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DEM Simulation of Liquefaction-Induced Lateral Spreading

Usama El Shamy
Southern Methodist University, Dallas, TX

Mourad Zeghal
Rensselaer Polytechnic Institute, Troy, NY

Ricardo Dobry
Rensselaer Polytechnic Institute, Troy, NY

Tarek Abdoun
Rensselaer Polytechnic Institute, Troy, NY

Sabanayagam Thevanayagam
University at Buffalo, SUNY Buffalo, NY

See next page for additional authors

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Usama El Shamy
Southern Methodist University
Dallas, TX 75275

Mourad Zeghal, Ricardo Dobry, and Tarek Abdoun
Rensselaer Polytechnic Institute
Troy, NY 12180

Sabanayagam Thevanayagam
University at Buffalo, SUNY
Buffalo, NY 14260

Ahmed Elgamal
University of California San Diego
La Jolla, CA 92093

ABSTRACT

This paper reports the results of a model-based simulation of 1g shake table tests of sloping saturated granular deposits subjected to seismic excitations. The simulation technique utilizes a transient fully-coupled continuum fluid discrete particle model of the water-saturated soil. The fluid (water) phase is idealized at a macroscale using an Eulerian averaged form of Navier-Stokes equations. The solid particles are modeled at the microscale as an assemblage of discrete spheres using the discrete element method. The interphase momentum transfer is accounted for using an established relationship. Numerical simulations were conducted to investigate the liquefaction induced lateral spreading of a mild-sloped semi-infinite deposit subjected to a dynamic base excitation. The employed model reproduced a number of response patterns observed in the 1g experiment. In addition, the simulation results captured the initiation of sliding at failure planes, the propagation of liquefaction front and associated large strain localization, and the redistribution of void space during shaking.

INTRODUCTION

Liquefaction of a water saturated soil results from the tendency of the solid grains to densify when subjected to shearing stresses. This densification is associated with a decrease in the volume of the soil skeleton. As pore water is relatively incompressible, excess pore pressures accumulate during shearing and lead to liquefaction. Lateral spreading of a gently sloping ground is a characteristic failure mechanism associated with soil liquefaction. The ground experiences significant lateral deformation associated with displacements as large as several meters. Such spreading occurs during seismic excitation as well as during the phase of excess pore pressure dissipation following the end of shaking.

The coupled (solid-fluid) response of saturated granular soils is commonly modeled using continuum formulations derived based on phenomenological considerations (e.g., Zienkiewicz *et al*., 1998; Arduino and Macari, 2001) or homogenization of the micromechanical equations of motion (Lewis and Schrefler, 1998). These formulations require a constitutive model to describe the relationship between effective stresses and strains of the solid phase. For liquefaction problems, constitutive models based on plasticity theory are most commonly used. Among them, the cap models (Desai and Siriwardane, 1984; Wood, 1990), the multi-yield surface plasticity model (Prevost, 1985), the bounding surface plasticity models (Dafalias and Herrmann, 1982), and the fuzzy-set plasticity models (Klisinski, 1988) have been used. Most of these models have been calibrated based on undrained cyclic triaxial test or simple shear test results. The finite element method is typically used to discretize the field equations (e.g., Zienkiewicz *et al*., 1998; Arduino and Macari, 2001; Yang and Elgamal, 2002). Continuum models for granular soil liquefaction are based on phenomenological observations of the two phases. However, there is a lack of physical experimental measurement of local microscale properties of the fluid and solid phases (e.g., porosity, permeability, relative fluid-particle velocity, etc.), and a number of assumptions are generally introduced to the field equations or to the constitutive models to overcome these shortcomings.

