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ABSTRACT

In this paper, the characteristics of the development of effective stresses and pore water pressure in elastic or elasto-plastic seabed with time under a sequence of wave loading are studied by virtue of numerical finite element analyses. Compared with analytical solution for elastic problem, the numerical procedure proposed can deal with complicated situation of seabed and can predict residual pore pressure. For inelastic seabed, bounding-surface hypoplasticity model is employed to describe the nonlinear stress-strain relationship of sand under cyclic loading. The development pattern of pore pressure under undrained condition obtained from laboratory experiments is not necessary in the numerical simulation.

INTRODUCTION

The analysis of stability and liquefaction potential of foundations are of practical significance in the design of offshore and coastal structures, such as oil platforms, breakwaters and pipelines, under wave-induced random loading. It has been received an intensive investigation in the past three decades. When waves propagate over the ocean surface, a sequence of water pressure is produced on the mudline or seafloor, which causes cyclic variations of effective stress and build-up, accumulation, partial drainage and re-distribution of excess pore pressure in the seabed. If the stormy waves last a long period, even liquefaction of porous deposits would occur. Generally, the wave-induced pore water pressure consists of two parts, i.e., transient pressure and residual or progressive pressure. In the past, a number of analytical and numerical investigations were focused on the elastic seabed response under progressive waves, standing waves or short-crested waves (Hsu and Jeng 1994; Jeng and Hsu 1996; Madsen 1978; Mei and Foda 1981; Thomas 1989, 1995; Yamamoto 1978). Based on different assumptions on permeability and thickness of seabed, the rigidity of footing and compressibility of pore water, a number of solutions have been obtained.

In this paper, the generalized formulation of two-dimensional Biot's theory of dynamic consolidation (Zienkiewicz *et al.* 1984) in conjunction with finite element method is employed to evaluate dynamic response of sandy seabed subjected to wave loading. The characteristics of the development of effective stresses and pore water pressure in elastic or elasto-plastic seabed with time under a sequence of wave loading are studied based on numerical computations. A comparative study with available analytical solutions for a certain elasticity problems is made to show the potential of elasto-plastic analysis in handling complicated situation of seabed and predicting residual pore pressure. It has been manifested by experimental studies that the principal stress direction of seabed soils under cyclic loading imposed by ocean waves is continuously changed. Traditional plasticity models are incapable to reproduce the real behavior of soils with the rotation of principal stress. Therefore, bounding-surface hypo-plasticity model (Wang *et al.* 1990) is utilized to describe dynamic nonlinear stress-strain relationship of sand. It is found that the model of pore pressure development under undrained condition is not essential.

FINITE ELEMENT FORMUALTION AND NUMERICAL METHOD OF GENERALIZED BOIT'S EQUATIONS

Governing Equations for Boundary-Value Problem

For a soil element subjected to wave loading, it is assumed that (1) The marine sediment is saturated, cohesionless, hydraulically transient and the flow in the porous seabed is governed by D'arcy law, and (2) The pore fluid is compressible while the compressibility of soil particle is overlooked. Then the generalized Boit's equation of dynamic consolidation presented by Zienkiewicz *et al.* (1984) is used. The overall equilibrium equation of the soil can be formulated

$$\sigma'_{ij,j} - p_{,i} - \rho \ddot{u}_i - \rho_f \ddot{w}_i = 0 \tag{1}$$

The equilibrium condition of fluid flow can be written as

$$-p_{,i} - \gamma_f k_{ij}^{-1} \dot{w}_j - \rho_f (\ddot{u}_i + \ddot{w}/n) = 0 \tag{2}$$

