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M. Ghazavi
K.N. Toosi University of Technology, Iran

M. Afzalirad
Islamic Azad University, Iran

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INVESTIGATION OF NON-UNIFORM PILE BEHAVIOUR UNDER TORSIONAL HARMONIC VIBRATION

M. Ghazavi
K.N. Toosi University of Technology
Tehran, Iran, E-mail: ghazavi_ma@kntu.ac.ir.

M. Afzalirad
Islamic Azad University, Ghaemshahr Branch
Ghaemshahr, Iran, E-mail: mobin_afzalirad@yahoo.com

ABSTRACT

Pile foundations for machines and structures are often subjected to horizontal or vertical harmonic vibrations. However, in some situations, piles also experience torsional harmonic vibrations. While there are some research work on tapered piles imposed to horizontal or vertical harmonic vibrations, the response of such piles till now is lacking. In this study, the elasto-dynamic theory has been used to derive the governing differential equation on a tapered pile experiencing torsional harmonic vibration. In this approach, the pile is assumed the pile has circular cross sectional area and consists of elastic material. It is further assumed that there is sufficient connection between the pile and the soil, so that the slippage cannot occur at the pile-soil contact surface. By using the developed method, the effect on the pile dynamic torsion amplitudes of the taper angle has been investigated. It has been found that the twist angle of the pile decreases with increasing the taper angle while the length and volume of the pile is kept constant. This reveals that the use of tapered piles leads to a better performance for foundations compared with the case of using straight-sided piles of the same length and volume.

INTRODUCTION

Pile foundations of machines and structures are usually under horizontal and vertical harmonic loads. They can also be subjected to torsional harmonic around their vertical axis. Although it has been paid some attention to the response results in horizontal and vertical axis, no investigation has been made to discover the torsional response of piles. Such foundations normally reduce permanent settlement and increase the foundation bearing capacity, therefore they can transfer machine vibration to high volume of soil. As a result, researchers basically focus on dynamic analysis of piles [Ghazavi and Bidgoli 2004].

Poulos (1975) has provided a theoretical analysis for torsional static response. Stole (1972) has performed static torsional tests on piles [Novak and Howell 1997]. By using analytical method based on liner elastic, Novak & Howell (1977) have gained some relations as mathematical closed form to calculate the pile torsional stiffness. Novak & Howell (1978) have extended his method for layered soil and with different characteristics. Rajapakse & Saha (1987) have expressed an elastic cylindrical harmonic response of bar with definite length in a half space, elastic and isotropic soil [Wang et al. 2008]. Militano & Rajapakse (1999) have reviewed a pile under torsion loadings and vertical harmonic [Cai et al. 2006]. Using taper piles has better performance than vertical piles which under dynamical loads has the same martial volume and its bearing capacity is much more too, but there is a little research on taper pile and diverse section especially under harmonic torsion loads.

Saha and Ghosh (1986) and Xie and Vaziri (1991) studied these kinds of piles under harmonic vertical loads. Ghazavi et al. (1997, 2004) have investigated the response of piles under vertical harmonic vibrations using finite element methods.

To the best knowledge of the authors, there is no research work on taper piles subjected to torsional vibration. This paper may be the first attempting to investigate this subject.

SUBJECT HYPOTHESIS

To investigate the torsional vibration effect on isolated pile, there are some assumptions:

1. Half space isotropic and liner viscoelastic soil are accompanied by hysterics damping.
2. Pile has been connected to soil perfectly.

3. Vertical pile is elastic and circular section, and section radius is considered across longitude variation.

PILE TORSION VIBRATION EMBEDDED IN SOIL

Based on the Newton second law, the governing differential equation for an element of the pile with size dz under torsional vibration (Fig. 1) is expressed as:

$$mr^2\frac{\ddot{\eta}}{2}.dz + c\dot{\eta}.dz = T - T + dT - \tau(2\pi r)r.dz \qquad (1)$$

$$mr^2\frac{\ddot{\eta}}{2}.dz + c\dot{\eta}.dz - dT + \tau(2\pi r)r.dz = 0 \qquad (2)$$