The alternative to continuum methods is to use particle-based techniques such as the Discrete Element Method (DEM). Pioneered by Cundall and Strack in 1979, DEM has since been utilized by a number of researchers in several engineering applications (e.g., Dobry and Ng, 1992; Tsuji *et al*., 1993; El Shamy and Zeghal, 2005a, b). In DEM, deformations of a soil skeleton reflect the movement of particles and subsequent rearrangement of their positions. During this process, the nonlinear behavior of soils results from particle sliding, rotation, and the formation and breakage of inter-particle contacts. Assemblies of discrete particles capture this behavior with relatively simple assumptions and a small number of physically measurable parameters at the microscale level.

This paper presents the results of model-based simulations of free-field, full-scale 1-g shake table tests of sloping deposits. These tests were conducted as part of an effort supported by the NEES program (NEES: George E. Brown, Jr. Network for Earthquake Engineering Simulation) of the US National Science Foundation. This project involves multiple tasks that include 1-g shake table modeling, centrifuge testing, continuum modeling and microscale computational simulations. The project is a cooperative research effort between five US universities: University at Buffalo SUNY (UB), Rensselaer Polytechnic Institute (RPI), University of California at San Diego (UCSD), and Southern Methodist University (SMU), as well as Japanese collaborators at Tokyo Institute of Technology (TIT) and the National Research Institute for Earth Science and Disaster Prevention (NIED).

The focus of this study is on the task of micromechanical modeling of the 1-g free-field tests. Specifically, it presents an attempt to reproduce the full-scale test in the large laminar box at UB. Detailed information on the laminar box and associated base shaking facility and instrumentation is presented by Thevanayagam *et al*. (2009). Computational simulations of liquefaction are achieved herein by using a transient fully-coupled continuum-discrete hydromechanical model to analyze the pore-fluid flow and solid phase deformation of saturated granular soils when subjected to dynamic excitation (El Shamy, 2004; Zeghal and El Shamy, 2004; El Shamy and Zeghal, 2005a, 2007). A brief description of the model is first presented followed by results of computational simulations of test SG-1. The main objective of this study is to provide information on the mechanisms associated with liquefaction and lateral spreading based on micromechanical considerations.

METHODOLOGY

Saturated granular soils were idealized as two overlapping media. The solid phase was modeled as an assemblage of discontinuous particles using the discrete element method, DEM (Cundall and Strack, 1979). The pore fluid was considered to be inviscid and incompressible, and was idealized using averaged Navier-Stokes equations of conservation of mass and momentum (e.g., Jackson, 2000):

$$\frac{\partial n}{\partial t} + \frac{\partial (n u_i)}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial (n u_i)}{\partial t} + \frac{\partial (n u_i u_j)}{\partial x_j} = -\frac{n}{\rho}\frac{\partial p}{\partial x_i} + n\rho g_i + d_i \tag{2}$$

along with appropriate boundary and initial conditions. In Eqs. (1) and (2), x_i (i=1,2,3) are Cartesian coordinates, n is porosity, u_i is fluid velocity, ρ is fluid density, p is fluid pressure, and g_i is gravitational force per unit mass. The term d_i represents averaged fluid-particle interaction force per unit of volume and was accounted for by using well established semi-empirical relationships proposed by Ergun (1952) and Comiti and Renaud (1989).

An explicit time-integration scheme was used to evaluate the coupled fluid-particle response. The fluid domain was discretized into parallelepiped cells and averaged Navier-Stokes equations were solved using a finite volume technique. Average drag forces exerted by the fluid on the particles within a specific cell were evaluated based on mean values of porosity, as well as of particle sizes and velocities within this cell. These forces were then applied to each of the individual particles proportionally to their volumes. Deformation of the solid phase subjected to the drag forces along with any external loads was subsequently computed using the DEM technique (Itasca, 2005). Details of the implemented continuum-discrete model are given in (El Shamy and Zeghal, 2005a and Zeghal and El Shamy, 2004).

