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Fifth International Conference on **Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics**  *and Symposium in Honor of Professor I.M. Idriss* May 24-29, 2010 • San Diego, California

# PRACTICAL SEISMIC DESIGN CONSIDERING NON-LINEAR SOIL- PILE- STRUCTURE INTERACTION

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# ABSTRACT

A substructure approach is proposed for the seismic analysis considering the soil-pile-structure interaction. Two software packages are available for practical applications, DYNAN program and SAP 2000 program. The nonlinearity of soil is considered approximately using a boundary zone model with non-reflective interface. The validation of model is confirmed with dynamic tests on piles in the field, and the results for a single pile are used to compare with the predictions in this study. The liquefaction for sand soil layer can be accounted for, and a case of liquefaction is discussed. The seismic response of a vacuum tower structure supported on pile foundation is examined in a high seismic zone, including response spectrum analysis and time history analysis. To illustrate the effects of soil-pile-structure interaction on the seismic response of structure, three different base conditions are considered, rigid base, i.e. no deformation of the foundation; linear soil-pile system; and nonlinear soil-pile system. The method and procedure introduced can be applied to the design of tall buildings, bridges, industrial structures and offshore platforms with soil-pile-structure interaction under seismic, blast, sea wave and other dynamic loads.

#### INTRODUCTION

Given recent progress, the importance of soil-structure interaction, or soil-pile-structure interaction, has been widely recognized. Now the real problem is how to account for the interaction in practice; for example, how to consider the nonlinearity of soil in an earthquake environment. A comprehensive method should not only be advanced in theory, but also verified by tests and applications.

A simple procedure based upon substructure method is adequate for routine design. The following assumptions are adopted in developing a more detailed method of analysis. The input ground motion is given at the level of pile heads and is not affected by the presence of the piles and their caps. Soilpile interaction analysis is conducted separately to yield the impedance of the pile foundation. The seismic response is obtained in the time domain using the input of earthquake records and in the frequency domain using the input of response spectra. This procedure is considered an efficient technique for solving the problem of nonlinear soil-pilestructure interactions (Han, 2002). The soil-pile system is simulated by a boundary zone model with a non-reflective interface. The model is an approximate but simple and realistic method that accounts for the nonlinearity of a soil-pile system. The validity of the computation method has been verified by dynamic experiments on full-scale pile foundations. The nonlinear features of the pile foundation and the group effects were examined.

In this study, a vacuum tower structure is examined in a seismic zone as a typical industrial structure supported on a pile foundation, including response spectrum analysis and time history analysis. The vacuum tower has a diameter of 8.5 m, a height of 35 m and a weight of 5,600 kN, and is set on a steel frame. There are 25 steel piles in the foundation. To illustrate the effects of soil-pile-structure interaction on the seismic response of the structure, three different base conditions are considered: rigid base, i.e. no deformation in the foundation: linear soil-pile system; and nonlinear soil-pile system. The case of the liquefaction of a sand layer is discussed.

#### NON-LINEAR SOIL-PILE SYSTEM

A number of approaches are available to account for dynamic soil-pile interaction but they are usually based on the assumptions that the soil behavior is governed by the law of linear elasticity or visco-elasticity, and the soil is perfectly bonded to a pile. In practice, however, the bonding between the soil and the pile is rarely perfect, and slippage or even separation often occurs in the contact area. Furthermore, the soil region immediately adjacent to the pile can undergo a large degree of straining, which would cause the soil-pile system to behave in a nonlinear manner. Many efforts to model the soil-pile interaction using the 3D Finite Element Method (FEM) have been made. However, it is too complex, especially for group piles in nonlinear soil. A rigorous approach to the nonlinearity of a soil-pile system is extremely difficult and time consuming.

As an approximate analysis, a procedure is developed using a combination of the analytical solution and the numerical solution, rather than using the general FEM. This procedure is considered as an efficient technique for solving the nonlinear soil-pile system (Han, 1997). The relationship between the foundation vibration and the resistance of the side soil layers was derived using elastic theory. Both theoretical and experimental studies have shown that the dynamic response of piles is very sensitive to the properties of the soil in the vicinity of the pile shaft. Novak and Sheta (1980) proposed including a cylindrical annulus of softer soil (an inner weakened zone or so called boundary zone) around the pile in plane strain analysis. One of the simplifications involved in the original boundary zone concept was that the mass of the inner zone was neglected to avoid the wave reflections from the interface between the inner boundary zone and the outer zone. To overcome this problem, Velestsos and Dotson (1988) proposed a scheme that can account for the mass of the boundary zone. Some of the effects of the boundary zone mass were investigated by Novak and Han (1990), who found that a homogeneous boundary zone with a non-zero mass yields undulation impedance due to wave reflections from the fictitious interface between the two media.

