalf of the pile group:

The variations of w' and k' with L/r_0 are shown in Figs. 6 and 7, respectively. From these figures, it is recognized that in the range of the frequency lower than the first natural frequency of the soil layer, the responses of the pile group don't differ from those of the single pile and that in the range of the frequency higher than the first natural frequency of the soil layer, the responses of the pile group differ from those of the single pile and that these curves are influenced by the distance between piles and the pile length.

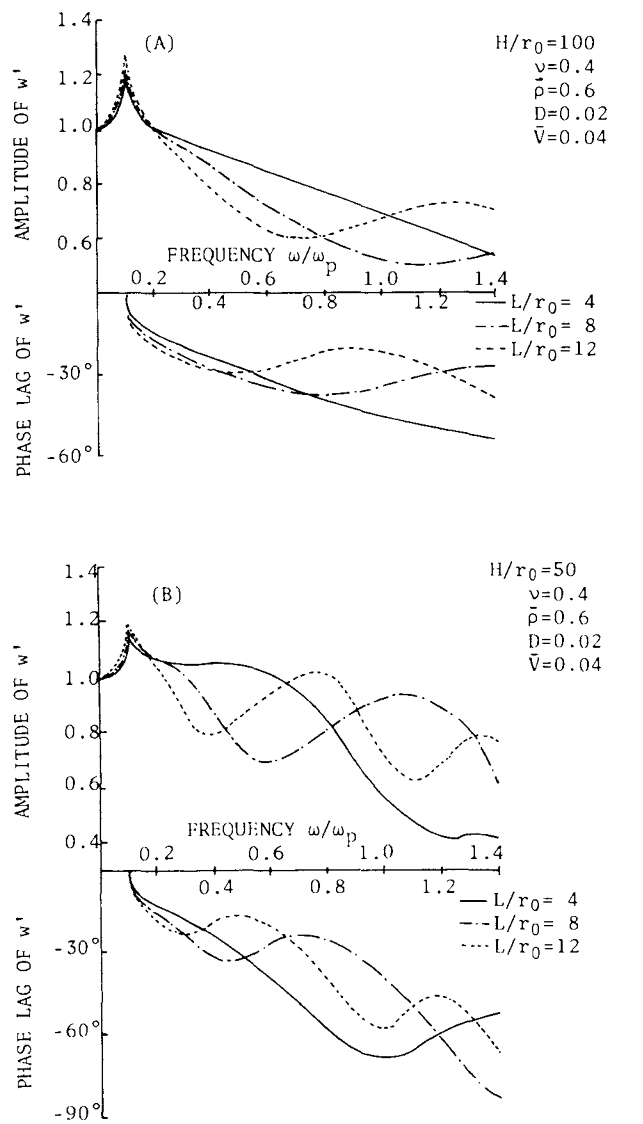

Fig. 6. Variations of Frequency Response of Amplitude and Phase Lag of Displacement with L/r_0

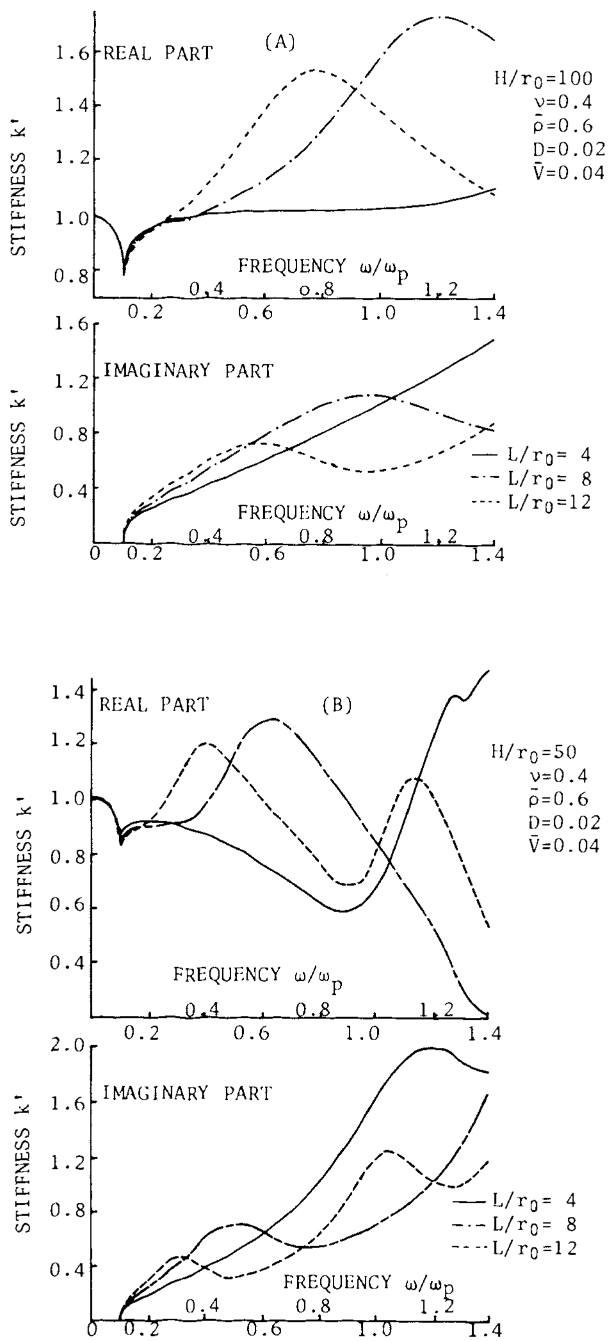

Fig. 7. Variations of Pile Stiffness vs. Frequency with L/r_0

CONCLUSIONS

The above analysis leads to the following conclusions;

(1) The responses of the floating pile differ from those of the end bearing pile and the shorter the pile length is, the more the responses of the displacement and the complex stiffness are affected by the excitation frequency. As the wave velocity ratio or the thickness of the soil layer is smaller, this tendency is remarkable.

(2) In the range of the frequency lower than the first natural frequency of the soil layer, the responses of the pile group don't differ from those of the single pile, while in the range of the frequency higher than the first natural frequency of the soil layer, the specific curves appear and differ from those of the single pile and are much affected by the distance among piles.

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