SIMULATION

Numerical simulations were conducted to assess the response mechanisms of saturated mildly-sloped deposits of granular soils when subjected to a dynamic base excitation. The total number of particles that can be reasonably modeled in a DEM simulation using current state-of-the-art serial computers is small in comparison to the number of grains comprised in an actual deposit. Therefore, a semi-infinite deposit was idealized by computationally pluviating particles within a parallelepiped domain having periodic boundaries (Cundall, 1988) in the two lateral directions (Fig. 1). Such boundaries allow particles to pass from the parallelepiped domain to fictitious adjacent ones and simulate an infinitely periodic system in the lateral directions. The periodic deposit consisted of relatively large spherical particles subjected to a high gravitational (g) field in order to get the number of particles to a computationally manageable size. The employed high g level mimics the conditions of centrifuge testing of small-scale geotechnical models.

A 182.5 mm high deposit of particles, having an average diameter of 6 mm, a shear modulus of 2.9 GPa and saturated with a viscous fluid, was subjected to a one-dimensional dynamic base excitation (in the *x*-direction) under a gravity field of 30-g. The lateral dimensions of the periodic deposit were 140 mm x 60 mm (in the *x* and *y* directions respectively, Fig. 1). The scaling laws associated with the employed high g level may be derived based on a dimensional analysis of governing field equations (Kutter, 1992). Thus, the analyzed model corresponds to a periodic prototype of a 5.5 m granular deposit with lateral dimensions of 4.2 m and 1.8 m. The deposit was saturated with a highly viscous fluid to compensate for the effects of employed 30-g field and large particle sizes (an average diameter of about 6 mm) and comply with the scaling of permeability. Under these conditions, the prototype fluid has a viscosity $\mu_f = 0.492$ Pa.s and the resultant initial permeability of the prototype deposit is about $8x10^{-4}$ m/s. Periodic lateral fluid boundaries were employed to ensure consistency with those of the solid phase. Additional boundary conditions include an impermeable wall condition (zero pressure gradient) at the base and zero pressure at the fluid surface. The employed model simulates to some extent the SG-1 full-scale test mentioned above. Note that the employed DEM simulation are for a submerged infinite deposit (through the use of lateral periodic boundaries) that is subjected to seepage parallel to the slope when the ground is gently inclined (in this case with a 1^o to the horizontal).

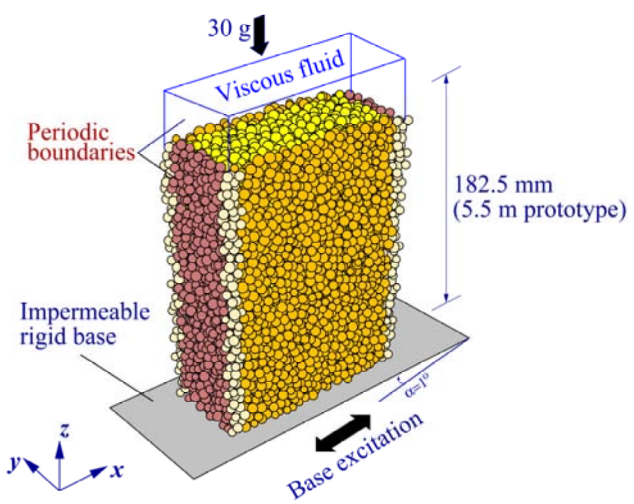

Fig. 1. Three-dimensional view of the particle deposit employed in the conducted simulation.

The above periodic deposit was subjected (under a 30-g field) to a sinusoidal base acceleration excitation having a frequency of 2 Hz similar to the one employed in the 1-g shake test at UB. The input motion was designed such that it starts with a small amplitude of 0.01 g that is predicted to be non-destructive (ND), aiming to determine the dynamic properties of the soil with no generation of excess pore pressures, for the first 5 s of shaking. Following this ND-phase, a stronger motion with an amplitude of 0.05 g was introduced for the next 10 s of motion. Average values of the fluid and solid phase parameters and variables were monitored during the course of the simulation within spherical control volumes along the central vertical axis of the deposit. Specifically, the process of solving the fluid phase equations produces averaged fluid and particle velocities, pore-pressure, and porosity within each fluid cell. Average stress tensor and coordination number are provided by PFC^{3D} within the spherical control volumes (Itasca, 2005). The sections below present an analysis of the deposit macro-scale characteristics and associated microscale mechanisms. Results are presented in prototype units exclusively. More computational details are given in (El Shamy et. al, 2009).