$$G^{*}(r) = \begin{array}{cc} G_{0}^{*} & r = r_{0} \\ G_{0}^{*} & r_{0} < r < R \\ G_{0}^{*} & r > R \end{array}$$
(1)

And

$$G_{i}^{*} = G_{i} (1 + i2 \beta_{i})$$

$$G_{o}^{*} = G_{o} (1 + i2 \beta_{o})$$
(2)

in which  $G_i$  and  $G_o$ = shear modulus of soil in the boundary zone and outer zone;  $r_o$  = radius of pile; R = radius of boundary zone; r = radial distance to an arbitrary point;  $\beta_i$  and  $\beta_o$  = damping ratio for the two zones; and i = root(-1).

The ideal model for the boundary zone should have properties smoothly approaching those of the outer zone to alleviate

wave reflections from the interface. Consequently, Han and Sabin (1995) proposed a model for the boundary zone with a non-reflective interface. The complex shear modulus, G (r), varies parabolically, as expressed by the function f(r). The properties of the soil medium in the boundary zone are defined by the complex-valued modulus.

Obviously, when the modulus ratio equals one, the soil behavior is linear. The shear modulus in the outer zone is a constant. As the modulus ratio  $G_i / G_o$  is less (or larger) than one, the soil behavior is nonlinear. For applications, this concerns the determination of the parameters of the boundary zone, such as the thickness, damping ratio in two zones and the modulus ratio. The thickness of boundary zone is assumed to be equal to the radius of pile, and damping ratio  $\beta_i = 2 \beta_o$ .

Thus, the parabolic function can be written as

$$f(r) = 1 - (1 - G_i^*/G_o^*) (r/r_o - 2)^2$$
(3)

The modulus ratio  $G_i/G_o$  is an approximate indicator for the nonlinear behavior of soil. The value of the modulus ratio depends on the density of excitation and vibration amplitudes (see Han 2002). Further dynamic tests on piles are needed to determine the value of the modulus ratio. The model of the boundary zone with a non-reflective interface has been widely accepted to approximately solve the problem of nonlinear soil. However, it should be explained that the method described here is not a rigorous approach to modeling the nonlinearity of a soil-pile system. It is an equivalent linear method with a lower value of  $G_i$  and a higher value of damping  $\beta_i$  in the boundary zone. With such a model the closed form solutions can be obtained for the impedance functions of a pile.

It can be seen that the parameters of soil used in dynamic loading are different from those used for static loading. For the latter, p-y or t-z curves are used to indicate the nonlinear behavior.

With the impedance of the soil layer, the element stiffness matrix of the soil-pile system can be formed in the same way as in the general finite element method. Then the overall stiffness matrix of a single pile can be assembled for different modes of vibration, including three translations and three rotations. The group effect of piles is accounted for using the method of interaction factors. The static interaction factors are based on Poulos and Davis (1980). The dynamic interaction factors are derived from the static interaction factors multiplied by a frequency variation, and the frequency variation of interaction factors is based on the charts of Kaynia and Kausel (1982). A computer program DYNAN has been developed for dynamic analysis of foundations (see website <u>www.ensoftinc.com</u>).

There are six degrees of freedom for the rigid mat, and lateral vibration is coupled to rocking vibration (Han, 1989). It should be explained that the foundations (or caps on piles) are assumed to be rigid. However, in most cases, the superstructures are flexible rather than rigid. The effects of soil-pile-structure interaction on dynamic response were discussed (see Han,

2008). The dynamic response of the superstructure can be calculated using a finite element program, such as SAP2000.

# DYNAMIC TESTS ON PILE FOUNDATION

To verify the validity of the boundary zone model, a series of dynamic experiments have been done on full-scale piles in the field (see El-Marsafawi et al, 1992). In this study the results from a set of tests on a single steel pile are used to verify the validity of the boundary zone model and to estimate the parameters of the boundary zone. The detail experimental setup is described in what follows (see Han & Novak, 1988).

The pile tested was a steel pipe with a diameter of 133 mm and length of 3.38 m. The pile was placed in a pit with a depth of 3.6 m and a diameter of 1.5 m. The pile cap was a concrete block 200 mm thick, with a mass of 250 kg. An exciter was fixed on the cap and its mass was 120 kg. The centre of gravity of the cap-exciter system was 3 mm below the cap surface.

