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Soil Model of Effective Stress for Seismic Loadings

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SYNOPSIS The Generalized Masing Curve, recently proposed, has been improved in this paper to involve the effect of pore pressure on shear modulus and damping ratio. Fraction cycle method for pore pressure generation can be adopted to avoid the iterative computation, which is necessary in equivalent cycle method.

INTRODUCTION

Most analyses of soil seismic response usually employ an iterative linear visco-elastic approach. However, there is a trend evolving nonlinear analytical methods. Up to now, the test data of soil properties for seismic loadings are available for uniform stress or strain cycles. In order to apply those to solving soil dynamic and liquefaction problems, a more valid viscoelastoplastic soil model of effective stress is in urgent need.

GENERALIZED MASING CURVE FOR TOTAL STRESS ANALYSIS

Many writers adopt nonlinear stress-strain curve of Masing type to describe hysteresis loops for unloading and reloading. First, the skeleton curve is constituted by shear modulus curve $G(\gamma)$:

$$\tau^{ep} = f(\gamma) = G(\gamma)\gamma \qquad \text{or} \qquad \gamma = F(\tau^{ep}) \tag{1}$$

in which, τ^{ep}-elastoplastic part of shear stress, γ-shear strain, and for unloading from point (τ_r, γ_r) by Masing Curve, i.e.,

$$\frac{\gamma - \gamma_r}{2} = F\left(\frac{\tau^{ep} - \tau_r}{2}\right) \tag{2}$$

Some of different function relations $F(\tau^{ep})$ or $f(\gamma)$ are adopted, such as hyperbola, R-O curve, N-linear and so on.

We express the influence of soil viscosity by Voight relation, then,

$$\tau = \tau^{ep} + \mu\frac{\partial\gamma}{\partial t} = f(\gamma) + \mu\frac{\partial\gamma}{\partial t} \tag{3}$$

in which μ- viscosity factor.

There is a problem in all of these methods, the hysteresis loop curve (2) is defined only by shear modulus curve $G(\gamma)$, but is not related to damping ratio curve $\eta(\gamma)$ at all. Thus, the damping ration $\eta_1(\gamma)$ corresponding to Masing Curve (2) generally disagrees with test damping ratio curve $\eta(\gamma)$.

We defined a damping ratio degradation factor:

$$k(\gamma) = \eta(\gamma)/\eta_1(\gamma) \qquad \text{or} \qquad K(\tau^{ep}) = k(F(\gamma)) \tag{4}$$

Fig.1,2 show some $G(\gamma)$, $\eta(\gamma)$ curves and the comparisons of damping ratio $\eta_1(\gamma)$ corresponding to Masing hysteresis loops with test curve $\eta(\gamma)$. The divergencies between them are obvious.

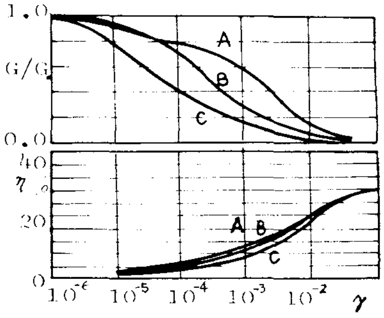

Fig.1. Shear Modulus and Damping Ratio

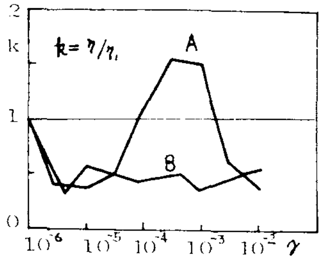

Fig.2. Damping Ratio Degradation Factor

Generalized Masing Curve was proposed by Wang et al. (1978, 1980a):

$$\gamma - \gamma_r = K(\tau_m)\left(2F\left(\frac{\tau^{ep} - \tau_r}{2}\right) - \frac{\gamma_m}{\tau_m}(\tau^{ep} - \tau_r)\right) + \frac{\gamma_m}{\tau_m}(\tau^{ep} - \tau_r) \tag{5}$$

in which (τ_m, γ_m) is the coordinates of maximum stress point. Fig.3 shows the meaning of G.M.C. . It is obvious that the damping ratio coresponding to G.M.C. is $k(\gamma)\eta_1(\gamma) = \eta(\gamma)$, which is consistent with test curve $\eta(\gamma)$.