Dynamic characteristics of soil deposit

As mentioned in the previous section, the use of the ND shaking phase of the input motion was mainly to obtain the dynamic characteristics of the soil deposit at low strains and with no buildup of pore fluid pressure. In view of the complexity involved in modeling the actual 1-g tests at UB in terms of the boundary conditions of the laminar box and the computationally challenging number of sand particles, it was decided to attempt to only match the dynamic properties represented by the initial shear wave velocity (and initial shear modulus) as the key parameters to obtain a reasonable DEM response pattern as close as possible to the 1-g model. The particle properties in DEM were modified until a close match was obtained between the shear wave velocity profiles in the DEM and UB deposits (Fig. 2).

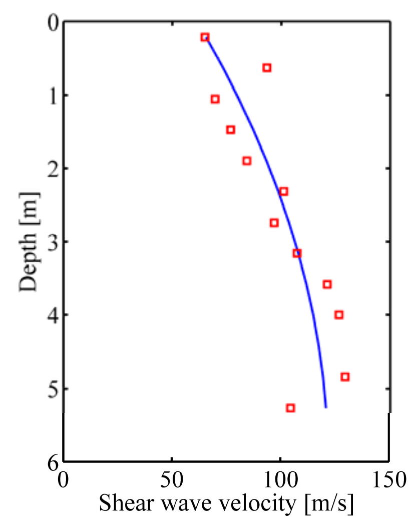

Fig. 2. Shear wave velocity profile obtained from DEM Simulation.

Liquefaction

Liquefaction is a phenomenon that is associated with a buildup of excess pore fluid pressure, Δu, and the resulting reduction in the amplitude of the initial vertical effective stress of the soil. It is, therefore, commonly defined as the instant at which the excess pore pressure ratio approaches a value of one indicating that the pore pressure has counterbalanced the effective stress and the inter-particle contact forces vanish. Investigation of the profiles of the computed excess pore pressure (Fig. 3) indicates that liquefaction occurred at the top of the deposit and progressed downwards.

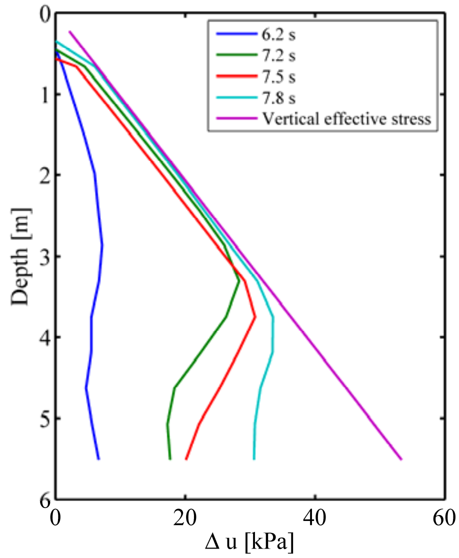

Fig. 3. Excess pore fluid pressure profiles.

