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# Shear Moduli and Damping of Cohesive Soils under Earthquake Loads

Wen-yao Hu
*Tongji University, Shanghai, China*

Tian-long Wang
*Tongji University, Shanghai, China*

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# Shear Moduli and Damping of Cohesive Soils under Earthquake Loads

**Hu Wen-yao and Wang Tian-long**

Zheng Da-tong Scientific Research Group, Research Institute of Structural Theory, Tongji University, Shanghai, China

SYNOPSIS To investigate the dynamic properties of soils under earthquake loading conditions, a series of cyclic triaxial compression tests was carried out on undisturbed saturated samples of cohesive soils from Tianjin. It has come to the conclusion that variation of damping ratio, as well as shear modulus, with strain amplitude may be expressed by an empirical formula. Its normalized form can be shown by a family of curves or a band zone with shrunken ends. New empirical equations used for evaluating elastic or initial shear modulus $G_0$ and maximum shear stress $\tau_{max}$ have been suggested. For the two parameters, some comparisons between field and laboratory test values and between the results of cyclic and static triaxial compression tests under conditions of the same loading levels have been made.

## INTRODUCTION

The stress conditions in a soil deposit during an earthquake, strictly speaking, is rather complicated. However, it is generally considered that the major cause for seismic damages to structures or soils is the shear waves which propagate upwards from the bedrock. Thus the shearing stress-strain relationship should be specially considered in study of soil behavior under earthquake loading conditions. In this connection, dynamic shear modulus and damping ratio may be the most useful indices for seismic design.

Soils exhibit a nonlinear stress-strain behavior. This behavior at "low amplitude" strain may be provided by field test or observation. However, it is difficult to determine this behavior at "high amplitude" strain by in-situ test. Although laboratory test has its limitation, it can be used to study the behavior of soils at strains from small to large or failed. And in most cases it can provide a lot of available data.

## MATHEMATICAL MODELS OF STRAIN-SOFTENING STRESS-STRAIN RELATIONSHIP

### 1. Relationship between Modulus and Strain

Laboratory tests have shown that the shape of the stress-strain relationship curve for any particular soil depends upon the type of loading and boundary restraining conditions. For the shape of the strain-softening shearing stress-strain curve of soils, some famous demonstrations based on laboratory tests have been presented (Kondner, 1963;Hardin, 1972). In general, it can be adequately represented by a hyperbolic curve. In this case, the normalized forms of stress vs. strain and of modulus vs. strain can be given by

$$\frac{\tau}{\tau_{max}} = \frac{\gamma/\gamma_r}{1+\gamma/\gamma_r} = 1 - \frac{1}{1+\gamma/\gamma_r} \qquad (1)$$

$$\frac{G}{G_0} = \frac{1}{1+\gamma/\gamma_r} \qquad (2)$$

in which $\tau$ and $\gamma$ are the cyclic shearing stress and strain, respectively; $\tau_{max}$ is the maximum shear stress; $G(=\tau/\gamma)$ is the secant shear modulus; $G_0$ is the maximum secant shear modulus or the initial or elastic shear modulus; and $\gamma_r$ ($=\tau_{max}/G_0$) is called "reference shearing strain". In cyclic triaxial compression test, quantities measured directly are values of axial stress and strain, but the stress-strain or modulus-strain relationship from this test can easily be converted into the expressed form of shearing stress vs. shearing strain or shear modulus vs. shearing strain. Since there is a simple relationship between shear and compression moduli and at the same time assuming that the stress-strain conditions at 45 degree plane of cylindrical specimen in the triaxial compression test are used to simulate the stress-strain conditions at horizontal plane of sample in simple or direct shear test, the conversion formulae will be

$$\frac{\sigma}{\sigma_{max}} = \frac{\varepsilon/\varepsilon_r}{1+\varepsilon/\varepsilon_r} \longrightarrow \frac{\tau}{\tau_{max}} = \frac{\gamma/\gamma_r}{\frac{1+\nu}{1+\nu_0}+\frac{\gamma}{\gamma_r}} \qquad (3)$$

$$\frac{E}{E_0} = \frac{1}{1+\varepsilon/\varepsilon_r} \longrightarrow \frac{G}{G_0} = \frac{1}{\frac{1+\nu}{1+\nu_0}+\frac{\gamma}{\gamma_r}} \qquad (4)$$

Where $\sigma$ and $\varepsilon$ are the cyclic axial stress and strain, respectively; $\sigma_{max}$ is the maximum normal stress; $E(=\sigma/\varepsilon)$ is the secant compression modulus; $E_o$ is the maximum compression modulus or the initial or elastic compression modulus; $\varepsilon_r$ $(=\sigma_{max}/E_o)$ is called "reference axial strain"; and $\nu_o$ and $\nu$ are Poission's ratios of soil at small strain and at any strain before failure, respectively.