$$mr^2\frac{\ddot{\eta}}{2} = \frac{mr^2}{2}\frac{\partial^2\eta}{\partial t^2} = \frac{\rho V r^2}{2}\frac{\partial^2\eta}{\partial t^2} = \rho\frac{\pi r^4}{2}\frac{\partial^2\eta}{\partial t^2} = \rho j\frac{\partial^2\eta}{\partial t^2} \qquad (3)$$

where r is the pile radius, T is torque, ρ is pile martial density, V is pile volume, m is pile mass, c is damping coefficient, J is the polar moment of circular section and η is the amplitude of pile rotation at depth z
The above equations have been derived considering torsional moment equilibrium around the vertical axis of the pile.

Using Eq. (3) instead of Eq. (2) gives:

$$\rho j\frac{\partial^2\eta}{\partial t^2} + c\frac{\partial\eta}{\partial z} - \frac{\partial T}{\partial z} + \tau(2\pi r)r = 0 \qquad (4)$$

The torque-twist correlation under torsional loading is expressed as:

$$T = G_P\, j\frac{\partial\eta}{\partial z} \Rightarrow \frac{\partial T}{\partial z} = \frac{d}{dz}(G_P\, j\frac{\partial\eta}{\partial z}) = G_P\, j\frac{\partial^2\eta}{\partial z^2} \qquad (5)$$

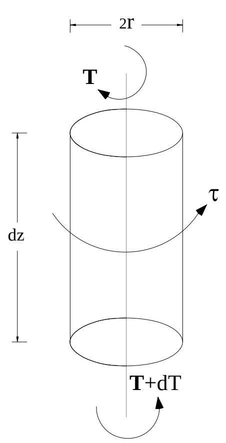

Fig 1: A pile element subjected to torque

Substituting Eq. (5) into Eq. (2) yields:

$$\rho_p\, j\frac{\partial^2\eta}{\partial t^2} + c\frac{\partial\eta}{\partial z} - G_p\, j\frac{\partial^2\eta}{\partial z^2} + \tau(2\pi r^2) = 0 \qquad (6)$$

where ρ_p is pile martial density, G_P: is pile material elastic shear module.
Novak (1974) and Novak & Howell (1978) suggested the following expression for the soil response to the torsional motion of a pile length of dz:

$$Gr^2(S_{\eta 1} + iS_{\eta 2})\eta(z,t)dz \qquad (7)$$

where G: is shear modules of the soil, $S_{\eta 1}$: is reaction stiffness parameter, $S_{\eta 2}$: soil reaction damping parameter and $S_{\eta 1}$, $S_{\eta 2}$ parameters are explained by:

$$S_{\eta 1}(a_0) = 2\pi(2 - a_0\frac{J_0(a_0)J_1(a_0) + Y_0(a_0)Y_1(a_0)}{J_1^2(a_0) + Y_1^2(a_0)}) \qquad (8)$$

$$S_{\eta 2}(a_0) = \frac{4}{J_1^2(a_0) + Y_1^2(a_0)} \qquad (9)$$

$$a_0 = \frac{r\varpi}{V_S}$$

(10)

where: a_0 is the dimensionless frequency, ϖ: is circular frequency, $V_S = \sqrt{G/\rho}$ shear wave velocity, $J_0(a_0)$ and $J_1(a_0)$ are Bessel functions of the first kind of order zero and one, respectively. $Y_0(a_0)$ and $Y_1(a_0)$ are Bessel functions of the second kind of order zero and one, respectively. Considering the damping hysterics, Eq. (7) can be written as:

$$G^* r^2(S_{\eta 1} + iS_{\eta 2})\eta(z,t)dz \qquad (11)$$

where:

$$G^* = G_1 + iG_2 \qquad (12)$$

$$\delta = G_1 / G_2 \qquad (13)$$

$$G^* = G_1(1 + i\tan\delta) \qquad (14)$$

where G_1 is real part and G_2 is imaginary part of shear modulus; G^* complex soil shear modulus, δ is loss angle, and $i = \sqrt{-1}$.
Liner hysterics model is common in mathematical and analytical calculations. By substituting Eq. (11) into Eq. (6), one can get:

$$\rho_p\, j\frac{\partial^2\eta(z,t)}{\partial t^2} + c\frac{\partial\eta(z,t)}{\partial z} - G_p\, j\frac{\partial^2\eta(z,t)}{\partial z^2} + G^* r^2(S_{W1} + iS_{W2})\eta(z,t) = 0$$

(15)

The above expressions presented so far are valid cylindrical piles with constant cross sections. Therefore, they must be modified for tapered piles.