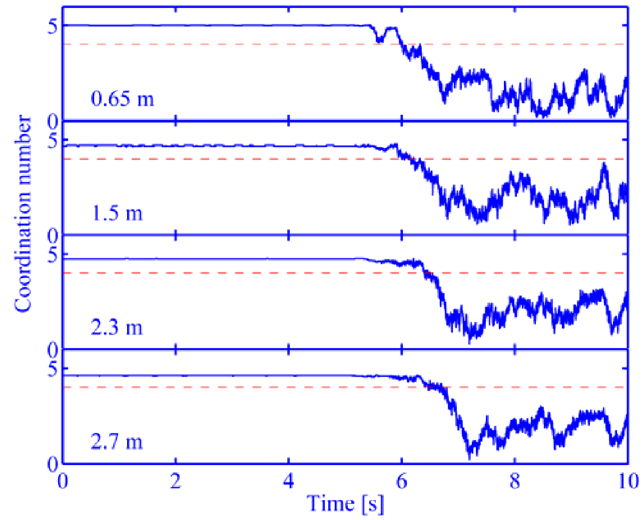

Fig. 5. Time histories of coordination number at selected depth locations.

The computed acceleration time histories suggest that the soil exhibited a dilative response as indicated by negative acceleration spikes after liquefaction at the 1.1 m and 2.9 m depth locations (Fig. 4). To complete the picture from a micromechanical point of view, the time histories of coordination number (average number of contacts per particle) were also investigated (Fig. 5). A limiting coordination number of 4 is needed for a stable assembly of frictional spheres (Edwards, 1998). Figure 5 also shows a progressive propagation of the liquefaction front with depth marked by the instants at which the coordination number significantly fell below the value of 4.

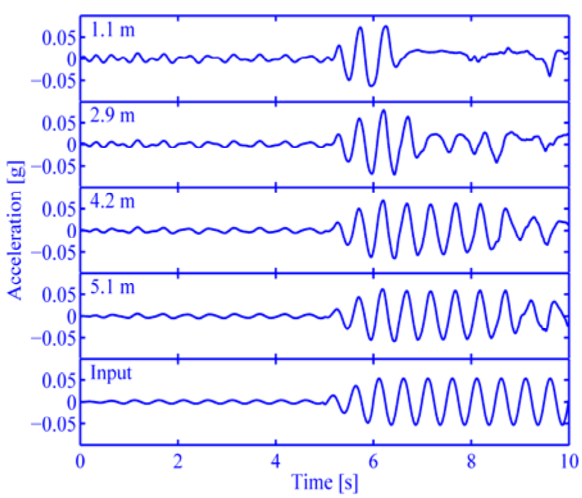

Fig. 4. Acceleration time histories.

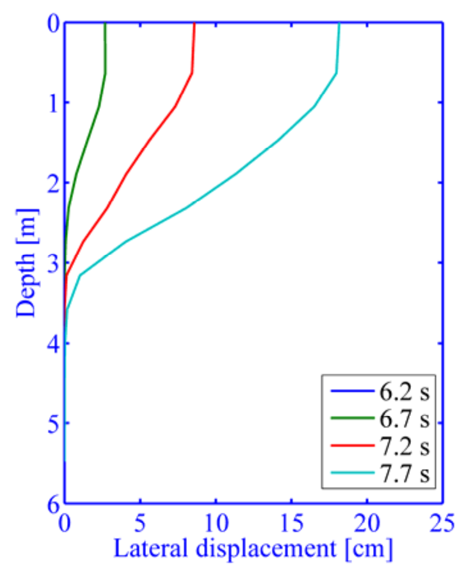

Fig. 6. Lateral displacement profiles.

<u>Lateral spreading</u>

The progressive accumulation of permanent (noncyclic) lateral displacement with depth is shown in Fig. 6. Only the top 4.0 m of this simulation accumulated lateral displacement. The instant at which lateral spreading was initiated is best captured through the inspection of the cyclic lateral displacement time histories at several depth locations. Lateral spreading is a reflection of a significant inter-particle sliding at a particular depth location. As sliding takes place, the cyclic motion vanishes and the displacement continues to increase with time as a result of the movement of the sliding mass downslope. This mechanism is shown in Fig. 7 with an arrow pointing at the initiation of sliding. A detailed analysis of the lateral displacement response in the 1-g test is provided in Dobry *et al*. (2009). A different, yet related, view of lateral spreading can be made by assessing the permanent shear strain profiles as discussed in the following section.