The washed medium sand was placed into the pit and compacted. The soil properties were measured in the laboratory and in situ for both the sandy-soil fill and the undisturbed natural deposit around the pit.

The shear wave velocity of sand soil measured was 93 m/s at the pile tip and the mass density was 1700 kg/m<sup>3</sup>. It was assumed that the distribution of shear modulus in sand is parabolic with depth.

The soil profile around the pit was established from ground surface to a depth of 20 m. The soil was homogeneous sandy clay. The shear wave velocity of the clay outside the pit was about twice that of the sand backfill and therefore the effect of the interface between the two media had to be assessed. In a low frequency domain, the differences in the dynamic deflections were quite small between the two cases, a horizontal homogeneous medium comprising of only the sand and a composite medium comprising of the inner zone of sand in the pit and the outer zone of clay.

Displacement pickups, strain gauges and compressive stress traducers were fixed along the pile shaft. Displacement and acceleration pickups were mounted on the pile cap.

A mechanical type exciter was used to produce the harmonic excitation in both the horizontal and vertical directions. The magnitude of the unbalanced force was changed by adjusting the angle of the eccentric masses. In this case, the unbalanced force was 0.0606  $\omega^2$  (N), where  $\omega$  is the circular frequency. The unbalanced force was increased with  $\omega^2$ .

The displacements were measured on the pile cap at different frequencies as the unbalanced force varied from low frequencies to high frequencies. The data from the horizontal amplitudes measured are shown by dots in Fig. 1.



Fig. 1 Theoretical predication vs. measured data for lateral vibration of single steel pile

The dynamic response of the pile is generated by the computer program to match the measured data. In the calculation, the distribution of shear modulus in sand soil is assumed to be parabolic with depth, and the shear wave velocity is 93 m/s at the pile tip.

The parameters of the boundary zone are estimated to indicate the non-linear properties of soil. The thickness ratio of the boundary zone versus the pile radius is one, and the damping ratio in the boundary zone is twice that of the outer zone. The shear modulus ratio  $G_i/G_o = 0.03$ ,

The real line in Fig.1 represents the response of pile with the boundary zone, and the nonlinear property of soil is accounted for. The dash line represent the response of pile with the linear property of soil,  $G_i/G_o = 1$ . It can be seen that the solid line matches the measured data very well, and the dynamic response calculated is very different for line soil and nonlinear soil. The frequencies corresponding to the peak value and the maximum amplitudes are varied with the cases of linear soil and nonlinear soil.

The modulus ratio  $G_i/G_o$  is an indicator showing the nonlinear behavior of soil approximately. The value of the modulus ratio depends on the density of the excitation and vibration amplitudes. Further dynamic tests on piles are needed to determine the value of modulus ratio.

The maximum displacement on the top of the pile cap is 1.88 mm at frequency 7.5 Hz, and the correspondent maximum acceleration is 0. 43 g. For steady-state vibration this represents a very intense motion.

The boundary zone model with a non-reflecting interface is valid for simulating nonlinear soil-pile interactions, even for the case of strong vibration.

# SEISMIC RESPONSE OF VACUUM TOWER STRUCTURE

A vacuum tower structure shown in Fig. 2 was built for a petrochemical plant in a seismically active area of Canada. Details of the steel structure are described in what follows. Four columns using WWF 400x243 (400 x 400 mm, weight of 243 kg/m) and a height of 20m are arranged in a rectangle with a center to center spacing of 8.55m. The vacuum vessel is supported directly by a top frame using beams of WWF 1400x358 (1400 x 400 mm, weight of 358 kg/m) on top and beams of W610x155 at bottom. There are three layers of beams beneath the top frame and the main beam is W460x82. The concrete mat foundation is 12 x 12 m and thickness is 1.2 m.

Fig.2 Vacuum tower structure View original model of as-built structure

The vacuum vessel is modeled as an elastic column with the mass distributed uniformly along its height. The steel structure is modeled using frame elements, and the mat foundation is

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modeled using shell elements, as shown in Fig. 3. The thickness of the vessel wall is 25.4 mm (one inch). The seismic response of the structure is calculated using the substructure method. The deflection of the structure, the base shear, and the overturning moments for different base conditions are investigated.