TORSIONAL VIBRATION OF NON-UNIFORM PILES

Suppose a pile element with a size of dz size of a taper pile embedded in soil and subjected to the torsional vibration as shown in Fig. 2. The equilibrium equation for torsion around the pile vertical axis gives:

$$mr^2\frac{\ddot{\eta}}{2}.dz + c\dot{\eta}.dz - dT + \tau(2\pi r)r.\frac{dz}{\cos\beta} = 0 \qquad (16)$$

Eq. (16) is derived using the Newton second law.
Eq. (2) is used herein for taper piles. This gives:

$$T = Gj\frac{\partial\eta(z,t)}{\partial z} \qquad (17)$$

where for taper pile, the polar moment section at a given point of the pile shaft is $j = \pi r_{(z)}^4/2$ where $r_{(z)}$ is the pile radius at z from the pile head as shown in Fig. 2 and given by:

$$r_{(z)} = r_1 + (L - Z)\tan\beta \qquad (18)$$

or:

$$r_m = r_{(\frac{L}{2})} = r_1 + (L - \frac{L}{2})\tan\beta = r_1 + (\frac{L}{2})\tan\beta$$
$$r_1 = r_m - (\frac{L}{2})\tan\beta \qquad (19)$$
$$r_{(Z)} = r_m + (\frac{L}{2} - Z)\tan\beta$$

Substituting Eq. (17) into Eq. (5) gives:

$$T = \frac{\pi}{2}r_{(z)}^4 G_P\frac{\partial\eta(z,t)}{\partial z} \qquad (20)$$
$$\frac{\partial T}{\partial z} = \frac{\pi}{2}r_{(z)}^4 G_P\frac{\partial^2\eta(z,t)}{\partial z^2} + 2\pi G_P(-\tan\beta)r_{(z)}^3\frac{\partial\eta(z,t)}{\partial z} \qquad (21)$$

where L is the length of pile, r_1 is the pile toe radius, r_2 is the pile head radius, β is the tape angle with respect to the vertical axis, and r_m is the pile radius at Z =L/2).

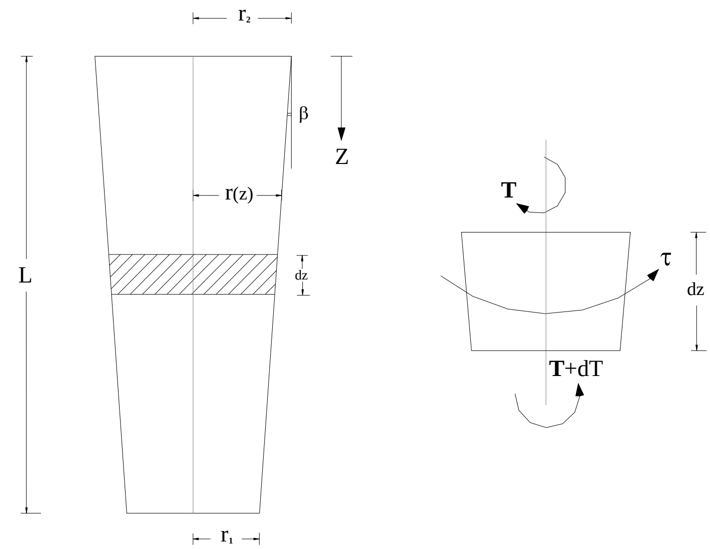

Fig 2: Non-uniform pile under torque

Substituting Eqs. (11) and (12) into Eq. (4) gives:

$$\rho_p\frac{\pi}{2}r_{(z)}^4\frac{\partial^2\eta(z,t)}{\partial t^2} + c\frac{\partial\eta(z,t)}{\partial t} + 2\pi G_P\tan\beta.r_{(z)}^3\frac{\partial\eta(z,t)}{\partial z} \qquad (22)$$
$$-\frac{\pi}{2}r_{(z)}^4 G_P\frac{\partial^2\eta(z,t)}{\partial z^2} + \frac{G^* r^2}{\cos\beta}(S_{\eta1} + iS_{\eta2})\eta(z,t) = 0$$