As to Poission's ratio of soil, it is often estimated and treated as a constant for the purpose of engineering calculation. Whitman (1970) suggested that, "For full saturated soils, Poisson's ratio may be taken as 0.5; for soils of low degree of saturation, 0.35 may be a reasonable value; and intermediate values may be selected for intermediate cases." The fact that the value of Poisson's ratio is quite close to 0.5 for fully saturated soils was also observed in our laboratory tests. Thus there is a valid reason for assumption of $\nu=\nu_o$ for cohesive soils in saturation or essential saturation. From that the expressions on both sides in Eq. (3) or (4) are equivalent and the conversions between them become quite convenient. The expressions in Eqs.(3) and (4) can be represented by curves shown in Fig.1.

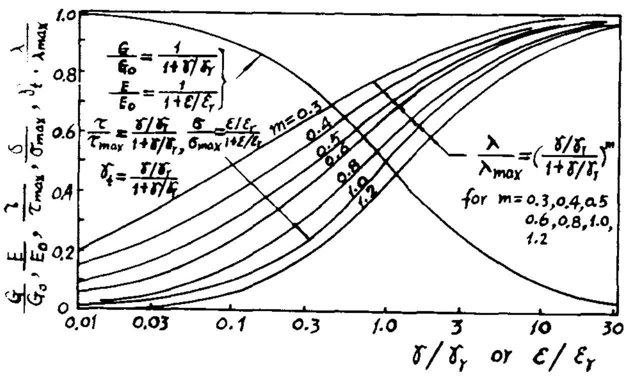



Fig.1. Stress, Modulus or Damping Ratio as a Function of Strain

2. <u>Relationship between Damping Ratio and Strain</u>

Tests show that variation of hysteresis damping of soils with strain amplitude exhibits more complicated and can hardly be described by a hyperbolic curve. It is found from tests that the following empirical expression

$$\frac{\lambda}{\lambda_{max}}=\left(1-\frac{G}{G_o}\right)^m=\left(\frac{\gamma/\gamma_r}{1+\gamma/\gamma_r}\right)^m \tag{5}$$

may approximately describe this variation. Herein $\lambda$ is the damping ratio at any strain before failure; $\lambda_{max}$ (maximum damping ratio) and m are the parameters determined by test. From Eq.(5) it is not difficult to see that the higher the value of m, the lower becomes the value of $\lambda$ with respect to any given $\gamma$ when the parameters $\lambda_{max}$ and $\gamma_r$ are kept constant. And so m may be defined as the shape parameter of curve of $\lambda/\lambda_{max}$ vs. $\gamma/\gamma_r$. Eq. (5) can be expressed by a family of curves or a band zone with shrunken ends, as shown in Fig.1. It is evident that Eq.(5) can be written as

$$\mathrm{Log}\,\frac{\lambda}{\lambda_{max}}=m\mathrm{Log}\,\gamma_t \tag{6}$$

in which

$$\gamma_t=\frac{\gamma/\gamma_r}{1+\gamma/\gamma_r}=1-\frac{G}{G_o} \tag{7}$$

and $\gamma_t$ is termed "transformed shearing strain". Eq.(6) can be represented by a series of radial straight lines or an oblique triangle zone in logarithmic plotting, as shown in Fig.2, and Eq.(7) by the corresponding curve shown in Fig.1.