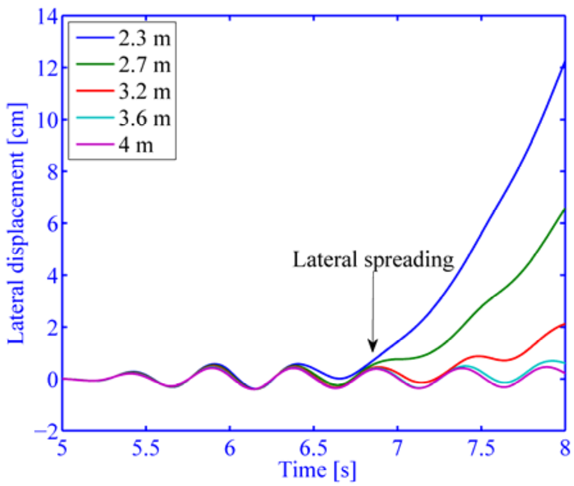

Fig. 7. Time histories of lateral displacements at different depth locations from DEM simulation of test SG-1.

Shear stress-strain response

The shear stresses resulting from the inter-particle contact forces were calculated within spherical control volumes and monitored during the course of the simulation (Itasca, 2005). Shear strains were calculated from the lateral displacement time histories (calculated by time integration of the average particle velocities within the fluid cells). The shear stress-shear strain histories are shown in Fig. 8. For depth locations up to about 3 m, the behavior was linear within the nondestructive phase of the shaking and quickly became nonlinear as shaking entered the second moderate phase and eventually shear stresses vanished and large strains accumulated. At large depth locations that did not liquefy (4.4 m in Fig. 8), the behavior remained mostly linear.

The effective stress path also shows that the deposit quickly liquefied as the shaking entered the second moderate phase (Fig. 9). It also shows indications of dilative behavior marked by the nonsymmetric loops formed as the mean effective confining stress approached zero at the shallow depth location of 0.65 m. As mentioned above, permanent (noncyclic) shear strain profiles describe the process of lateral spreading as it highlights zones of large accumulation of shear strains and consequently reflect the initiation of sliding. Figure 10 depicts the distribution of permanent shear strain with depth for selected time instants.

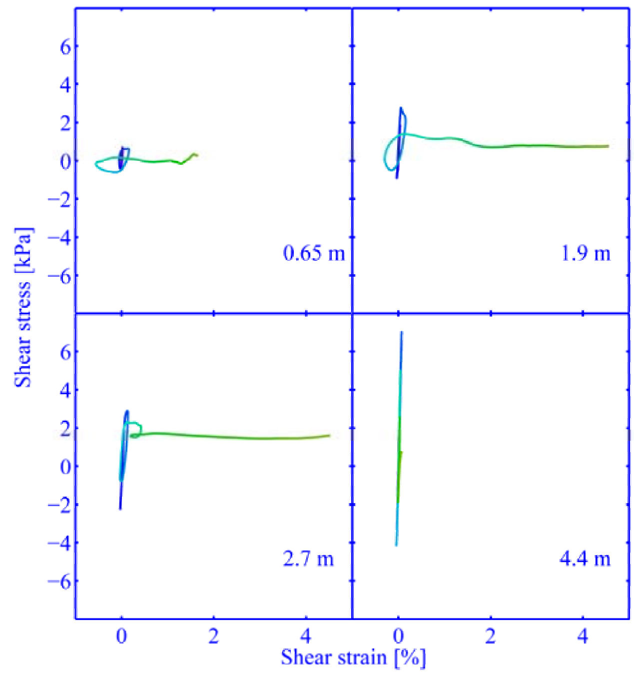

Fig. 8. Shear stress-shear strain loops calculated from DEM simulation of test SG-1.