The angle of twist of the pile can be decomposed as:

$$\eta(z,t) = \eta(z)e^{i\varpi t} \qquad (23)$$

where η(z) is the complex amplitude of pile rotation at depth z, ω is circular frequency and t represents time. Considering Eq. (23) gives:

$$\frac{\partial\eta(z,t)}{\partial t} = \eta(z)i\varpi e^{i\varpi t} \qquad (24)$$
$$\frac{\partial^2\eta(z,t)}{\partial t^2} = \eta(z)i^2\varpi^2 e^{i\varpi t} = -\eta(z)\varpi^2 e^{i\varpi t} \qquad (25)$$

$$\frac{\partial\eta(z,t)}{\partial z} = \frac{d\eta(z)}{dz}e^{i\varpi t} \qquad (26)$$

$$\frac{\partial^2\eta(z,t)}{\partial z^2} = \frac{d^2\eta(z)}{dz^2}e^{i\varpi t} \qquad (27)$$

Eqs. (24) to (27) can further give:

$$\rho_p\frac{\pi}{2}r_{(z)}^4[-\eta(z)\varpi^2 e^{i\varpi t}] + c[\eta(z)i\varpi e^{i\varpi t}] + 2\pi G_P\tan\beta.r_{(z)}^3[\frac{d\eta(z)}{dz}e^{i\varpi t}]$$
$$-\frac{\pi}{2}r_{(z)}^4 G_P\frac{d^2\eta(z)}{dz^2}e^{i\varpi t} + \frac{G^* r_{(z)}{}^2}{\cos\beta}(S_{\eta1} + iS_{\eta2})\eta(z)e^{i\varpi t} = 0$$
$$(28)$$

$$\rho_P \frac{\pi}{2} r_{(z)}^4[-\eta(z)\varpi^2] + c[\eta(z) i\varpi] + 2\pi G_P \tan\beta . r_{(z)}^3 [\frac{d\eta(z)}{dz}]$$
$$-\frac{\pi}{2} r_{(z)}^4 G_P \frac{d^2\eta(z)}{dz^2} + \frac{G^* r_{(z)}{}^2}{\cos\beta}(S_{\eta 1}+iS_{\eta 2})\eta(z)=0$$
(29)

$$2\pi G_P \tan\beta . r_{(z)}^3 \frac{d\eta(z)}{dz} - \frac{\pi}{2} r_{(z)}^4 G_P \frac{d^2\eta(z)}{dz^2} + \left[\frac{G^* r_{(z)}{}^2}{\cos\beta}(S_{\eta 1}+iS_{\eta 2}) - \rho_P \frac{\pi}{2} r_{(z)}^4 \varpi^2 + ci\varpi\right]\eta(z)=0 \quad (30)$$

$$2\pi G_P \tan\beta . r_{(z)} \frac{d\eta(z)}{dz} - \frac{\pi}{2} r_{(z)}^2 G_P \frac{d^2\eta(z)}{dz^2} + \left[\frac{G^*}{\cos\beta}(S_{\eta 1}+iS_{\eta 2}) - \rho_P \frac{\pi}{2} r_{(z)}^2 \varpi^2 + \frac{ci\varpi}{r_{(z)}^2}\right]\eta(z)=0 \quad (31)$$

This equation may be solved using numerical approach such as finite difference.