GENERALIZED MASING CURVE FOR EFFECTIVE STRESS ANALYSIS

Owing to the application of cyclic stress, pore water pressure of saturated sand increases. The generation of pore pressure causes the degradation of soil stiffness, even in the same magnitude of shear strain amplitude. Many authors adopt a simple formula to evaluate variations of soil stiffness, i.e.,

$$\frac{G_0(\sigma')}{G_0(\sigma_0')} = \left(\frac{\sigma'}{\sigma_0'}\right)^{1/2} \qquad (6)$$

in which G -elastic shear modulus, σ_0'-initial effective stress, σ'-effective stress.

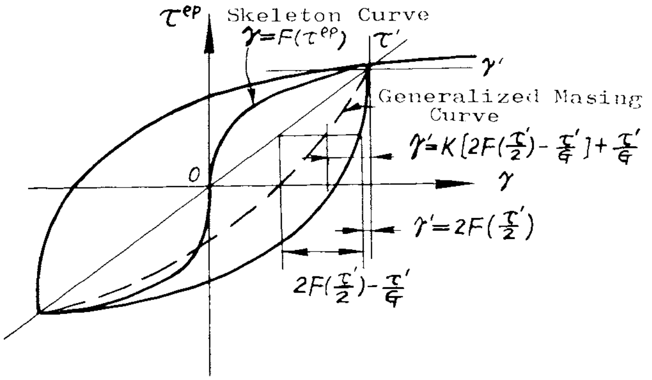

Fig.3. the Meaning of Generalized Masing Curve

Now we suggest a general presentation to evaluate the degradation of shear modulus, which is due to increased pore pressure. Suppose $\tilde{G}(\gamma,\sigma')$ is secant shear modulus at any shear strain amplitude and average effective stress level, it can be expressed as

$$\tilde{G}(\gamma,\sigma') = \alpha(\sigma')G(\gamma) \qquad (7)$$

in which $G(\gamma)$-shear modulus at initial pore pressure and $\alpha(\sigma')$-modification factor of pore pressure for soil stiffness,

$$\alpha(\sigma') = c+(1-c)\left(\frac{\sigma'}{\sigma_0'}\right)^a \qquad (8)$$

in which a,c are experiment constents. Similarly, it is found that the damping ratio in effective stress analysis is a fuction of average pore water pressure in every cycle, too. It can be expressed as

$$\tilde{\eta}(\gamma,\sigma') = \beta(\sigma')\eta(\gamma) \qquad (9)$$

in which $\eta(\gamma)$-damping ratio at initial pore pressure, $\beta(\sigma')$-modification factor of pore pressure for damping ratio,

$$\beta(\sigma') = d+(1-d)\left(\frac{\sigma'}{\sigma_0'}\right)^b \qquad (10)$$

in which b,d are experiment constents. Some of test curves of factors $\alpha(\sigma')$ and $\beta(\sigma')$ are shown in Fig.4,5. It is found, for some silty sand in a certain range of strain amplitude by Li et al. (1980), that a = 1.0, b = 0.80.

If we apply the factor of "damping ratio degradation" in effective stress analysis as the same in total stress analysis, then it has a new meaning: damping ratio decreases, while pore pressure increases. That is,

$$\tilde{k}(\gamma) = \tilde{\eta}(\gamma)/\eta_1(\gamma) = \beta(\sigma')k(\gamma)$$

$$\text{or } \tilde{K}(\tau) = \beta(\sigma')K(\tau) \qquad (11)$$

Because

$$\gamma = \tau^{ep}/\tilde{G} = (\tau^{ep}/G)/\alpha = F(\tau^{ep})/\alpha(\sigma') = \tilde{F}(\tau^{ep},\sigma') \qquad (12)$$

We can improve Generalized Masing Curve (5) to involve the pore water pressure, i.e., G.M.C. for effective stress analysis is pressented as

$$\gamma - \gamma_r = \tilde{K}(\tau_m)\left(2\tilde{F}\left(\frac{\tau^{ep}-\tau_r}{2}\right)-\frac{\gamma_m}{\tau_m}(\tau^{ep}-\tau_r)\right)+\frac{\gamma_m}{\tau_m}(\tau^{ep}-\tau_r) \qquad (13)$$

in which $\gamma_m = \tilde{F}(\tau_m,\sigma') = F(\tau_m)/\alpha(\sigma')$. It has the same form as formula (5), but the fuctions $\tilde{K}$, $\tilde{F}$, γ_m depend on pore water pressure.