Mass balance condition of flow is given

$$\dot{u}_{i,i} + \dot{w}_{i,i} + \dot{p}/Q = 0 \tag{3}$$

Constitutive law of soil can be expressed as below

$$\mathrm{d}\sigma'_{ij} = D_{ijkl}\mathrm{d}\varepsilon_{kl} \tag{4}$$

in which σ'_{ij} is the effective stress of soil (tension positive), p is the pore pressure, u_i is the average displacement of the solid matrix, w_i is the average relative displacement of fluid with respect to solid skeleton, k_{ij} is known as the permeability matrix, generally it is anisotropic in x- and z-directions, $Q = k_f / n$, k_f is the bulk modulus of fluid and n is the porosity of soil. For convenience, it is specified that the change of porosity in the whole course can be neglected, ρ and ρ_f are the density of soil and pore fluid respectively, $\gamma_f = \rho_f g$, g is the gravity acceleration, D_{ijkl} is the tangent modulus matrix, which is defined by the constitutive relation used. The total displacement of pore fluid is defined as

$$U_i = u_i + w_i / n \tag{5}$$

Substituting Equation. (5) into Equations (1) and (2) leads to

$$\sigma'_{ij},_j - p_{,i} - \rho\ddot{u}_i + n\rho_f\left(\ddot{U}_i - \ddot{u}_i\right) = 0 \tag{6}$$

$$-p_{,i} - \gamma_f k_{ij}^{-1} n(\dot{U}_j - \dot{u}_j) - \rho_f\ddot{U}_i = 0 \tag{7}$$

Subtracting n times of Equation (7) from Equation (6) yields

$$\sigma'_{ij},_j - (1-n)p_{,i} + (n\rho_f - \rho)\ddot{u}_i + n^2\gamma_f k_{ij}^{-1}\left(\dot{U}_j - \dot{u}_j\right) = 0 \tag{8}$$

While initial pore pressure is not taken into consideration, integrating Equation (3) gives

$$p = -Q\left[nU_{i,i} + (1-n)u_{i,i}\right] \tag{9}$$

Considering Equation (9), Equations (8) and (7) can be rewritten as

$$\begin{aligned}&\sigma'_{ij},_j + n(1-n)QU_{m,mi} + (1-n)^2 Qu_{m,mi}\\ &+ (n\rho_f - \rho)\ddot{u}_i + n^2\gamma_f k_{ij}^{-1}\left(\dot{U}_j - \dot{u}_j\right) = 0\end{aligned} \tag{10}$$

$$n^2 QU_{m,mi} + n(1-n)Qu_{m,mi} - \gamma_f k_{ij}^{-1} n(\dot{U}_j - \dot{u}_j) - \rho_f\ddot{U}_i = 0 \tag{11}$$

This set of simultaneous equations is the $u \sim U$ form of generalized formulations of Boit's dynamic consolidation equations. Accordingly, the finite element formulation can be developed by applying Galerkin's weighted-residual procedure. The following interpolation schemes of u and U are introduced

$$u_i = N_k^u \tilde{u}_{ki}, \quad U_i = N_k^U \tilde{U}_{ki}$$

where N_k^u and N_k^U respectively represent the shape function for variables u and U. Applying Galerkin's weighted-residual procedure to Equations (10) and (11) leads to numerical formulation of finite element method (FEM)

$$\begin{aligned}&\int_A N_I^u\left[\sigma'_{ij},_j + n(1-n)QU_{m,mi} + (1-n)^2 Qu_{m,mi}\right]\mathrm{d}A\\ &+ \int_A N^u\left[(n\rho_f - \rho)\ddot{u}_i + n^2\gamma_f k_{ij}^{-1}\left(\dot{U}_j - \dot{u}_j\right)\right]\mathrm{d}A = 0\end{aligned} \tag{12}$$

$$\begin{aligned}&\int_A N_I^U\left[n^2 QU_{m,mi} + n(1-n)Qu_{m,mi}\right]\mathrm{d}A\\ &- \int_A N_I^U\left[\gamma_f k_{ij}^{-1} n(\dot{U}_j - \dot{u}_j) + \rho_f\ddot{U}_i\right]\mathrm{d}A = 0\end{aligned} \tag{13}$$

On basis of Equations (12) and (13), the matrix form of finite element formulation can be obtained through integration as below

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{Bmatrix} \ddot{\tilde{u}} \\ \ddot{\tilde{U}} \end{Bmatrix} + \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix}\begin{Bmatrix} \dot{\tilde{u}} \\ \dot{\tilde{U}} \end{Bmatrix} + \begin{bmatrix} K_1 & K_2 \\ K_3 & K_4 \end{bmatrix}\begin{Bmatrix} \tilde{u} \\ \tilde{U} \end{Bmatrix} + \begin{Bmatrix} R_1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \tag{14}$$

where

$K_f = \begin{bmatrix} k_x^{-1} & 0 \\ 0 & k_z^{-1} \end{bmatrix}$, $\quad M_1 = (\rho - n\rho_f)\int_A N_I^u N_k^u \mathrm{d}A$,