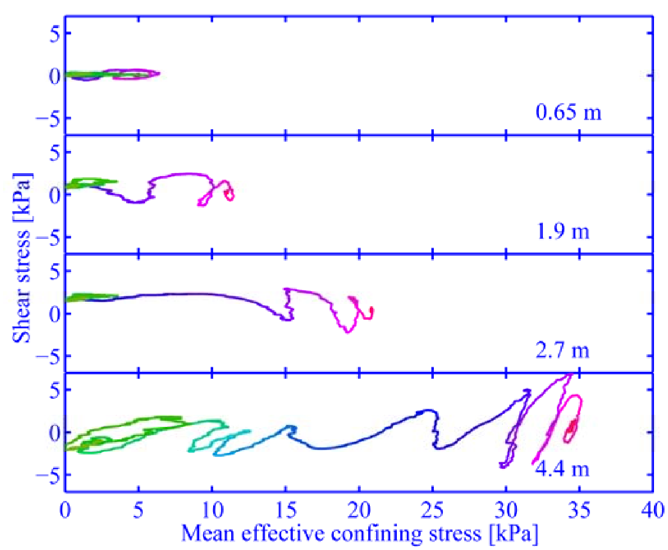

Fig. 9. Evolution of effective stress path from DEM simulation of test SG-1.

It is interesting to note that the permanent shear strain profiles in Fig. 10 reflect a trend that has been consistently observed in many centrifuge tests. Sharp and Dobry (2002) reported that a transition zone of limited thickness where shear strains are greater than either above or below the depth location of the transition zone. They also noted that this zone contains the liquefaction front and travels down as shaking progresses. Inspection of Fig. 10 shows a clear formation of that zone between the depths of 1.1 m~1.6 m for the 7.2 s profile and between the depths of 2.5 m~3.0 m for the 8.2 s time instant.

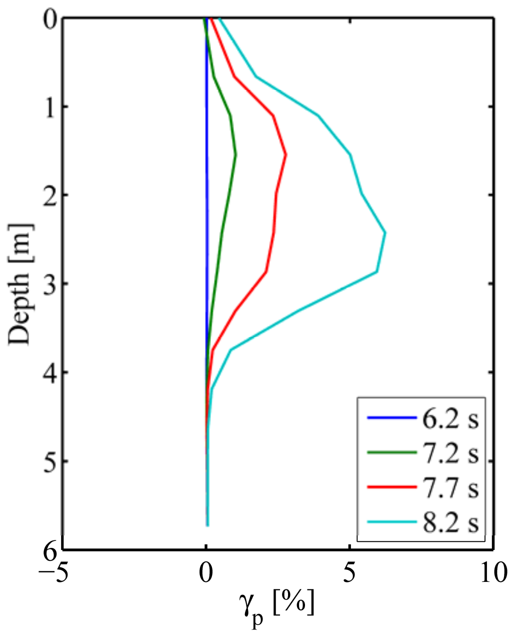

Fig. 10. Permanent shear strain profile in test SG-1.

