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Analysis of Aseismic Reliability Considering the Uncertainties Both Structural Parameters and Earthquake Loadings for the Gravity Type Earth-Retaining Wall

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ANALYSIS OF ASEISMIC RELIABILITY CONSIDERING THE UNCERTAINTIES BOTH STRUCTURAL PARAMETERS AND EARTHQUAKE LOADINGS FOR THE GRAVITY TYPE EARTH-RETAINING WALL

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ABSTRACT

A simple numerical deterministic approach for evaluating stochastic properties of structural systems with uncertain parameters under random seismic excitations (“double-random problem”) is presented. The method is based on the impulse response function method, the perturbation law and random central differential law. On the other hand, the equivalent linear model is used to account for the nonlinearity of soil. On the basis of the above random response analysis method, the aseismic reliability of earth-retaining wall is calculated and the analysis flow of dynamic reliability under the stationary random seismic action is also presented. From the above analysis, we can see that it is equally important both structural parameters and seismic excitations, the maximum value response including the displacement, moment and resultant force will increase with the heightening of reliability index and the aseismic design by linear model will result risk than non-linear model. The simulation analysis result verifies that the proposed method is applicable to the aseismic reliability analysis by contrast with the Richard-Elms displacement method and field actual measurements, which also provides a practical way for analyzing the aseismic reliability of other geotechnical engineering structures.

INTRODUCTION

Earthquake-resistance design of structures using probabilistic analysis and performance-based design criteria is an emerging field of structural engineering. These analysis and design methodologies are aimed at improving the existing practice and design codes for better prediction of the structural performance. As earth-retaining wall is among the most frequently encountered geotechnical structures, the ability to predict their safety under seismic conditions is of paramount importance in geotechnical practice.

Geotechnical engineers have long used the lumped factor of safety approach in the design of foundations and retaining walls. A more recent alternative is the limit state approach using partial factors. Yet another approach based on a target reliability index by the first-order reliability method (FORM) is perhaps more popular at present. But, these approaches all have not been considered the uncertainties of earthquake loading. The seismic earth pressure on earth-retaining wall is based on the pseudo-static analysis known as the Mononobe-Okabe (M-O) method. The reliability of earth-retaining wall under deterministic seismic loading had been investigated by Grivas D A, Vlavianos V J, Souflis C, etc. However, few of them took account of the random properties of the seismic excitations, meanwhile the structural parameters are also random. The influences of the two random factors on the earth-retaining wall are sometimes equally important. So, it is essential to take account of uncertainties both structural systems and earthquake excitations.

The objective of this paper is to perform stochastic response and reliability analysis of earth-retaining wall considering the uncertainties both structural parameters and earthquake loadings. This thesis presents methods of the impulse response functions on the basis of δ function, using general purpose FEM software for structure analysis, and the perturbation law and central differential law for evaluating the stochastic properties of structural systems with uncertain parameters under random seismic excitations. On the other hand, the equivalent linear model is used to account for the nonlinearity of soil.

FORMULATIONS

Calculation of Impulse Functions

The unit impulse function *δ(t)* should satisfy the following condition:

$$\int_a^b \delta(x-x_0)\mathrm{d}x = \begin{cases} 0, (a,b<x_0 \text{ or } a,b>x_0) \\ 1, (a<x_0<b) \end{cases} \quad (1)$$

Under the action of *δ(t)*, the response of a structure system is called impulse response function *h(t)*. To get *h(t)* through numerical analysis software, we can use a discrete series of *δ(t)* as following:

$$\delta(t) = \{1/\Delta t, 0, 0, 0, \cdots\cdots\} \quad (2)$$

As the excitation of the structure system, in which Δt is the time step in the dynamic analysis. It can be proved that the discrete Fourier Transform of Equation (2) is consistent to theoretical value of *δ(t)*. This discrete series can be used as the input of FEM software to obtain the impulse response function at any point of earth-retaining wall.

Stochastic Response and Reliability Analysis of Structures

Stationary Random Response Analysis Based on the Perturbation Law and Random Central Differential Law. For the multi-degree-of freedom systems in the stationary random excitation of earthquake loadings, the equation of motion is expressed as:

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\} = -[M]\{E\}\ddot{x}(t) \quad (3)$$

Where, *[M], [C], [K]* is individual the mass, damping and rigidity matrix, which can be divided into the sum of mean value and random variance with zero mean:

$$[M]=[\bar{M}]+[\tilde{M}], \quad [C]=[\bar{C}]+[\tilde{C}],$$
$$[K]=[\bar{K}]+[\tilde{K}] \quad (4)$$

If the random properties of the mass, damping and rigidity matrix will be ignored, we can get the stationary random response equation:

$$[\bar{M}]\{\ddot{y}\}+[\bar{C}]\{\dot{y}\}+[\bar{K}]\{y\} = -[\bar{M}]\{E\}\ddot{x}(t) \quad (5)$$