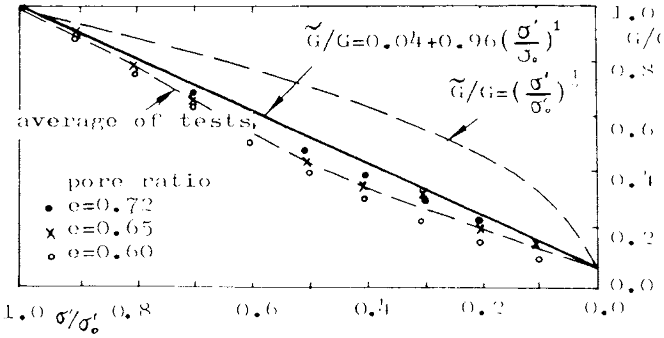

Fig.4. Shear Modulus-Effective Stress Curve

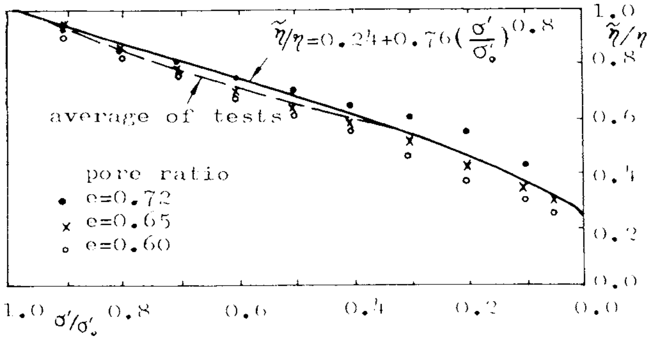

Fig.5. Damping Ratio-Effective Stress Curve

STRESS-STRAIN RELATION FOR IRREGULAR CYCLIC LOADINGS

Suppose (τ_m, γ_m) is "maximum stress point" before a given moment, (τ_r, γ_r) is "reversal point", then for very time step Δt a judgement must be made:

(1) If $|\tau^{ep}| \geqslant \tau_m$, then stress point goes up to skeleton cure $\gamma = \tilde{F}(\tau^{ep}, \sigma')$.

(2) If $|\tau^{ep}| < \tau_m$, then stress point is on unloading and reloading curve (13).

(3) If the G.M.C. originated from (τ_r, γ_r) meets

the G.M.C. originated from (τ_m, γ_m), and max $|\gamma|$ less than γ_m , then the stress point goes over to the second one.

The three rules are sufficient to limit a stress point under skeleton curve, when a irregular cyclic loading is applied to a soil deposit.

FRACTION CYCLE METHOD FOR PORE PRESSURE GENERATION

Up to now, many authors apply a concept of equivalent uniform cycles to the problems of seismic response of soil masses. It is necessary to perform a iterative computation, while the equivalent cycle method is adopted. The fraction cycle method for pore pressure generation is applied to evaluation of pore pressure direct from irregular shear wave. When the quarter cycle method is used, one might evaluate the waved increase of pore pressure.

The main points of half cycle method may briefly be described as follows: (1) pore pressure ratio-cycle ratio (r_u -r_N) and stress ratio-number of cycles to liquefaction (τ/σ_0' -N_l) are basic experimental curves. But before performing τ/σ_0' -N_l curve must be transformed to τ/σ_0' -l curve, in which $l={}^1/2N_l$ is called as percentage of damage of half cycle, N_l is the number of cycles causing pore pressure ratio of 100%. (2) computing the increments of percentage of damage Δl according to the increments of shear stress $\Delta\tau$, adding it to the initial one, then the sum $\Sigma\Delta l$ is the percentage of damage at this moment. (3) compute pore pressure ratio from r_u -r_N curve by substituting $r_N = \Sigma\Delta l$.