SOLUTION OF DIFFERENTIAL EQUATION BY FINITE DIFFRENCE METHOD

The finite difference technique is used to solve Eq. (31). This method replaces partial differential equation supervisor and equations which defines limited situations with finite differential equations [Smith 1978, Chapra 1998]. The use of finite difference method gives partial derivations as:

$$(\frac{\partial^2\eta}{\partial z^2})_0 = \frac{\eta_2+\eta_4-2\eta_0}{\Delta z^2} \quad (32)$$

$$(\frac{\partial\eta}{\partial z})_0 = \frac{\eta_2-\eta_0}{\Delta z} \quad (33)$$

Eqs. (32) and (33) can be substituted into Eq. (31). It is also assumed that the end part of the pile is embedded in a stiff layer, thus the end condition should be considered fixed. This situation is simulated by assuming:

$$\eta(0)=1 \quad (34)$$
$$\eta(l)=0 \quad (35)$$

PARAMETRIC ANALYSIS

Two piles having slenderness ratios of 20 and 100 are considered in this section. Figs. 3 to 12 show the results. The slenderness ratio, L/r_m, is defined as the pile length divided by the pile average radius. Table 1 presents the soil and pile characteristics.

Table 1. Soil and pile characteristics

Soil density (ρ_s) (kg/m^3)	Pile density (ρ_p) (kg/m^3)	Damping coefficient of pile (c)
1800	2500	0.05
Average radius r_m (m)	Circular frequency ϖ	Damping coefficient of soil $tag\delta$
0.50	2	0.00

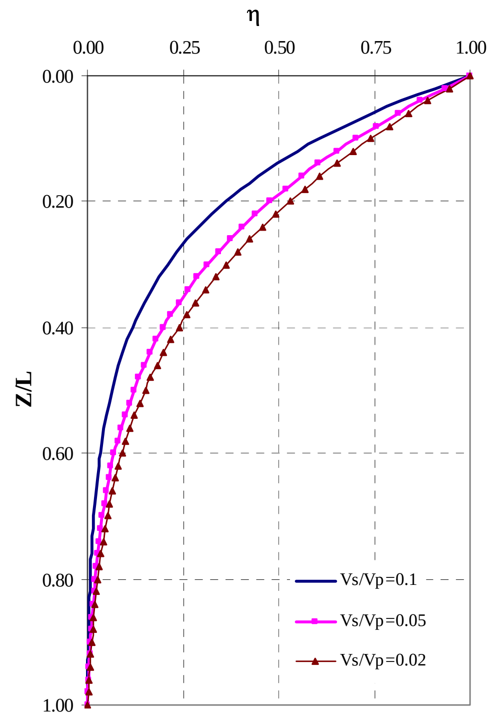

Fig. 3. Variation of pile rotation amplitude with depth for various shear wave velocity ratio V_S/V_P (slenderness ratio=20, taper angle = 1^o, $\tan\delta$ =0.0).

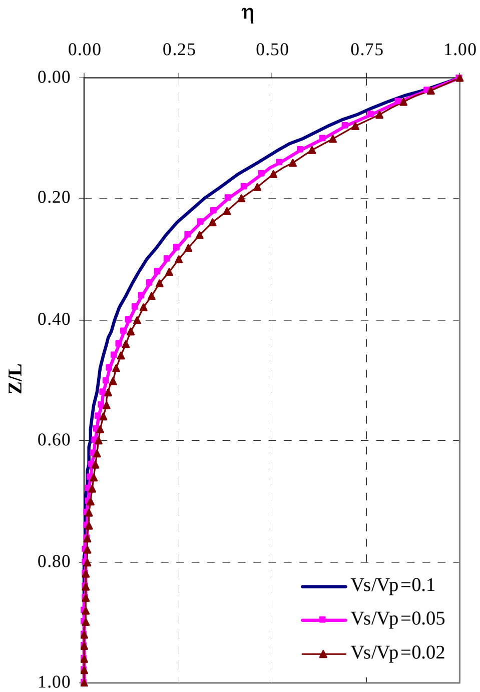

Fig. 4. Variation of pile rotation amplitude with depth for various shear wave velocity ratio V_S/V (slenderness ratio=20, taper angle = 3^o, $\tan\delta$ =0.0.