Soil Conditions and Pile Foundation

The structure is in a seismically active area, and the range of peak horizontal ground acceleration is equal to 0.13 g. At the site, the surface soil is soft clay with a depth of 2m, followed by a layer of saturated fine sand with a depth of 2 m, then some clay, sand, and finally bedrock. The depth to the bedrock is about 30 m. Soil properties vary with depth and are

characterized by the shear wave velocity Vs and unit weight  $\gamma$ , as shown in Table 1.

Depth	Soil	γ	Vs
(m)		$(kN/m^{3})$	(m/s)
0 -2	Soft Clay	18	130
2 - 4	Fine Sand	18	140
4 - 12	Stiff Clay	20	300
12 - 16	Silty Sand	19	240
16 - 20	Silty Clay	18	300
20 - 25	Shale	18	200
25 - 30	Sand	20	300
Below	Bedrock	21.5	370

Table 1. Soil Propertie
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The piles are steel HP 360 x 108, (346 x 370 mm) with a length of 30 m driven to bedrock. Twenty-five piles in a square pattern are fixed to the mat foundation, with spacing of 2.75 m and a spacing ratio of 7.6.

The stiffness and damping of the pile foundation are calculated for different base conditions. In the first case a nonlinear soil-pile system is assumed, and the boundary zone model is used around the piles. The parameters of the weakened zone are selected as:  $G_i / G_o = 0.3$ ,  $t_{m}/r_o = 1.0$ ,  $\beta_i = 2 \times \beta_o$ . In the second case, a linear soil-pile system is assumed, the soil layers are homogeneous, and there is no weakened zone.

Table 2. Stiffness and Damping of Pile (f = 1.0 Hz)

	Stiffness			Damping		
Soil – Pile Interaction	K <sub>x</sub>	Kz	Kφ	C <sub>x</sub>	Cz	$C_{\phi}$
	10 <sup>6</sup>	$10^{6}$	10 <sup>8</sup>	$10^{4}$	$10^{4}$	$10^{5}$
	(kN/m)	(kN/m)	(kN.m/ra)	(kN/m/s)	(kN/m/s)	(kN.m/rad/s)
Linear Soil	1.283	3.215	1.333	1.244	1.803	6.411
Nonlinear	0.646	2.877	1.160	0.998	1.005	3.171
Liquefaction	0.1799	2.527	1.006	0.749	0.943	2.787

Where,  $K_x$ ,  $K_z$ , and  $K_{\phi}$  are stiffness in the horizontal, vertical and rocking directions, respectively, and  $C_x$ ,  $C_z$ , and  $C_{\phi}$  are damping constants in the same directions.

In the third case, liquefaction is assumed in the saturate fine sand layer, and the top layer of soft clay has not yielded. Both stiffness and damping are frequency dependent. Since the fundamental period of the structure is close to 1.0 seconds, the stiffness and damping are calculated at a frequency of f = 1.0Hz, and the results are shown in Table 2. It can be seen that both stiffness and damping in the nonlinear case are lower than in the linear case. For example, the horizontal stiffness in the nonlinear case is about half that of the linear case. In the case of liquefaction, the values of horizontal stiffness are reduced significantly, and significant damage is possible.

## Time History Analysis

A record of horizontal ground acceleration from an earthquake is employed for the time history analysis. The peak value of acceleration is 0.13 g as shown in Fig. 4. The time step is 0.005 seconds, and the duration is 80 seconds in the earthquake record. To investigate the influence of foundation flexibility on the superstructure, the seismic analysis of the vacuum tower structure is conducted for three different foundation conditions: a rigid base, and linear and nonlinear soil-pile systems. For the rigid base case, the stiffness of the foundation is assumed to be infinite with no deformation occurring in the footing. Initial seismic analysis was done this way forty years ago, when soil-structure interaction was not considered. For cases of linear and nonlinear soil-pile systems, the values of stiffness and damping shown in Table 2 are used.



Fig. 4 Horizontal ground acceleration from an earthquake record

The analysis is done using a finite element model as shown in Fig. 3. The seismic response and natural frequency of the structure are different for the three base conditions. The deflection, base shear, and overturning moment are shown in Table 3, and the natural periods of structure are shown in Table 4.

Table 3 shows that the earthquake forces for the fixed base condition are larger than those for cases of soilstructure interaction that accounts for a flexible base. The theoretical prediction for a structure fixed on a rigid base without soil-structure interaction does not represent the real seismic response, since the stiffness is overestimated and the damping is underestimated. From Table 4, it can be seen that the structure with a flexible base has longer natural periods than with a fixed base. A comparison shows that the maximum values and time histories for the seismic forces and seismic response are different depending on whether the foundation is considered as a fixed or flexible base. The soil-pile-structure interaction should be considered for the seismic analysis.