$M_2 = n\rho_f \int_A N_I^U N_k^U \mathrm{d}A$, $\quad C_1 = n^2\gamma_f \int_A N_I^u K_f N_k^u \mathrm{d}A$,

$C_2 = -n^2\gamma_f \int_A N_I^u K_f N_k^U \mathrm{d}A$, $\quad C_3 = (C_2)^{\mathrm{T}}$,

$C_4 = n^2\gamma_f \int_A N_I^U K_f \mathrm{d}A$, $\quad K_1 = (1-n)^2 Q\int_A N_{I,i}^u N_{k,j}^u \mathrm{d}A$,

$K_2 = (1-n)nQ\int_A N_{I,i}^u N_{k,j}^U \mathrm{d}A$, $\quad K_3 = (K_2)^{\mathrm{T}}$,

$K_4 = n^2 Q\int_A N_{I,i}^U N_{k,j}^U \mathrm{d}A$ $\quad R_1 = \int_A N_{I,j}^u \sigma'_{ij} \mathrm{d}A$,

$F_1 = \oint N_I^u(\sigma_n + np_n)\mathrm{d}s$, $\quad F_2 = -n\oint N_I^U p_n \mathrm{d}s$

Bounding-Surface Hypoplasticity Model of Sand

Linear elastic stress-strain relation of soils is widely used to elastic analysis of dynamic response of seabed under wave loading. However soils of seabed mostly display nonlinear or inelastic deformation behavior under cyclic or/and transient loading. Therefore, elasto-plastic dynamic constitutive relationship for sand should be applied for evaluating nonlinear dynamic response of seafloor. Bounding-surface hypoplasticity model developed by Wang *et al.* (1990) is capable to simulate behavior of sandy soils under complicated sequence of loading with continuous rotation of principal stress direction. It is featured that unlike conventional elasto-plastic models, the plastic strain rate depends not only on the current state of stress, but also on the stress increments. This modification makes successful simulation under rotational shear possible. In addition, instead of the concept of plastic yield point, it is postulated that plastic strain will occur from the beginning of loading. This postulation will simplify dynamic calculation since the determination of yield point in each element is not necessary. Bounding-surface hypoplasticity model has been illustrated by Wang *et al.* (1990). The incremental elasto-plastic stiffness matrix could be written as following

$$D_{ijkl} = E_{ijkl} - \frac{P_{ij}^r Q_{kl}^p - P_{ij}^p Q_{kl}^r}{A_r B_p - A_p B_r}$$

$$E_{ijkl} = K\delta_{ij}\delta_{kl} + G\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\right)$$

$$P_{ij}^{r} = \frac{2G}{H_r} n_{Dij} + \frac{K}{K_r} \delta_{ij}$$

$$P_{ij}^{p} = \left(\frac{2G}{H_p} r_{ij} + \frac{K}{K_p} \delta_{ij} \right) h(p - p_m) h(dp)$$

$$Q_{kl}^{r} = A_p n_{N_{kl}} - A_r \delta_{kl}, \qquad Q_{kl}^{p} = B_p n_{N_{kl}} - B_r \delta_{kl}$$

$$A_r = \frac{1}{2G} + \frac{1}{H_r} n_{D_{ij}} n_{N_{ij}}, \; B_p = \frac{1}{K} + \frac{1}{K_p} h(p - p_m) h(dp)$$

$$B_r = \left[\frac{1}{2G} + \frac{1}{H_p} h(p - p_m) h(dp) \right] r_{ij} n_{N_{ij}}, \; A_p = \frac{1}{K_r}$$

where G, K, H_r, H_p, K_r, K_p, n_N, n_D are the model parameters which are illustrated by Wang *et al.* (1990).