Fig.6 shows τ/σ_0' -N_l curve after Martin and Seed (1979), one of the curves (σ_0'=1720psf) was transformed into l-τ curve as Fig.7. In Fig.8 OABCD is a irregular half cycle.

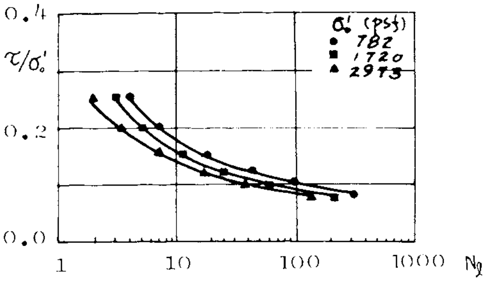

Fig.6. Shear Stress Ratio-Number of Cycles to Liquefaction

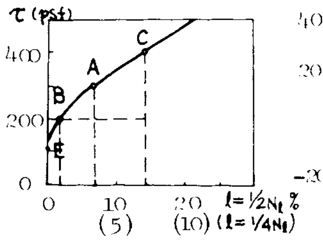

Fig.7. Shear Stress-Percentage of Damage

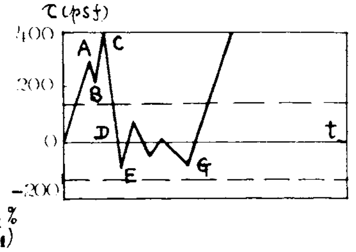

Fig.8. Irregular Stress cycle

It is also assumed that the increase of pore pressure appears only in the stress intervals, in which the absolute value of shear stress increases. The stress path is shown in l-τ graph in Fig.7. Because the percentage of damage of interval D-G in Fig.8 is negligible, the cycle ratio before point G is $r_N = l_A + |l_C - l_B|$.

Fig.9 is an example of equivalent number of uniform cycles after Martin (1979). The computation results of cycle ratio-time curve is shown in Fig.10.

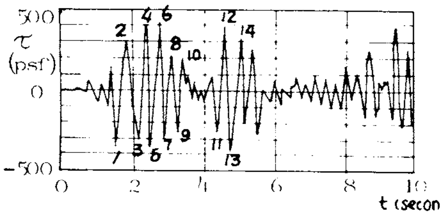

Fig.9. Shear Stress Wave

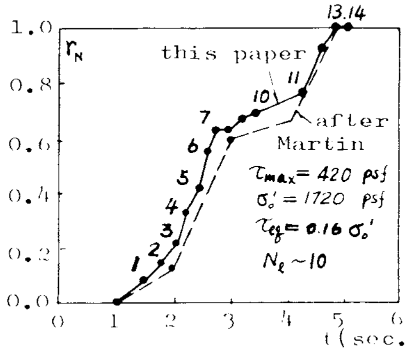

Fig.10. Cycle Ratio-Time

In application of quarter cycle method, it is assumed that the increase of stress is attended by increase of pore pressure and the decrease of stress is attended by less decrease of pore pressure. The data of cyclic triaxial tests are divided into plus pore pressure ratio r_{u+} and minus pore pressure ratio r_{u-} , it is evident that the pore pressure ration r_u is equal to r_{u+} -r_{u-} . Fig.12 shows an examples of quarter cycle method, half cycle method and equivalent cycle method, in which r_u , r_{u+} , r_{u-} are taken from Fig.11, which expresses the experements of burnt coal dust.