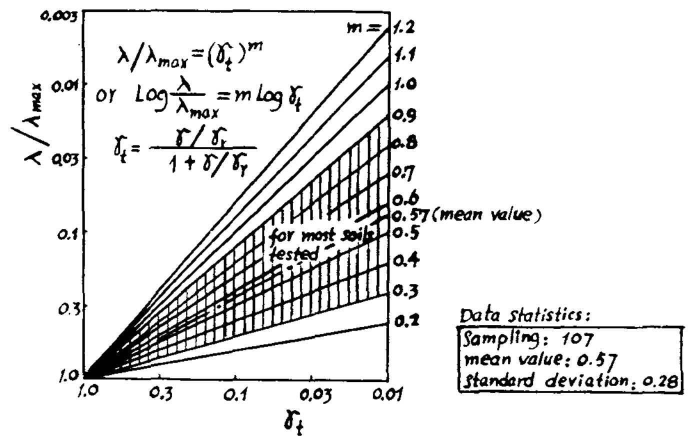



Fig.2. $\lambda/\lambda_{max}$ vs. $\gamma_t$

PARAMETERS OF EMPIRICAL EQUATIONS

There are four independent parameters, namely, $G_o$, $\tau_{max}$, $\lambda_{max}$ and m.

1. <u>Initial or Elastic Shear Modulus $G_o$</u>

Soils exhibit the behavior similar to linear stress-strain relationship only at small strain amplitude. Laboratory tests have shown that there is an insignificant change in $V_s$ (shear wave velocity) for tests run at shearing strain amplitude of $10^{-5}$ or less (Richart, Jr., 1977). Wave velocity tests usually develop shearing strain in the field of $10^{-6}$ or less. Laboratory tests of the resonant column type may also be controlled to develop strain on the order of $10^{-6}$. Laboratory tests of cyclic loading type generally develop the least effective strain of the order of $1\times10^{-4}$ and the "presumptive" initial or elastic modulus may be presumed by extrapolation on the basis of law obtained from testing. In addition, relationship between $G_o$ and some of physical or mechanical property indices which have a significant influence on it may be established based on the results of laboratory or field tests. Therefore, there is an opportunity to check laboratory values against field values or to compare one of laboratory values with another.

2. Maximum Shear Stress $\tau_{max}$

This parameter is often determined by laboratory tests. For example, laboratory tests of cyclic loading type can provide not only the value of $G_o$ but also the value of $\tau_{max}$. Herein $\tau_{max}$ is not necessarily identical with the value of shearing strength, but they may be relatively close to each other. In order to fully utilize the data of static tests and reasonably select the value of this parameter, an establishment of relationship between static and cyclic shearing strength may be of benefit. Sometimes the cyclic shearing strength for a given soil can be considered to be its static shearing strength adding corrections to take account of dynamic effects.

3. Damping Ratio Parameter $\lambda_{max}$ and m

In effect, it is difficult to systematically study the variation of damping with strain amplitude under in-situ conditions. Most of this sort of studies were carried out in laboratory. A feasible means of determining the two parameters is the laboratory tests of cyclic loading type.

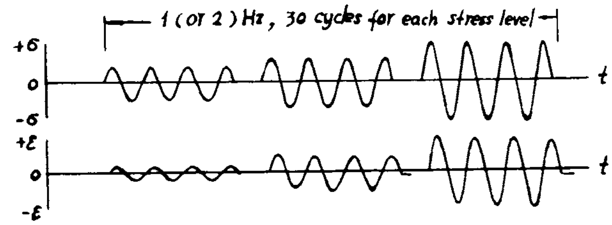



Fig.3. Sketch of Cyclic Stress and Strain

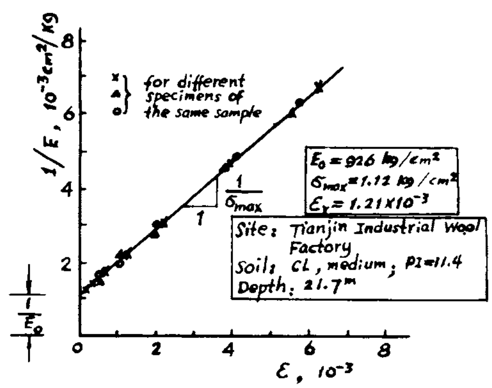



Fig.4. Typical Results of 1/E vs. $\varepsilon$ in the Case of Equal Consolidation Pressures

## TEST RESULTS AND DISCUSSIONS

The types and properties of soils tested are shown in Table I. The cylindrical specimens of $\phi$40X80 mm were resaturated and reconsolidated