The power spectrum of the response is:

$$[S_{yy}(\omega)] = [H^*(\omega)][S_{xx}(\omega)][H(\omega)]^T \quad (6)$$

and the mean square of the response is:

$$\mu_v = \sigma_v^2 = 2\int_0^{+\infty} S_{yy}(\omega)\mathrm{d}\omega \quad (7)$$

As the uncertainties of structural parameters, the dynamic response also has the random properties. Based on the above assumptions that the random characteristic of the uncertain structural system can be written as:

$$\eta_k = \bar{\eta}_k + \tilde{\eta}_k \quad (8)$$

Then, the random properties of the response is:

$$\mu_v = \bar{\mu}_v + \tilde{\mu}_v \quad (9)$$

and the mean square of the response is:

$$\sigma_{\mu_v}^2 = \mathrm{co}\ \ (\mu_v\mathrm{y}\mu_v) = \sum_{k=1}^{m}\sum_{q=1}^{m}\left(\frac{\partial\bar{\mu}_v}{\partial\eta_k}\right)\left(\frac{\partial\tilde{\mu}_v}{\partial\eta_q}\right)\rho_{kq}\sigma_{\eta_k}\sigma_{\eta_q} \quad (10)$$

If $\bar{\mu}_v >> \sigma_{\mu_v}$, then the coefficient of the variation is:

$$\xi_{\mu_v} = \sigma_{\mu_v} / \bar{\mu}_v \quad (11)$$

If only considering the effect of the k_{th} variation, then

$$\xi_{\mu_v k} = \left(\frac{\partial\bar{\mu}_v}{\partial\eta_k}\sigma_{\eta_k}\right) \Big/ \bar{\mu}_v \quad (12)$$

So, the sum of the coefficient of the variation is:

$$\xi_{\mu_v}^2 = \sum_{k=1}^{m}\sum_{q=1}^{m}\rho_{kq}\xi_{\mu_v k}\xi_{\mu_v q} \quad (13)$$

According to the definition of mathematics, derived number $f'(a)$ is the limit of difference quotient *[f(a+h)-f(a)]/h* when $h \to 0$. If the precision don't need too high, we can get the approximation value of derived number using the difference quotient:

$$f'(a) \approx [f(a+h) - f(a-h)]/2h \quad (14)$$

In this paper, the sensitivity of the random response will be calculated by the upper equation.

2.2.2 Dynamic Reliability Analysis of Structures

2.2.2.1 Dynamic Reliability Analysis Based on the First Excursion Failure Criterion. The spectrum moment of the stationary response can be written as:

$$\lambda_i = \int_{-\infty}^{+\infty}\omega^i\varphi(\omega)\mathrm{d}\omega \quad (15)$$

Where , λ_i is the spectrum moment of the i_{th} order.

The shape factor of the power spectrum is:

$$q = \sqrt{1-\lambda_1^2/(\lambda_0\lambda_2)} \tag{16}$$

Assuming the passage of the response process to a given level as a Poisson process, we can obtain the probability in which the response is within the limit of *(-b, b)* as following:

$$P(b,-b) = \exp\left\{-\frac{\omega_2 T}{\pi}\exp\left[-\frac{b^2}{2\sigma^2}\right]\right\} \tag{17}$$

A modified expression with Malkov assumption is:

$$P(b,-b) = \exp\left\{-\frac{\omega_2 T}{\pi}\exp(-\frac{r^2}{2})\frac{1-\exp\left[-\sqrt{\frac{\pi}{2}}qr\right]}{1-\exp(-\frac{r^2}{2})}\right\} \tag{18}$$

Where, T is the duration of ground motions, $\omega_2 = \sqrt{\lambda_2/\lambda_0}$, $\sigma = \sqrt{\lambda_0}$, $r = b/\sigma$.

For some reason, the bound maybe a random variable, the dynamic reliability of structures can be expressed as：

$$P_s = 1+\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\int_0^\tau\cdots\int_0^\tau M_n(t_1\cdots t_n)dt_1\cdots dt_n \tag{19}$$

Where, $M_n(t_1\cdots t_n)dt_1\cdots dt_n$ represent the probability of $y(t)$, assume that:

$$M_n(t_1\cdots t_n) = \prod_{i=1}^{n} M(t_i) \tag{20}$$

Where, $M(t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}|\dot{y}(t)| f_{y\dot{y}}(R,\dot{y},t)d\dot{y} f_R(\nu)d\nu$

$f_R(\nu)$ represent the probability function of R (the resistance of structures):

$$f_R(\nu) = \frac{1}{\sqrt{2\pi}\sigma_R}\exp\left[\frac{(\nu-m_R)^2}{2\sigma_R^2}\right] \tag{21}$$

Where, m_R, σ_R represent the mean value and standard deviation of R .