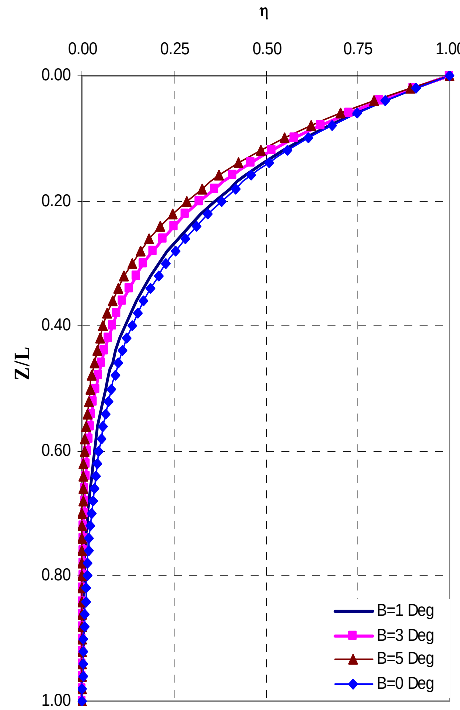

Fig. 6. Variation of pile rotation amplitude with depth for various taper angle (V_S/V_P=0.1, slenderness ratio=20, $\tan\delta$ =0.0)

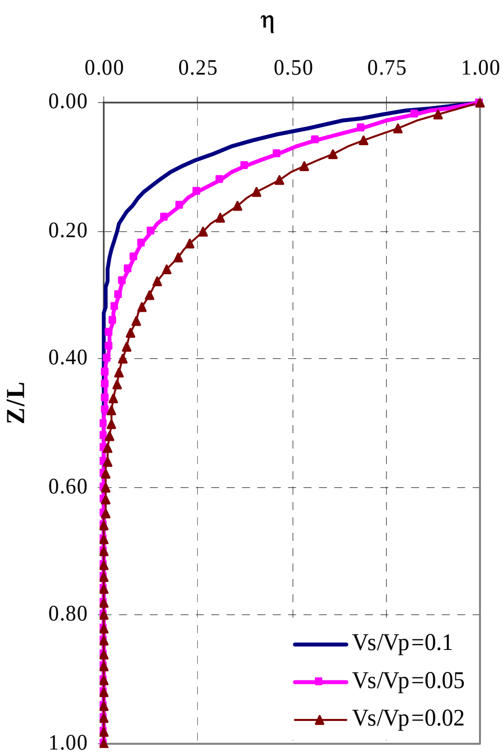

Fig. 5. Variation of pile rotation amplitude with depth for various V_S/V_P (slenderness ratio=100, taper angle = 1^o, $\tan\delta$ =0.0)

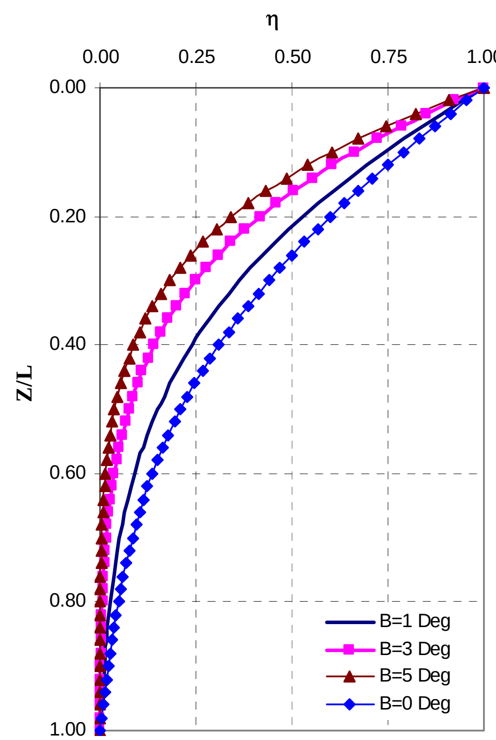

Fig. 7. Variation of pile rotation amplitude with depth for various taper angle (V_S/V_P=0.05, slenderness ratio=20, $\tan\delta$ =0.0)