Table I. Types and Main Properties of Soils Tested

| No | Type of Soil Tested | Site | Depth (m) | W (%) | $\gamma$ (g/cm³) | e | $S_r$ (%) | LL | PL | PI | LI |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | CH, very soft | Y.H. | 4.4-8.4 | 49.1 | 1.74 | 1.341 | 100 | 46.1 | 24.5 | 21.6 | 1.0 |
| 2 | | H.G. | 3.0-5.0 | 46.3 | 1.76 | 1.279 | 99 | 45.0 | 25.0 | 20.0 | 1.1 |
| 3 | | H.G. | 16.0-18.0 | 42.2 | 1.84 | 1.118 | 100 | 41.0 | 22.0 | 19.0 | 1.1 |
| 4 | CI, very soft | Y.H. | 11.7-13.0 | 39.4 | 1.78 | 1.130 | 95 | 34.2 | 18.5 | 15.7 | 1.0 |
| 5 | | Y.H. | 12.0-16.0 | 30.2 | 2.01 | 0.7[illegible]6 | 100 | 2[illegible].7 | 16.5 | 9.2 | 1.0 |
| 6 | | A.F.B. | 9.7-10.7 | 30.7 | 1.90 | 0.866 | 96 | 28.2 | 17.0 | 11.2 | 1.2 |
| 7 | | H.G. | 7.0-8.0 | 34.4 | 1.89 | 0.993 | 100 | — | — | — | — |
| 8 | | H.G. | 8.0-13.0 | 38.2 | 1.86 | 1.021 | 100 | 36.5 | 20.2 | 16.3 | 1.1 |
| 9 | CI, soft | H.G. | 15.7-16.7 | 26.4 | 2.01 | 0.771 | 100 | 27.8 | 17.0 | 10.8 | 0.9 |
| 10 | CL, soft | H.G. | 21.7-23.7 | 24.8 | 1.99 | 0.691 | 97 | 26.7 | 17.0 | 9.7 | 0.8 |
| 11 | | Y.H. | 30.7 | 22.5 | 2.05 | 0.613 | 99 | 24.0 | 15.0 | 9.0 | 0.8 |
| 12 | CH, medium | Y.H. | 9.0-10.0 | 31.6 | 1.91 | 0.881 | 98 | 46.0 | 24.5 | 21.5 | 0.3 |
| 13 | | A.F.B. | 17.7 | 36.0 | 1.82 | 1.042 | 94 | 42.5 | 23.0 | 19.5 | 0.7 |
| 14 | | Y.H. | 24.5-25.4 | 41.2 | 1.79 | 1.163 | 97 | 50.8 | 26.5 | 24.3 | 0.6 |
| 15 | CI, medium | Y.H. | 18.7-21.7,26.7 | 27.9 | 1.95 | 0.789 | 96 | 32.2 | 18.0 | 14.2 | 0.7 |
| 16 | | H.G. | 8.7 | 26.6 | 1.99 | 0.727 | 99 | — | — | — | — |
| 17 | | H.G. | 23.0-25.0 | 27.3 | 1.96 | 0.760 | 97 | 34.6 | 19.0 | 15.6 | 0.5 |

Notes: (1) Y.H.,H.G.and A.F.B. represent Yong-he Bridge, Han-gu bridge and Bureau of Agriculture and Forestry,respectively;
(2) CH,CI and CL represent cohesive soils with high, intermediate and low liquid limit, respectively;
(3) property indices in the Table denote their mean values

under assigned pressures before cyclic loading test. For some samples, the consolidation pressures are evaluated on the basis of geostatic pressures so as to try to restore their original ground stress conditions. The consolidation pressures on the other samples are assigned by testing requirements. For example, in order to investigate relationship between initial modulus $G_o$ and mean consolidation pressure $\sigma'_m$, the consolidation pressures applied to different specimens of the same sample are different.

Type DSZ-100 dynamic triaxial compression apparatus with electromagnetic excitation, made in China, was used for this research. A sinusoidal load, with a frequency of 1 or 2 Hz, was applied to the specimen in its axial direction during testing, and cyclic loading tests were performed under undrained condition. The cyclic loading program and the waves recorded are sketched in Fig.3.