When $y(t)$ is stationary gauss process with zero mean, the unite probability density function of $y(t)$ and $\dot{y}(t)$ is:

$$f_{y\dot{y}}(y,\dot{y}) = \frac{1}{2\pi\sigma_y\sigma_{\dot{y}}}\exp\left\{-\frac{1}{2}\left(\frac{y^2}{\sigma_y^2}+\frac{\dot{y}^2}{\sigma_{\dot{y}}^2}\right)\right\} \tag{22}$$

Where, $\sigma_y = \sqrt{\lambda_0}$, $\sigma_{\dot{y}} = \sqrt{\lambda_2}$

Associate with the above formulas, we can get:

$$M(t) = -\frac{1}{\pi}\frac{\sigma_{\dot{y}}}{\sqrt{\sigma_y^2+\sigma_R^2}}\exp\left[-\frac{m_R^2}{2(\sigma_y^2+\sigma_R^2)}\right] \tag{23}$$

we can obtain the probability：

$$\begin{aligned} P_s &= \exp\left[-\int_0^\tau M(t)dt\right] \\ &= \exp\left\{-\frac{\tau}{\pi}\frac{\sigma_{\dot{y}}}{\sqrt{\sigma_y^2+\sigma_R^2}}\exp\left[-\frac{m_R^2}{2(\sigma_y^2+\sigma_R^2)}\right]\right\} \end{aligned} \tag{24}$$

2.2.2.2 Dynamic Reliability Analysis Based on the Maximum Response. We assume the maximum response can be converted to σ_y :

$$y_m = R_y\sigma_y \tag{25}$$

Where, R_y is the peak factor.

Thereout, the mean value and variance of the maximum response can be expressed:

$$E[y_m] = E[R_y]\cdot E[\sigma_y] \tag{26}$$

$$V[y_m] = V[R_y]V[\sigma_y]+(E[R_y])^2V[\sigma_y]+V[R_y](E[\sigma_y])^2 \tag{27}$$

Where,

$$E[R_y] = \sqrt{2\ln(\nu t)}+\frac{0.5772}{\sqrt{2\ln(\nu t)}} \tag{28}$$

$$V[R_y] = \frac{\pi^2}{6}\cdot\frac{1}{2\ln\nu t} \tag{29}$$

$$E[\sigma_y] = \sqrt{E[\sigma_y^2]} \tag{30}$$

$$V[\sigma_y] = \frac{V[\sigma_y^2]}{4E[\sigma_y^2]} \tag{31}$$

$$\nu = \frac{E[\sigma_{\dot{y}}]}{2\pi E[\sigma_y]} \tag{32}$$

The probability distribution of the maximum response is usually expressed as:

$$F_{y_m}(y) = \exp\{-\exp[-\beta(y-\gamma)]\} \quad (33)$$

β , γ are factors that can be confirmed by $E[y_m]$ and $\sqrt{V[y_m]}$:

$$\beta = \frac{1.2825}{\sqrt{V[y_m]}} \quad (34)$$

$$\gamma = E[y_m] - \frac{0.5772}{\beta} \quad (35)$$

We can obtain the failure probability：

$$p_f = 1 - F_{y_m}(y_0) \quad (36)$$

Equivalent Linear Model

In this paper, Hardin-Drnevich constitutive model is applied accounting for the nonlinearity of soil. When the cycle equivalent shear strain is γ_e, the equivalent dynamic shear modules and damping ratio is:

$$G(\gamma_e) = \frac{1}{1+\gamma_e/\gamma_r} G_{\max} \quad (37)$$

$$\lambda(\gamma_e) = \frac{\gamma_e/\gamma_r}{1+\gamma_e/\gamma_r} \lambda_{\max} \quad (38)$$

Where, G_{max} is the maximum shear module; λ_{max} is the maximum equivalent damping ratio; γ_r is the reference shear strain.

The iterative operations are used in the equivalent linear analysis. First, a troop of shear modulus and damping ratio are evaluated in each soil stratum, and the linear random response analysis is done in this condition. Second, calculate the mean square of the response σ_r of each soil stratum, determine the value of cycle equivalent shear strain γ_e,, in this paper,

$$\gamma_e = \sqrt{\pi/2}\sigma_\gamma$$

then determine the new value of shear modulus and damping ratio by equation is (37) and (38). Thirdly, a new linear random response analysis is done again. Finally, through many iterative operations, still the minus value of shear modulus satisfy the precision of the convergence, then, the final shear modulus in calculation will be the equivalent linear shear modulus of the soil.

NUMERICAL EXAMPLE AND PARAMETER STUDY

Finite Element Model and Parameters

A vertical reference wall, 6m high and 0.5m wide at the top, with the foundation and backfill properties shown in Fig.1 has been selected for a detailed study. The wall section has been designed for factor-of-safety (FOS) against sliding, overturning, bearing capacity failure, and eccentricity under static condition. A base width of 3.57m has been determined. In order to considering the boundary effect under earthquake excitation, the viscoelasticity artificial boundaries are used in the FEM model. The random Parameters of materials are shown in Table 1.