Dynamic Computational Procedure

The finite element equations of the system can be rewritten in the following compacted form

$$[M]\{\ddot{D}\} + [C]\{\dot{D}\} + [K]\{D\} + \{R\} = \{F\} \tag{15}$$

This equation can be numerically solved by step-by-step integration procedure in time domain. Using Newmark's time integration scheme, Equation (15) will be reduced to

$$\left[\tilde{K}\right]\{\Delta D\}_{t+\Delta t} = \left\{\Delta \tilde{F}\right\}_{t+\Delta t} \tag{16}$$

where $\left[\tilde{K}\right]$ and $\left\{\Delta \tilde{F}\right\}_{t+\Delta t}$ are respectively the effective stiffness matrix and effective load vector of the system at time t, $\{\Delta D\}_{t+\Delta t}$ is the incremental generalized displacement vector form time t to time $t + \Delta t$,

$$\{\Delta D\}_{t+\Delta t} = \{D\}_{t+\Delta t} - \{D\}_t, \quad \left[\tilde{K}\right] = a_1[M] + a_4[C] + [K] + [K_0]$$

$$\left\{\Delta \tilde{F}\right\}_{t+\Delta t} = \{F\}_{t+\Delta t} - \{R\}_t - [K]\{D\}_t - [M]\left(- a_1\{D\}_t + a_2\{\dot{D}\}_t + a_3\{\ddot{D}\}_t\right) - [C]\left(- a_4\{D\}_t + a_5\{\dot{D}\}_t + a_6\{\ddot{D}\}_t\right)$$

in which

$$a_1 = \frac{1}{\alpha \Delta t^2}, \quad a_2 = -\frac{1}{\alpha \Delta t}, \quad a_3 = 1 - \frac{1}{2\alpha}, \quad a_4 = \frac{\beta}{\alpha \Delta t},$$

$$a_5 = 1 - \frac{\beta}{\alpha}, \quad a_6 = \Delta t - \frac{\beta \Delta t}{2\alpha}$$

where α and β are Newmark's integration parameters. $[K_0]$ is defined by D_{ijkl} and explained by Wang *et al.* (1990). For bounding-surface hypoplasticity model, the tangent modulus coefficient matrix D_{ijkl} is nonlinearly related to strain rate. Therefore an iterative method for numerically solving Equation (16) is essential at each time interval. After the incremental generalized displacements $\{\Delta D\}_{t+\Delta t}$ are achieved from Equation (16), the velocity and acceleration of the system at time $t + \Delta t$ will be determined according to Newmark's integration scheme. Then the development of pore water pressure is easily to be defined through Equation (9). In fact the pore water pressure consists of two components, i.e., transient part and residual part. The numerical computations indicate that the used nonlinear iterative algorithm together with Newmark's time integration scheme is stable and convergent.

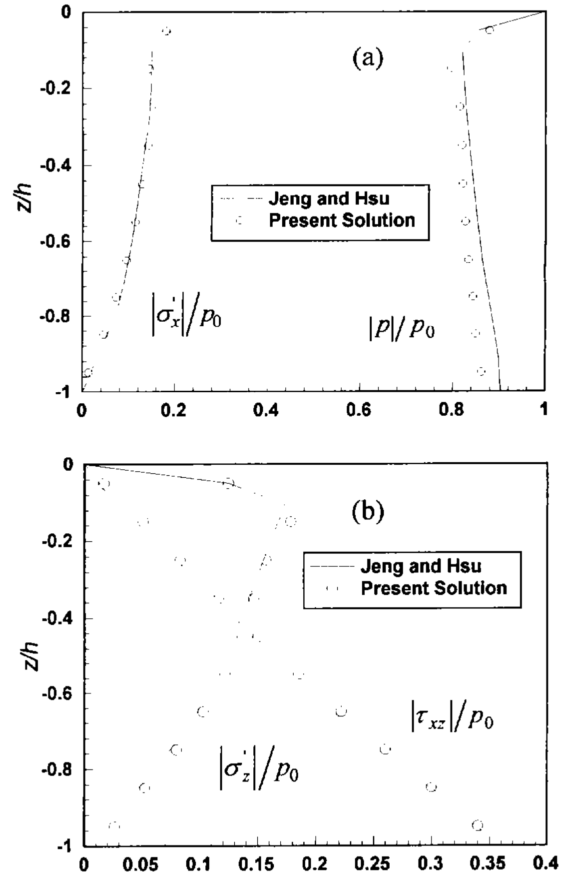

Fig. 1. Comparison of amplitudes of effective stresses, shear stress and pore pressure