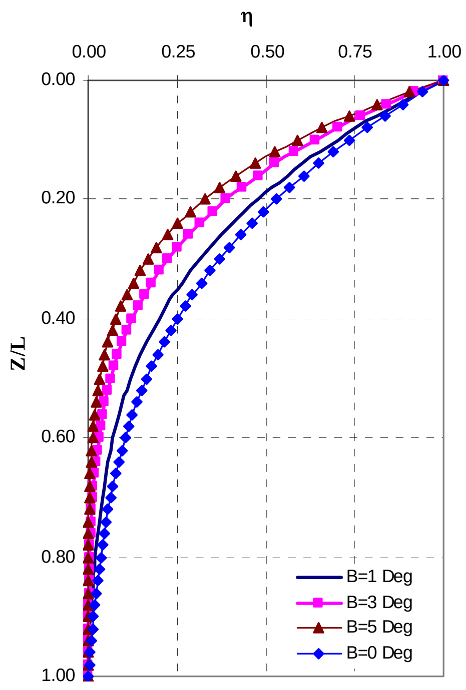

Fig. 8. Variation of pile rotation amplitude with depth for various taper angle (V_S/V_P=0.02, slenderness ratio=20, $\tan\delta$ =0.0)

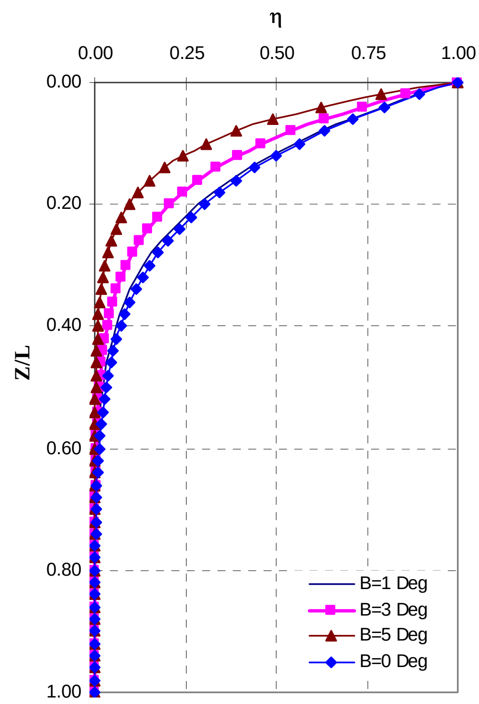

Fig. 10. Variation of pile rotation amplitude with depth for various taper angle (V_S/V_P=0.10, slenderness ratio=100, $\tan\delta$ =0.0)

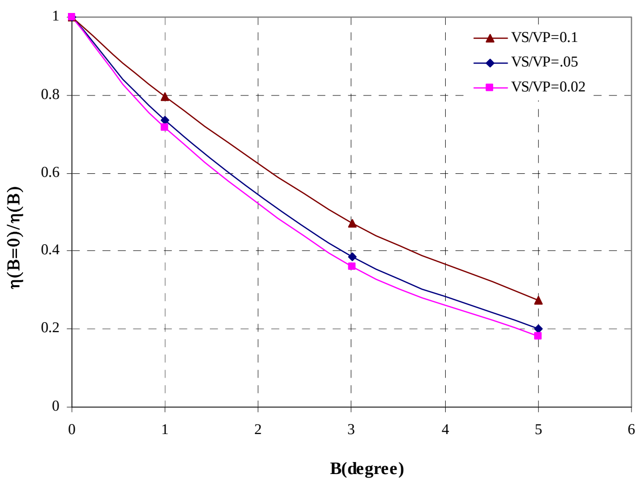

Fig. 9. Variation of normalized amplitude of pile rotation with taper angle (slenderness ratio=20m, $\tan\delta = 0.0$)