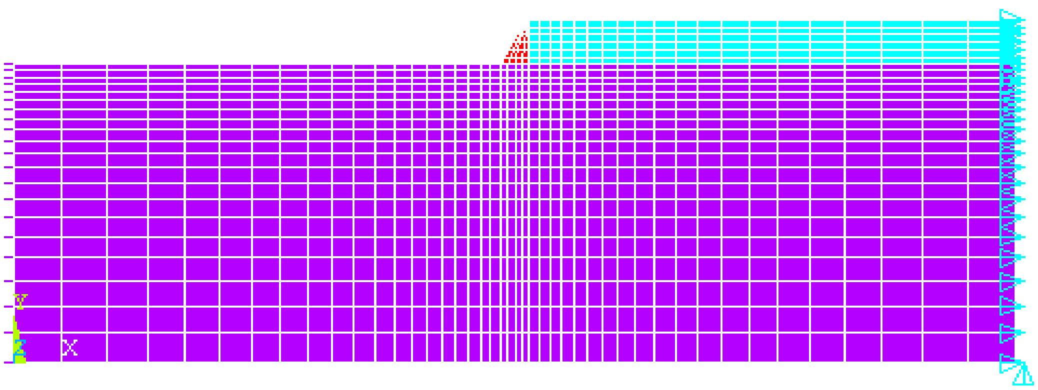

Fig.1. Finite element model of retaining wall

Table 1. Random Parameters in FEM Model

Material Properties	Mean Value			Standard Deviation		
	Foundation Soil	Backfill Soil	Retaining Wall	Foundation Soil	Backfill Soil	Retaining Wall
Density / (kg·m^{-3})	1944	2160	2300	194.4	216	230
Young's modulus / pa	2.964 E8	4.394 E8	2E 10	2.964 E7	4.394 E7	2 E9
Poisson ratio	0.3	0.3	0.2	0.03	0.03	0.02
Damping ratio	0.05	0.05	—	0.005	0.005	—

Random Model for the Seismic Excitation

A model following Kanai and Tajimi is adopted as the stationary power spectrum of the seismic acceleration.

$$S(\omega)=\frac{1+4\xi_g^2\dfrac{\omega^2}{\omega_g^2}}{\left(1-\dfrac{\omega^2}{\omega_g^2}\right)^2+4\xi_g^2\dfrac{\omega^2}{\omega_g^2}}\cdot S_0 \qquad (39)$$

$$S_0=\frac{4\xi_g\sigma_A^2}{\pi\omega_g(1+4\xi_g^2)} \qquad (40)$$

A model following Ou-jinping is also adopted as the stationary power spectrum of the seismic acceleration.

$$S(\omega)=\frac{1+4\xi_g^2\dfrac{\omega^2}{\omega_g^2}}{\left(1-\dfrac{\omega^2}{\omega_g^2}\right)^2+4\xi_g^2\dfrac{\omega^2}{\omega_g^2}}\cdot\frac{4\xi_g\sigma_A^2}{\pi\omega_g(1+4\xi_g^2)}\cdot\frac{\omega_g^2}{\omega_h^2+\omega^2} \qquad (41)$$

In this paper, these models are used and the characteristic parameters of ground motion during earthquakes including ω_g, ξ_g and σ_A are individual 16.5, 0.8 and 1.3531. ω_g is the natural frequency of the site, ξ_g is the damping ratio, σ_A is the mean square root of the acceleration. The Earthquake intensity is 9 and the site soil type is medium soil.

Choose of the Perturbation Step Size and Equivalent Linear Model of Soil

We can know from section 2.2.1, as long as we get the effect of the k_{th} random parameters which cause the coefficient of the variation, then, the sum of the coefficient of the variation will be calculated. Random parameters in calculation are shown in Table 1. The perturbation step size is individual 0.01σ, 0.1σ and σ. The parameters including λ_{max} and γ_r in the equivalent linear model of soil are individual 28% and 2.5×10^{-4}, otherwise, the maximum shear module G_{max} in the foundation and backfill soil is individual 1.14×10^2 MPa and 1.69×10^2 MPa.

Results and Discussions

Stationary Random Seismic Response Regardless of the Parameter Random with Linear Earth-retaining Wall. The impulse response function of the displacement at the top of the wall is shown in Fig.2, the time interval Δt is 0.01s, the total time is 10s and it's corresponding transfer function and response power spectrum also are given in Fig.3 and Fig.4. The Stationary mean square roots response of earth pressure along the back of wall is shown in Fig.5, the linear mean square roots response of resultant force is 339.3374 KN and the action point is 1.811m, and in contrast with the result by M-O method is 2m and the resultant force is 263.606 KN. Stationary mean square roots response including displacement, moment, horizontal resultant force and vertical resultant force for the linear model is shown in Table 2.