For the case of equal consolidation pressure applied to different specimens of the same sample, typical results of 1/E vs. $\varepsilon$ are shown in Fig.4. Tests have shown that, if the soil in the same sample is homogeneous and manipulation during testing is careful, all data from different specimens of the same sample will fall on a single straight line, as shown in Fig.4; otherwise the data will be scattered. Nevertheless, the data for each of these specimens nearly always fall on the same straight line. It follows that the strain-softening stress-strain curve for a particular soil can be adequately represented by a hyperbolic curve. For the case of different consolidation pressures applied to each of the specimens of the same sample, typical results are shown in Fig.5. From that rela-

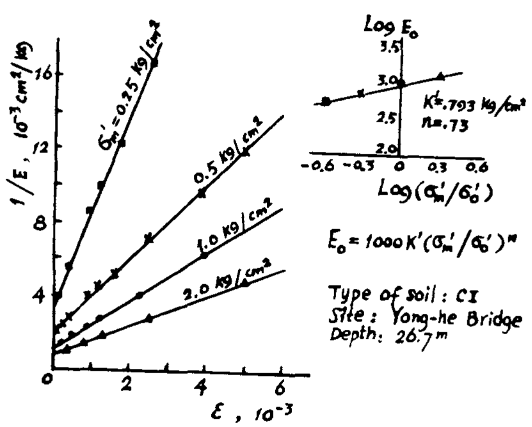



Fig.5. Typical Results of 1/E vs. $\varepsilon$ in the Case of Different Consolidation Pressures

tionship between initial modulus $G_o$ and mean consoliation pressure $\sigma'_m$ for a given soil can be provided. The results of this testing are gathered in Fig.6. Tests indicate that variation of $G_o$ with $\sigma'_m/\sigma'_o$ may be aproximately represented by a straight line in logarithmic plotting, i.e., this relationship can be expressed

$$G_o = 1000K\left(\frac{\sigma'_m}{\sigma'_o}\right)^n \tag{8}$$

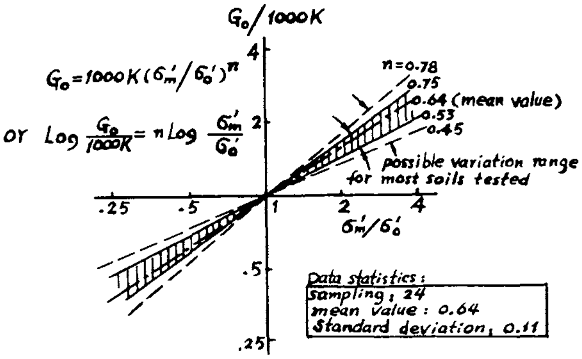



Fig.6. $G_o$/1000K vs. $\sigma'_m/\sigma'_o$

Which was first presented by Seed and Idriss (1970). In Eq.(8), $\sigma'_o$ is a pressure parameter which may be taken as unit pressure (1 kg/cm$^2$ or 1 ton/m$^2$, etc.); K and n are the parameters determined by tests. From Eq.(8) it can be seen that 1000K equals $G_o$ or K is equal to $G_o$/1000 for the case of $\sigma'_m = \sigma'_o$. It may be noted from Fig.6 that the value n runs only within a narrow range for different soils. Besides, tests found that the value n slightly increases with compressibility of soil. It is suggested that the value n may be taken as 0.6 for the purpose of engineering calculation.

It has been shown by tests that initial or elastic modulus $G_o$ essentially depends upon the mean consolidation pressure $\sigma'_m$ and the void ratio e for a given soil of normal consolidation or light overconsolidation. The test results are summed up in Fig.7.

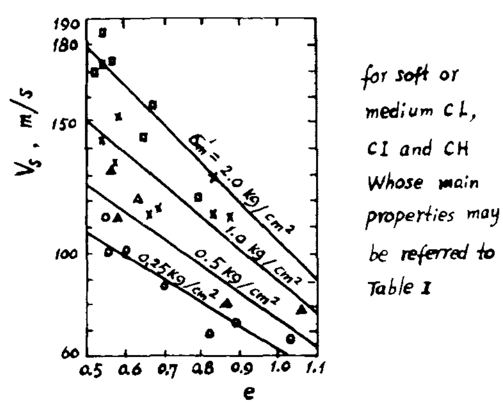



Fig.7. $V_s$ vs. e

From that we have

$$\underset{(m/s)}{V_s} = (212-124.4e)\ \underset{(kg/cm^2)}{(\sigma'_m)^{0.25}} \tag{9}$$