NUMERICAL RESULTS AND COMPARATIVE STUDIES

The fundamental theory and computational technique mentioned above are numerically implemented based on finite element method. The numerical simulations of seabed subjected to continuous progressive and standing wave loading are made. The linear elastic solution and elasto-plastic analysis are achieved respectively based on elastic constitutive relationship and bounding-surface hypoplasticity model for a seabed consisting of sandy layer.

Verification of Numerical Solution

For verifying the numerical solution computed by the present theory and formulation, an idealized seabed (Jeng and Hsu 1996) is analyzed as an example. The seabed with thickness of

h =25m and water depth of d =70m is subjected to ocean wave loading characterized by the length of L =324m and period of T =15.0s of wave. Based on hydraulic dynamics, the amplitude of water pressure on mudline is given by $p_0 = \dfrac{\gamma_f H}{2\cosh(2\pi d/L)}$, where H is the wave height. The seabed is composed of fine sand with shear modulus of G =10MPa and Possion's ratio of $\nu = 1/3$. The porosity of soil is n =0.3 and permeability coefficients are $k_x = k_z = 10^{-4}$ m/s. For this elastic problem, an analytical solution was developed by Jeng and Hsu (1996) in which Biot's static theory of consolidation was used. Hence in the present numerical analysis, inertial components of both pore water and sand skeleton are overlooked in the governing equations, i.e., $a_1 = a_2 = a_3 = a_6 = 0$. Both analytical solutions given by Jeng and Hsu (1996) and numerical solutions are shown in Fig. 1. It can be observed that there is a slight difference of the distribution of normalized pore water pressure $|p|/p_0$ along depth between analytical solutions (Jeng and Hsu 1996) and numerical results. The numerical results of distribution of the amplitudes of effective stresses along depth agree well with the analytical solution (Jeng and Hsu 1996).

Pore Pressure Development and Rotation of Principal Stress in Elasto-Plastic Seabed

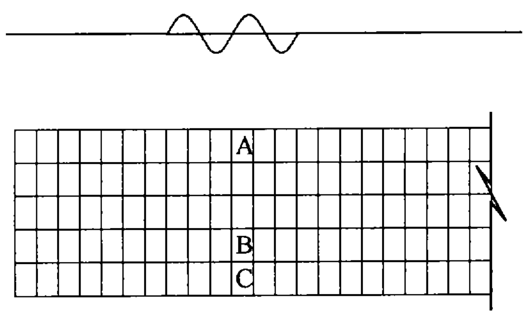

Fig. 2. Seabed subjected to progressive wave loading

While inelastic behavior of soils is considered, the seabed will display nonlinear characteristics in development of pore water pressure and displacement different from the elastic seabed. The seabed consisting of an uniform sand is analyzed by using bounding-surface hypoplasticity model. The parameters of wave are L =50.0m, H =3.0m, T =6.14s and the depth of water is d =10.0m. The finite element discretization is shown in Fig. 2. The constants of the bounding-surface hypoplasticity model are given by Wang *et al.* (1990), i.e., $e_{in} = 0.85$, $R_f^c = 1.2$, $c = 0.8$, $R_p^c = 0.813$, $G_0 = 70$, κ =0.01, $h_r^c = 0.28$, $h_r^e = 0.14$, $k_r^c = 0.272$, $k_r^e = 0.095$, $a = 1$, $b = 1$, $d_0 = 1.2$, $h_p = 35$, $\lambda = 0.02$, $\alpha = 0$, $z_D = 0.5$, $z_N = 0.85$ and permeability of sand is isotropic with $k_x = k_z = 10^{-5}$ m/s. The time integration parameters are chosen as Δt =0.2s, α =0.3, β =0.5. Loading duration is 20s (i.e., 100 time steps). The