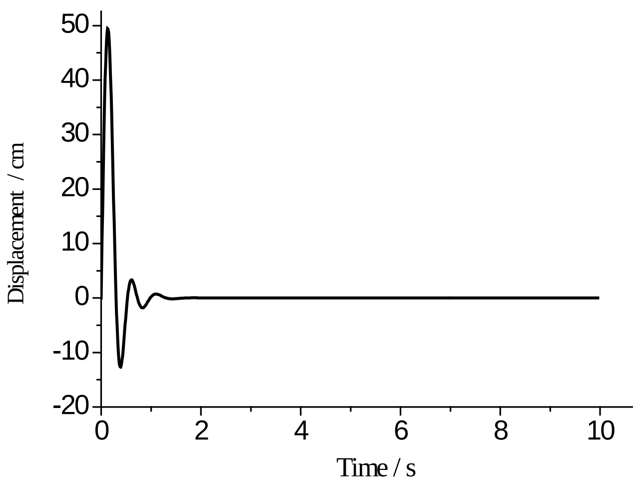

Fig.2. Impulse-response function

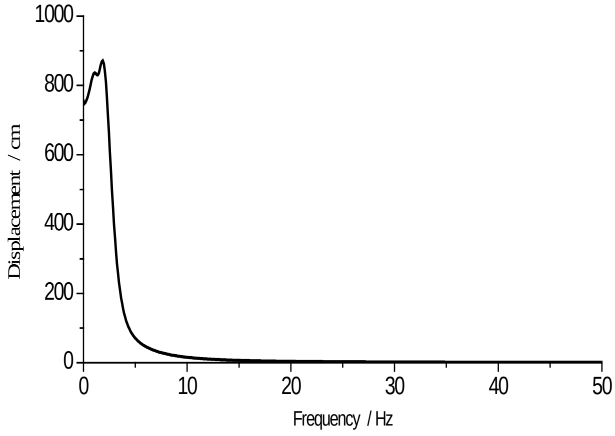

Fig.3. Transfer function

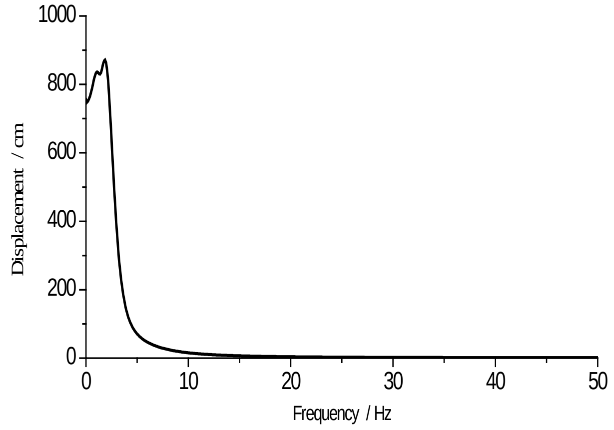

Fig.4. Response Power spectrum

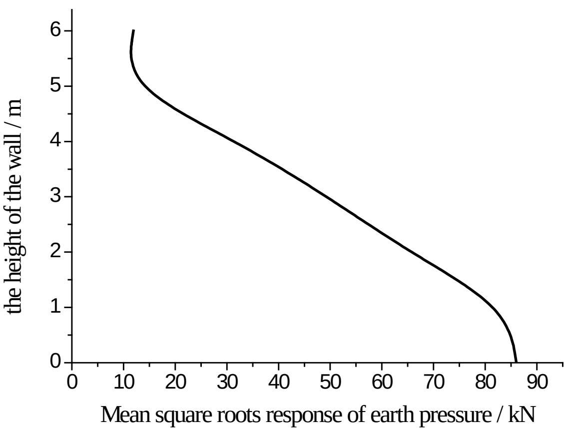

Fig.5. Linear mean square roots response of earth pressure along the back of wall

Stationary Random Seismic Response Considering the Uncertainties Both Structural Parameters and Earthquake Loadings for Linear Earth-retaining Wall.

Table 2. Stationary Mean Square Roots Response for the Linear Model

Mean Square Roots Response	Linear Model
Displacement at the top of wall / cm	0.0149
Displacement at the middle of wall / cm	0.0143
Displacement at the base of wall / cm	0.0137
Displacement at the toe of wall / cm	0.0136
Moment based on the heel of wall / (KN·m)	614.5729
Horizontal resultant force on the base / KN	683.8612
Vertical resultant force on the base / KN	280.733

Stationary Random Response Analysis Based on the Perturbation Law (Method 1). Coefficient of the variation with the horizontal displacement root-mean-square response is shown in Table 3~4. The perturbation step size is individual 0.01σ, 0.1σ and σ. we can see the influence degree of all the factors. The foundation soil's young's modulus, density and damp ratio are the primary effect factor.

Stationary Random Response Analysis Based on Monte-Carlo (M-C) Simulation (Method 2). The Result of M-C simulation (number=1000) can be found from Table 5. In this paper, the simulation number of 100, 1000 and 10000 is individual analyzed. Curve with horizontal displacement root-mean-square response and simulation number at the top of the wall is shown in Fig.6.

Result Comparision of the Two Methods. Result comparison of horizontal displacement root-mean-square response is shown in Table 6. We can see that the error is individual 2.95%, 6.30% and 5.42% in Method 1 if take the M-C simulation (number=10000) as the standard. So, The result verifies that the proposed method is applicable to the analysis of uncertain structural system under the random seismic excitations.

Parameters Study and Sensitivity Analysis. Fig.7~Fig.8 shows the results of sensitivity analysis. We can see that the foundation soil's young's modulus, density, damp ratio and backfill soil's density are the primary effect factor according to the result of horizontal displacement root-mean-square response. But, the Foundation soil's young's modulus, Poisson ratio and damping ratio are negatively correlated to horizontal displacement root-mean-square response.