Since $G_o = \rho V_s^2$ ($\rho$ is the mass density of soil), the empirical formula used for evaluating initial shear modulus $G_o$ for soft or medium saturated cohesive soils may be given by

$$\underset{(kg/cm^2)}{G_o} = 158\, \underset{(g/cm^3)}{\gamma}(1.70-e)^2 (\underset{(kg/cm^2)}{\sigma'_m})^{0.5} \qquad 10(a)$$

or

$$\underset{(kg/cm^2)}{G_o} = 426\left(\frac{1+0.37e}{1+e}\right)(1.70-e)^2 (\underset{(kg/cm^2)}{\sigma'_m})^{0.5} \qquad 10(b)$$

in which $\gamma$ is the unit weight of soil. It should be emphasized that the value K in Eq.(8) corresponds to the void ratio at $\sigma'_o$ and Eq.(9) or (10) to the void ratio at $\sigma'_m$. For this reason the power of $\sigma'_m$ in Eq.(9) or (10) is different from that of $\sigma'_m / \sigma'_o$ in Eq.(8).

Results of $G_o/G$ vs. $\gamma$ are shown in Fig.8, from which the variation range of $\gamma_r$ for soils tested can be seen.

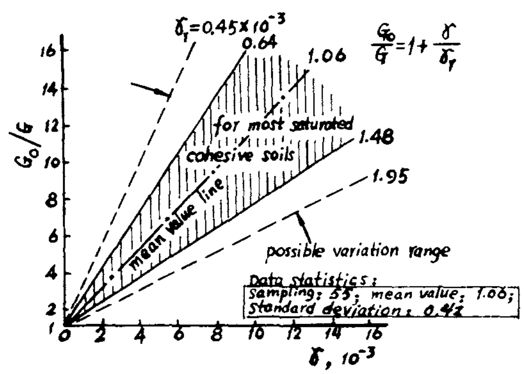


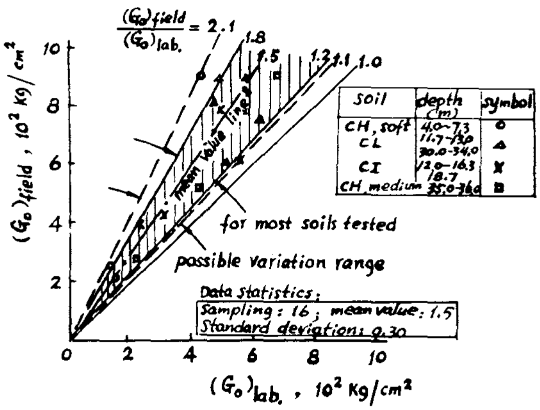

Fig.8. $G_o/G$ vs. $\gamma$

In regard to the value $\sigma_{max}$ or $\tau_{max}$ ($= \sigma_{max}/2$), it seems mainly dependent of consolidation pressure in the case of isotropic consolidation. Moreover, there is a linear relationship between both of them. This relationship for soils tested may be expressed

$$\sigma_{max} (\text{or } 2\ \tau_{max}) = 0.15 + 0.47\ \sigma'_m \qquad (11)$$

In comparison of the value $G_o$ based on in-situ shear wave velocity test with those from triaxial compression test, it shows that the field values are nearly always greater than the laboratory values, as shown in Fig.9. The ratio of both of them runs about from 1.2 to 1.8 and the mean ratio is about 1.5. Herein the laboratory values do not take "secondary time effects" into account; otherwise they will be closer to the field values.

Comparing values of $\tau_{max}$ from cyclic triaxial compression tests with those based on static triaxial compression tests, under the same levels of loading as well as the same conditions of "consolidation quick test", shows that the

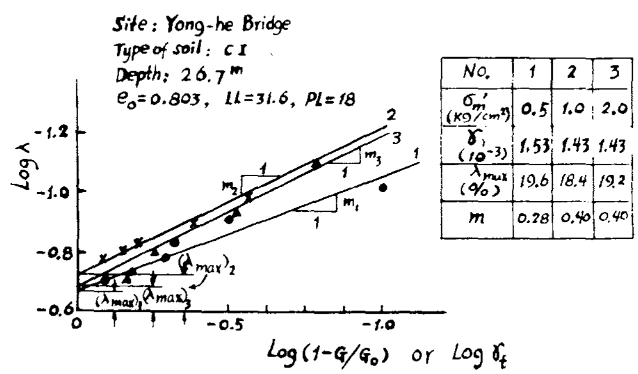



| Soil | depth (m) | symbol |
|---|---|---|
| CH, soft | 4.0~7.3 | o |
| CL | 11.7~13.0, 30.0~34.0 | ▲ |
| CI | 12.0~16.3, 18.7 | x |
| CH, medium | 35.0~36.0 | ■ |

Fig.9. $(G_o)$field vs. $(G_o)_{lab}$.

former seems always lower than the latter. The ratio of static value to dynamic value runs about from 1.2 to 14 and this ratio tends to the low value as plasticity of soil increases. Unfortunately, this test can not provide dynamic correction factor available for seismic design.