Table 3. Maximum Values of Seismic Response and Seismic
Forces of Tower Structure

Base	Amplitude at	Base Shear	Moment
Conditions	Top (mm)	(kN)	(kN-m)
Fixed	22.05	807	19,630
Base			
Linear	26.30	598	14,980
Soil			
Nonlinear	26.05	545	14,120
Soil			

Table 4. Natural Period of Tower Structure (Second)

Model	Shape	Fixed Base	Linear Soil	Nonlinear Soil
1	Lateral X1	0.769	0.967	1.004
2	Lateral Y1	0.767	0.962	0.991
3	Lateral X2	0.184	0.191	0.197
4	Lateral Y2	0.173	0.187	0.190
5	Vertical	0.161	0.181	0.189
6	Torsional	0.122	0.130	0.140

## Response Spectrum Analysis

Elastic dynamic analysis of a structure utilizes the peak dynamic response of all modes having a significant contribution to total structural response. Peak modal responses are calculated using the ordinates of the appropriate response spectrum curve corresponding to the modal periods. Maximum modal contributions are combined in a statistical manner to obtain an approximate total structural response.

The equivalent lateral seismic force, V, is calculated in accordance with the formula in NBCC 2005.

$$V = S (T_a) M_v I_E W / (R_d R_o)$$
(4)

$$S(T_a) = F_v S_a(T_a)$$
<sup>(5)</sup>

where,  $T_a$  = fundamental lateral period, 0.75 second is calculated for fixed base;  $S_a$  ( $T_a$ ) = ground acceleration. The values of  $S_a$  ( $T_a$ ) are given for different locations. For the location of vacuum tower,  $S_a$  (0.75) = 0.13 g.  $F_v$  = site coefficient 1.37 is used based on the soil properties.  $M_v$  =

and

higher model factor 1.0 is used here.  $I_E$  = important factor, 1.0 is used here.  $R_d$ ,  $R_o$  = ductility factor and over-strength factor respectively, 1.5 and 1.3 are used for conventional construction of moment frames and braced frames. The weight of steel frame is 1,963 kN, and total weight (including vessel) is W = 7,563 kN.

The most difficult part of the entire RSA (Response Spectrum Analysis) procedure is calculating the scaling factor. The unscaled RSA base shear is calculated using a finite element program RISA - 3D. Thus, Scale Factor is equal to V/Unscaled RSA base shear. The spectra are normalized using modal participation. In the calculation for scale factor, 15 vibration modes are calculated making the modal participation to be over 90%.

A local response spectrum is used in the analysis. The following values from NBC 2005 are used, considering the location of vacuum tower structure:  $S_a (0.2) = 0.28$  g,  $S_a (0.5) = 0.17$  g,  $S_a (1.0) = 0.090$  g, and  $S_a (2.0) = 0.053$  g. The response spectrum analysis is done for fixed base. The seismic response and seismic forces are calculated, the amplitude at top of tower = 20.9 mm, base shear = 776 kN and overturn moment = 19,936 kN-m. Comparing to the data in Table 3, it is interesting to note that the seismic response and seismic forces spectrum analysis are close to those from the time history analysis at the same base condition.

## CONCLUSIONS

The dynamic analysis is required based on NBCC 2005 for structures with stiffness and mass irregularity, such as the vacuum tower under earthquake loads. The parameters used in equivalent static forces can be checked against the time history analysis and the response spectrum analysis.

The problem of nonlinear soil - pile system can be solved approximately using the model of boundary zone, and the validation is confirmed with the dynamic tests on pile foundation. The nonlinear properties of soil can be accounted for in the dynamic analysis of soil-pile-structure interaction.

For seismic response, the soil - pile interaction is an important factor which affects the stiffness and damping of foundation. The liquefaction developed in a layer of saturated fine sand can reduce the horizontal stiffness significantly, and further damage is possible.

The theoretical prediction for a structure fixed on a rigid base without the interaction does not represent the real seismic response, since the stiffness is overestimated and the damping is underestimated.

The approximate and practical method proposed in this study is workable. The method and procedure can be applied to the design of tall buildings, bridges, industrial structures and offshore platforms with soil-pile-structure interaction under seismic, blast, sea wave and other dynamic loads.

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