Table 3. Coefficient of the Variation with the Horizontal Displacement Root-mean-square Response at the Top of the Wall

Location	Random Parameters	Perturbation Step Size		
		0.01σ	0.1σ	σ
Foundation soil	Density	6.752×10^{-2}	5.739×10^{-2}	5.739×10^{-2}
	young's modulus	6.752×10^{-2}	8.440×10^{-2}	8.474×10^{-2}
	Poisson ratio	0	0	1.688×10^{-3}
	damping ratio	6.752×10^{-2}	4.727×10^{-2}	4.794×10^{-2}
Backfill soil	Density	3.376×10^{-2}	3.376×10^{-2}	3.545×10^{-2}
	young's modulus	0	3.376×10^{-3}	3.714×10^{-3}
	Poisson ratio	0	0	2.701×10^{-3}
	damping ratio	0	0	2.026×10^{-3}
Retaining wall	Density	0	0	1.350×10^{-3}
	young's modulus	0	0	0
	Poisson ratio	0	0	0

Table 4. Coefficient of the Variation with the Horizontal Displacement Root-mean-square Response at the Base of the Wall

Location	Random Parameters	Perturbation Step Size		
		0.01σ	0.1σ	σ
Foundation soil	Density	7.353×10^{-2}	5.88×10^{-2}	5.588×10^{-2}
	young's modulus	7.353×10^{-2}	8.088×10^{-2}	7.978×10^{-2}
	Poisson ratio	0	0	2.206×10^{-3}
	damping ratio	7.353×10^{-2}	5.147×10^{-2}	4.853×10^{-2}
Backfill soil	Density	3.677×10^{-2}	3.309×10^{-2}	3.346×10^{-2}
	young's modulus	0	3.677×10^{-3}	2.941×10^{-3}
	Poisson ratio	0	0	1.103×10^{-3}
	damping ratio	0	0	2.206×10^{-3}
Retaining wall	Density	0	0	7.353×10^{-4}
	young's modulus	0	0	0
	Poisson ratio	0	0	0

Table 5. Result of M-C Simulation (number=1000)

Root-mean-square Response	Mean Value	Standard Deviation	Coefficient of the Variation (%)
Displacement at the top of wall / cm	1.5451	0.18177	11.76
Displacement at the middle of wall / cm	1.4777	0.17016	11.52
Displacement at the base of wall / cm	1.4157	0.16081	11.36
Displacement at the toe of wall / cm	1.4131	0.1605	11.36
Moment based on the heel of wall / (KN · m)	346.41	69.305	20.01
Horizontal resultant force on the base / KN	627.00	117.20	18.69
Vertical resultant force on the base / KN	698.42	139.73	20.00
Displacement at the top of wall / cm	287.50	45.909	15.97

Table 6. Result Comparison of Horizontal Displacement Root-mean-square Response

Method	Mean Value	Standard Deviation	Coefficient of the Variation (%)	Error (%)
Method 1 ($\triangle = 0.01\sigma$)	1.481	0.18024	12.17	2.95
Method 1 ($\triangle = 0.1\sigma$)	1.481	0.17402	11.75	6.30
Method 1 ($\triangle = \sigma$)	1.481	0.17565	11.86	5.42
Method 2 (100)	1.4058	0.16336	11.62	7.34
Method 2 (1000)	1.5451	0.18177	11.76	6.22
Method 2 (10000)	1.5375	0.19278	12.54	standard

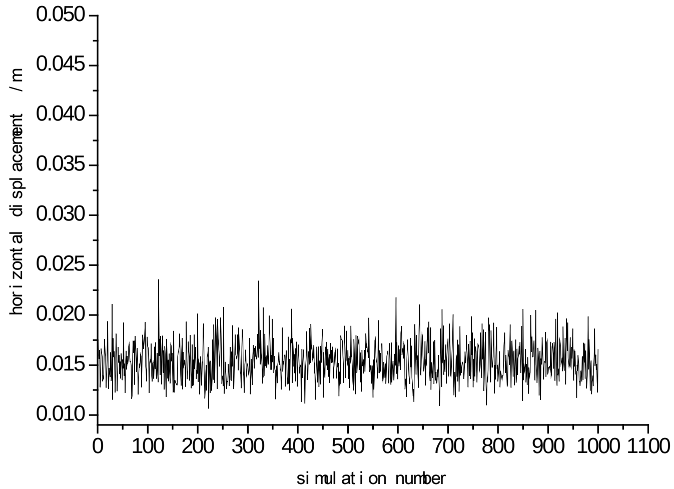

Figure 6. Curve with horizontal displacement root-mean-square response and simulation number at the top of the wall