Typical results of variation of damping ratio with "transformed strain" are given in Fig.10.




| No. | 1 | 2 | 3 |
|---|---|---|---|
| $\sigma'_m$ (Kg/cm²) | 0.5 | 1.0 | 2.0 |
| $\gamma_r$ ($10^{-3}$) | 1.53 | 1.43 | 1.43 |
| $\lambda_{max}$ (%) | 19.6 | 18.4 | 19.2 |
| m | 0.28 | 0.40 | 0.40 |

Fig.10. Typical Results of $\lambda$ vs. $\gamma_t$

The data appear somewhat scattered, but for a particular soil it may be aproximately represented by a straight line in logarithmic plotting. Tests indicate that the value m runs in a considerably wide range, as shown by the shade zone in Fig.2 for different soils or consoildation pressures and that the value of $\lambda_{max}$ seems to be a relatively stable one. For soils tested, the value $\lambda_{max}$ may be taken as 20% for purpose of practical analyses.

It has been justified by most investigators that the strain-softening stress-strain relationship can be adequately represented by a hyperbolic curve. For the law of damping ratio varying with strain amplitude, however, considerable difference exists in the results from different authors. Fig.11 shows comparison of results of this testing with those from the other authors. It shows that the difference is appreciable but their tendency is in agreement.

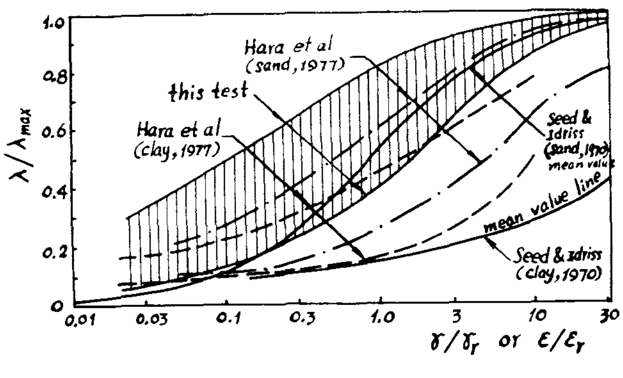



Fig.11. Comparison of Results about Variation of Damping Ratio with Strain

In cyclic triaxial compression test, the effective least strain is of the order of $1 \times 10^{-4}$ and soils begin to fail at cyclic strain of the order of $1 \times 10^{-2}$ or more for most saturated cohesive soils. On this basis the various conclusions mentioned above were drawn.

CONCLUSIONS

The strain-softening stress-strain relationship for a particular soil in a certain range of cyclic strain amplitude can be sufficiently represented by a hyperbolic curve. And thus variation of modulus with cyclic strain may be adequately expressed by an empirical equation.

The relationship between shear wave velocity and void ratio may approximately be represented by a straight line for a given consolidation pressure, and an empirical formula used in evaluating value of $G_o$ for undisturbed saturated cohesive soils has been suggested.

Parameter $\tau_{max}$ or $\sigma_{max}$ is mainly dependent of consolidation pressure under isotropic consolidation condition and an empirical relationship between $\sigma_{max}$ (or $\tau_{max}$) and mean consolidation pressure $\sigma_m'$ was offered.

Comparing the values of $G_o$ from in-situ shear wave velocity test with those from laboratory test shows that the former is nearly always greater than the latter and the ratio of both of them runs about from 1.2 to 1.8.

Tests show that the cyclic maximum shear stress $\tau_{max}$ is almost always lower than the static one, and the ratio of them runs in a considerably wide range.

An empirical equation concerning variation of damping with strain has been suggested. Tests indicate that the value m runs within a wide range and the value $\lambda_{max}$ seems to be considerably stable. For the practical purpose, the value $\lambda_{max}$ may be taken as 20% for most saturated cohesive soils.

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