Void redistribution

The mechanism of pore-pressure buildup resulting from ground shaking of submerged slopes has been arguably attributed to void redistribution in the soil deposit and associated development of upward pore-fluid flow that results in a pressure gradient capable of causing liquefaction (e.g., Malvick *et al*., 2006; Boulanger and Truman, 1996; Whitman, 1985). The mechanism suggests that during shaking of the slope, the lower zone of the deposit contracts (densifies) while the upper layer of the deposit dilates (expands). Densification results in reduction of void space and a tendency to squeeze water out of the pores. Dilation, on the other hand, reduces the pore-pressure in the voids and therefore helps to attract water towards the dilating zones near the surface. Some researchers also suggest that as water is squeezed from the reconsolidated sand rushing into the surface, it may be trapped by relatively low permeability layers near the surface and forms a water film that significantly contributes to the initiation of lateral spreading (e.g., Kokusho, 1999; Adalier and Elgamal, 1992; Elgamal *et al*., 1989; Scott and Zuckerman, 1972). While this rather intuitive mechanism is appealing, the authors are not aware of any computational technique that is capable of verifying and quantifying this mechanism. The microscale analysis presented herein provides a unique opportunity to quantify the void redistribution mechanism with its ability to track local changes in porosity (which reflects volumetric change) as well as the induced pore-fluid pressures and velocities. The time histories of volume change at different depth locations are shown in Fig. 11. This figure indicates that significant dilation (increase in volume) took place near the surface at the depth locations of 0.2 m and 0.65 m prior to liquefaction. This was accompanied with contraction taking place at the 1.1~3.2 m depth range. The combined action of contraction and dilation resulted in upward pore-fluid flow relative to the averaged particle velocities with amplitudes that were generally larger near the shallow layers as a result of accumulation of pore fluid squeezed out from the underlying layers. Details of pore fluid flow characteristics and resulting drag forces are given in El Shamy *et al*. (2009).

CONCLUSIONS

The use of DEM-based simulations continues to prove that it is a powerful emerging technique for analyzing soil systems. The employed coupled fluid-particle model in this study was able to reproduce a number of response mechanisms observed in the 1-g shake table experiments. Specifically, it captured the initiation of sliding, the propagation of liquefaction front and associated large strain localization, and the redistribution of void space during shaking. During dynamic excitation of a mildly-sloped ground, significant dilation (increase in volume) takes place near the surface prior to liquefaction and is accompanied with contraction taking place at deeper locations. The combined action of contraction and dilation results in upward pore-fluid flow relative to the solid particle that contributes to the upward drag forces leading to liquefaction.

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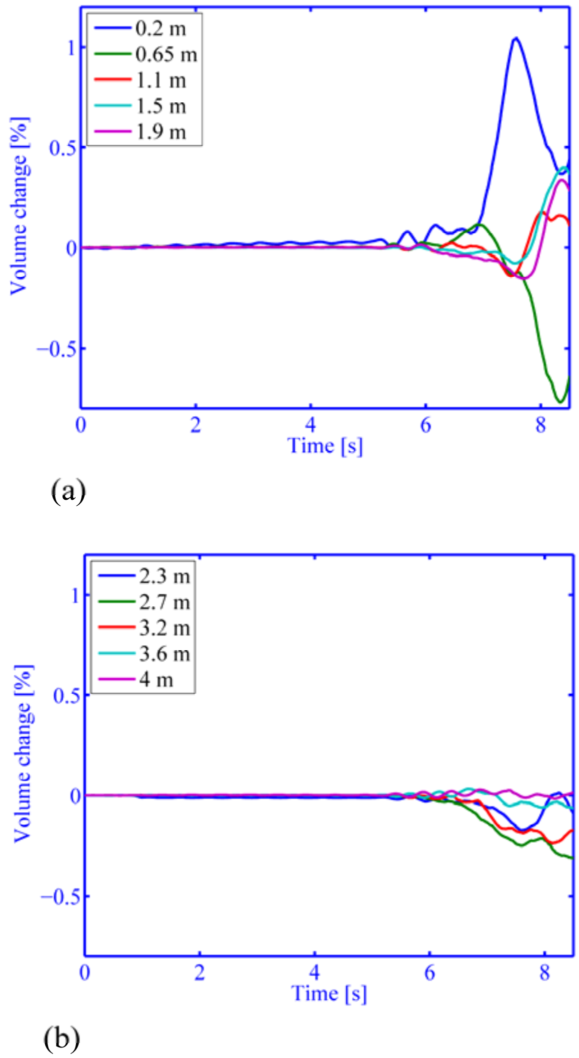

Fig. 11. Time histories of local volume changes at several depth locations (the positive sign indicates volume increase): (a) top portion of the deposit; and (b) bottom portion of the deposit.

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