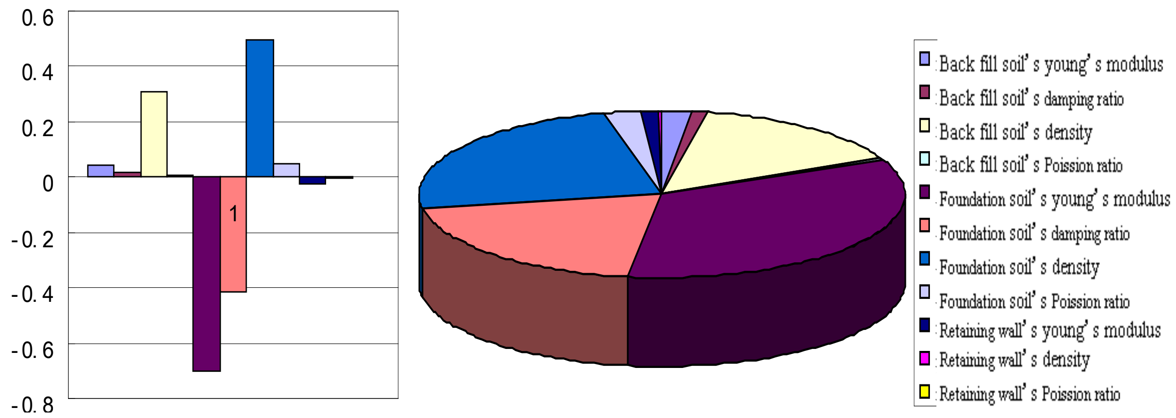

Fig.7. Sensitivity analysis of horizontal displacement root-mean-square response at the top of the wall

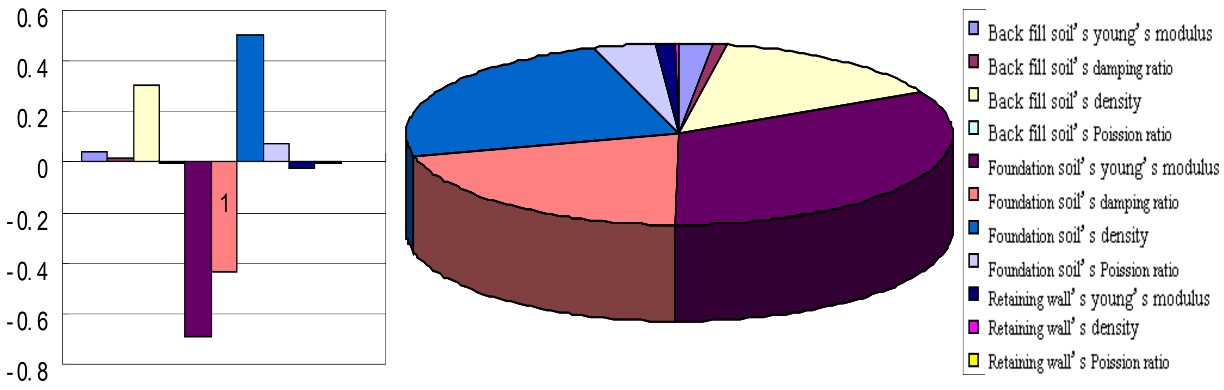

Fig.8. Sensitivity analysis of horizontal displacement root-mean-square response at the bottom of the wall

Stationary Random Seismic Response for Non-linear Earth-retaining Wall. Stationary mean square roots response for the non-linear model is shown in the Table 7. From the Table 2 and Table 7, we can see that the stationary mean square roots response for the linear and non-linear model is very difference. The mean square roots response of displacements is 0.0149 cm, with linear analysis and 0.0677 cm with non-linear analysis. The result shows that the use of a nonlinear soil model is necessary. The coefficient of the variation of the displacement with the different perturbation step size of non-linear analysis is shown in Table 8. The total coefficient of the variation with step size of 0.01σ, 0.1σ and σ is 0.1753, 0.1754 and 0.1758, respectively.

Dynamic Reliability Analysis.

Dynamic Reliability Analysis Based on the First Excursion Failure Criterion. It can also be found from Table 9 and Table 10 that the maximum value response of retaining wall under all kinds of reliability index based on the distribution of Poisson and Markov. The value of linear model is in contrast with the Non-linear model. So we can see from the tables that the displacement, moment and resultant force will increase with the heightening of reliability index, and the aseismic design by linear model will result risk than non-linear model.

Table 7. Stationary Mean-square-roots Response for the Non-linear Model

Mean-square-roots Response	Non-linear Model
Displacement at the top of wall / cm	0.0677
Displacement at the middle of wall / cm	0.05523
Displacement at the base of wall / cm	0.04311
Displacement at the toe of wall / cm	0.04304
Moment based on the heel of wall / (KN · m)	902.0916
Horizontal resultant force on the base / KN	817.1494
Vertical resultant force on the base / KN	291.158

Table 8. The Coefficient of the Variation of the Displacement at the Top of the Wall of Non-linear Analysis