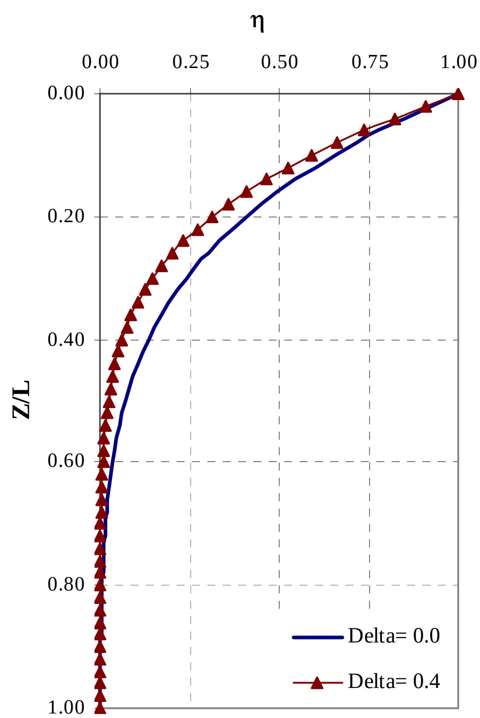

Fig. 11. Variation of amplitude of pile rotation with depth for various delta (V_S/V_P=0.10, slenderness ratio=20, $\tan\delta$ =0.0, 0.40)

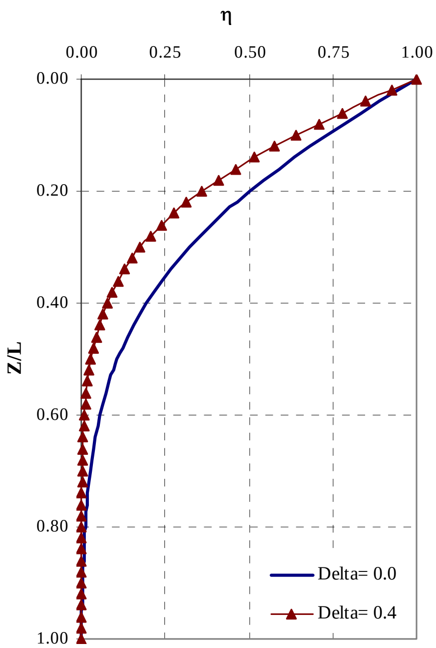

Fig. 12. Variation of amplitude of pile rotation with depth for various delta (V_S/V_P=0.05, slenderness ratio=20, $\tan\delta$ =0.0, 0.40)

Figs. 3 to 5 show the variation of amplitude with velocity ratio (soil stiffness). It can be seen that the pile twist amplitude quickly diminishes with increasing the depth. It also varies with velocity ratio and the taper angle. The amplitude of the pile rotation decreases with increasing the wave velocity ratio.

Figs. 6 to 10 illustrate the variation of the pile rotation amplitude with depth, V_S/V_P, slenderness ratio, taper angle, and δ. The amplitude is considerably dependent on the taper angle. As seen, the rotation amplitude of the pile decreases with increasing the taper angle. The effect of taper angle on the pile rotational amplitude is shown Fig 9, as well

Figs. 5 and 10 indicate that for a pile having a slenderness ratio of 100, if the pile toe is fixed, the pile head twist for Z/L greater than 0.4 tends to approach zero, whereas in for a pile having a slenderness ratio of 20, the pile head twist angle approaches zero at Z/L greater than 0.6 (Figs. 3 to 6).

The effect of the material damping of the soil is shown in Figs. 11 and 12. As observed, with increasing the soil damping, the twist amplitude of the pile decreases.

CONCLUSIONS

A simple approach has been presented for tapered piles subjected to torsional harmonic vibrations. The main conclusions may be summarized as follow:

- Taper piles offer less twist angle under torsional harmonic vibration than straight sided piles of the same volume and length.
- With increasing the shear wave velocity of the soil, the tapered pile twist angle decreases.
- With increasing the soil damping, the pile twist angle decreases.
- With increasing the taper angle, the pile torsion decreases.
- For a pile having a slenderness ratio of 100, if the pile toe is fixed, pile head twist angle for Z/L greater than 0.4 tends to approach zero, whereas in for a pile having a slenderness ratio of 20, the pile head twist angle approaches zero at Z/L greater than 0.6.

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