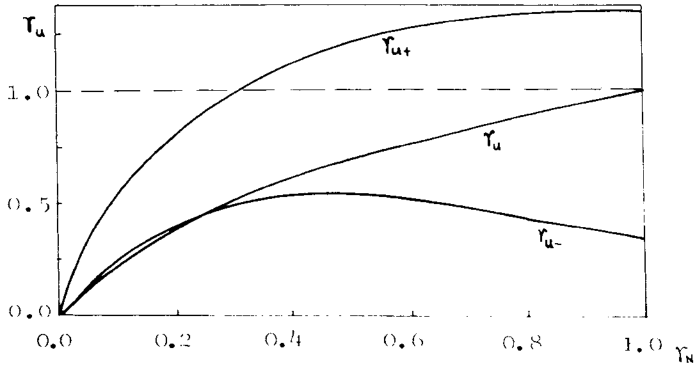

Fig.11. Pore Pressure Ratio r , r and r

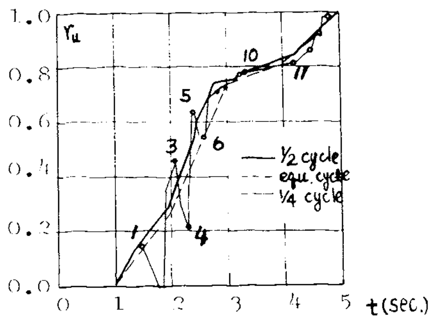

Fig.12 Pore Pressure Ratio-Time

BASE OF FRACTION CYCLE METHOD

If we suppose that the variations of pore pressure are mainly depended on the stress increments and concerned with current pore pressure and stress levels, that is

$$dU = c(\sigma_0')h_1(U)\cdot h_2(T)\cdot dT \qquad (14)$$

in which $U=u/\sigma_0'$ -pore pressure ratio, $T=\tau/\sigma_0'$ -shear stress ratio, $c(\sigma_0')$, $h_1(U)$, $h_2(T)$, unkown fuction. Then it is found that (1980b)

$$h_1(U) = dr_u/dr_N, \qquad c(\sigma_0')h_2(T) = dl/dT \qquad (15)$$

i.e., they are the derivatives of test curves $r_u - r_N$ and $1-\tau/\sigma_0'$.

EXAMPLE PROBLEMS OF LIQUEFACTION

The soil models have been combined with a computer program LSR-1980, which includes dissipation of pore water pressure. Four test curves and four parameters are basic, named $G-\gamma$, $\eta-\gamma$, r_u-r_N, $\tau/\sigma_0'-1$, a,b,c,d. Fig.13 shows the developments of pore pressure of a sand deposit, excited at an uniform cycles of linear acceleration wave at 15m deep. The frequency of the wave is 2.5, and peak value is 100 gal. The differences of the two precedures in Fig.13 are due to the parameters a,b,c,d. In the left a=0.5, b=0.8, c=0.05, d=0.0001, in the right a=b=1, c=0.05, d=10 .

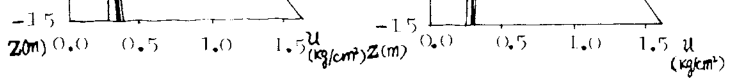

Fig. 13. Development of Pore Pressure in time

The hysteresis loops of a soil element of the deposit at 5.25m deep are shown in Fig.14. Examples of seismic loadings are given in another paper (1980c).

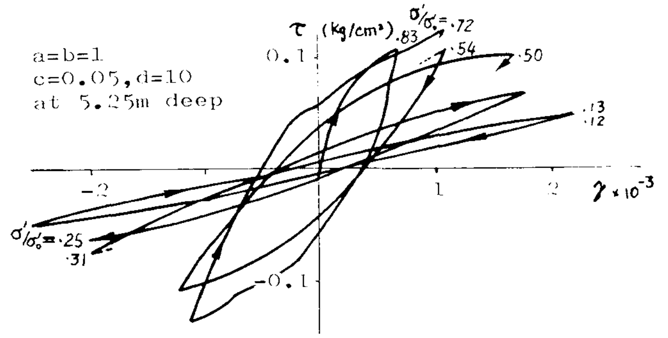

Fig.14. Hysteresis Loops of Effective Stress

CONCLUSION

The soil model of effective stress, introduced in this paper, is a visco-elastoplastic soil model considering the effect of increased pore pressure on stiffness and damping. It makes the cycle dynamic response evaluation consistent with the experimental curves, such as shear modulus and damping ratio vs shear strain and pore pressure, pore pressure ratio vs cycle ratio and stress ratio vs number of cycles to liquefaction, etc. The generation of pore pressure is evaluated by fraction cycle method, which is simple and reasonable. The soil models have been applied to the computer programs SRHSL-1978 and LSR-1980 respectively for total and effective stress analysis of site seismic respones.

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