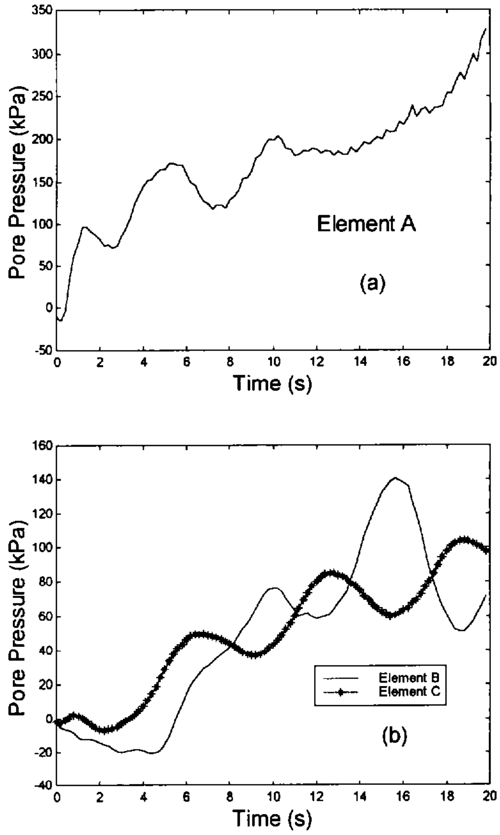

Fig. 3. Development of pore pressure in Element A, B and C

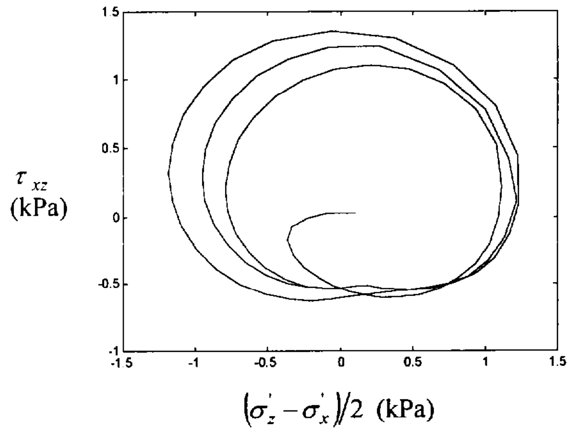

Fig. 4. Stress path in Element A

development of pore pressure near the seabed surface (e.g., Element A) and bottom (e.g., Element B and Element C) are shown in Fig. 3. As seen from Fig. 3, excess pore water pressure is accumulated in the whole seabed quickly. At the beginning of loading, negative pore water pressure appears in the three elements, depending on location of the elements. The re-distribution of residual pore water pressure occurs obviously after several wave loading.

It has been recognized that at a fixed depth of elastic seabed under progressive loading, the maximum devetoric stress, i.e., $\frac{\sigma_1 - \sigma_3}{2} = \sqrt{\left(\frac{\sigma_z' - \sigma_x'}{2}\right)^2 + (\tau_{xz})^2}$, keeps constant while the direction of the principal stress change continually (Ishihara and Towhata 1983). It is manifested in Fig.4 that this tendency seems valid for plastic seabed. The rotation of principal axis can be observed from the stress path at a Gaussian integration point in Element A.

CONCLUSIONS

Usually, the model describing development of pore water pressure of sandy soil under undrained condition measured in laboratory experiments is required in analyses of seabed dynamics. Since it is difficult to produce enough wave height to cause fluctuation of pore pressure in the soil layer and the partial drainage can not taken into consideration, few studies on the pattern describing development of pore pressure are achieved. However, such a model is not necessary in the analysis method presented in this paper.

An iterative Newmark's time integration scheme is effectively incorporated in the finite element procedure to predict dynamic response of elastic or inelastic seabed under complicated loading. For elasto-plastic problem, the bounding-surface hypoplasticity model for sand is employed to consider nonlinear behavior of seabed sands. The constant stiffness approach for iterations is used to reduce efficiently the computational efforts.

It has been found that the development of pore pressure for plastic seabed has different features from that in elastic seabed. The rotation of principal stresses can be rationally simulated. Further studies will be paid for reasonably consider liquefaction triggering and post-liquefaction behavior of sand layer.

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