Location	Random Parameters	Perturbation Step Size		
		0.01σ	0.1σ	σ
Backfill soil	Density	5.98E-2	5.75E-2	5.956E-2
	Young's modulus	0	1.477E-4	1.846E-4
	Poisson ratio	2.22E-3	2.3634E-3	4.756E-3
	Damping ratio	1.477E-3	1.329E-3	1.322E-3
Foundation soil	Density	9.9E-2	1.008E-1	9.876E-2
	Young's modulus	1.123E-1	1.121E-1	1.132E-1
	Poisson ratio	4.43E-3	1.9941E-3	3.95E-3
	Damping ratio	6.87E-2	6.88E-2	6.9136E-2
Retaining wall	Density	1.477E-3	1.6987E-3	1.721E-3
	Young's modulus	0	7.386E-5	5.908E-5
	Poisson ratio	0	0	0

Table 9. Maximum Value Response of Retaining Wall under all kinds of Reliability Index Based on the Distribution of Poisson

Model	Linear Model					Non-linear Model				
Reliability index	1.5	2.0	2.5	3.0	4.0	1.5	2.0	2.5	3.0	4.0
Displacement at the Top of wall / cm	3.89	4.48	5.10	5.74	6.87	15.56	18.58	21.59	24.70	30.09
Displacement at the Middle of wall / cm	3.72	4.30	4.88	5.50	6.59	12.68	15.14	17.60	20.13	24.54
Displacement at the Base of wall / cm	3.56	4.11	4.68	5.27	6.31	9.87	11.80	13.72	15.70	19.14
Total earth pressure at the back of wall / KN	939	1067	1201	1343	1596	1055	1251	1448	1652	2008
Moment based on the heel of wall / (KN · m)	1704	1937	2179	2435	2893	2120	2515	2910	3320	4034
Horizontal resultant Force on the base / KN	1888	2148	2419	2705	3214	1918	2276	2634	3006	3653
Vertical resultant force on the base / KN	778	884	995	1112	1321	689	816	943	1075	1305

Table 10. Maximum Value Response of Retaining Wall under all kinds of Reliability Index Based on the Distribution of Markov

Model	Linear Model					Non-linear Model				
Reliability index	1.5	2.0	2.5	3.0	4.0	1.5	2.0	2.5	3.0	4.0
Displacement at the Top of wall / cm	3.18	4.21	4.86	5.43	6.53	14.86	16.98	20.19	22.80	28.69
Displacement at the Middle of wall / cm	3.12	3.95	4.32	5.02	6.24	12.08	13.74	16.30	18.73	22.24
Displacement at the Base of wall / cm	2.84	3.81	4.01	4.83	5.98	8.97	10.53	12.12	14.20	17.84
Total earth pressure at the back of wall / KN	853	975	1092	1243	1483	983	1142	1257	1456	1896
Moment based on the heel of wall / (KN · m)	1598	1863	2017	2311	2753	1995	2351	2692	3091	3872
Horizontal resultant Force on the base / KN	1621	2015	2243	2521	3082	1842	2086	2483	2867	3390
Vertical resultant force on the base / KN	695	703	894	1006	1210	579	783	814	957	1145

Dynamic Reliability Analysis Based on the Maximum Response. Mean and variance of maximum random seismic response for the Non-linear retaining wall considering the random of structural parameters is shown in table 11. It can also be found from Table 12-13 that the maximum horizontal displacement, Force and moment value of different reliability index for the Nonlinear retaining wall considering the random of structural parameters, we can see that the displacement, moment and resultant force will increase with the heightening of reliability index, and the aseismic design by Kanai model will result risk than Ou-jinping model.

Table 11. Mean and Variance of Maximum Random Seismic Response for the Nonlinear Retaining Wall Considering the Random of Structural Parameters

Maximum Random Seismic Response	Kanai Model		Ou-jinping Model	
	Mean Value	Variance	Mean Value	Variance
Horizontal displacement at the Top of wall / cm	4.02	3.91	2.56	2.53
Horizontal displacement at the Middle of wall / cm	3.55	3.59	2.26	2.33
Horizontal displacement at the Base of wall / cm	3.11	3.32	1.98	2.16
Horizontal resultant force on the base / KN	1118.08	839.95	701.28	531.50
Vertical resultant force on the base / KN	413.33	289.65	257.62	182.06
Total earth pressure at the back of wall / KN	589.65	439.93	419.99	210.93
Moment based on the heel of wall / (KN · m)	1111.24	818.13	695.52	516.23

Table 12. Maximum Horizontal Displacement Value of Different Reliability Index for the Nonlinear Retaining Wall Considering the Random of Structural Parameters

			At the Top of Wall / cm	At the Middle of Wall / cm	At the Base of Wall / cm
Kanai Model	Reliability Index (β)	1.5	10.40	9.41	8.53
		2.0	13.74	12.49	11.37
		2.5	17.74	16.16	14.77
		3.0	22.50	20.55	18.82
		4.0	32.32	29.58	27.18
Ou-jinping Model	Reliability Index (β)	1.5	6.69	6.07	5.51
		2.0	8.85	8.07	7.36
		2.5	11.43	10.45	9.57
		3.0	14.52	13.30	12.21
		4.0	20.87	19.16	17.65

Table 13. Maximum Force and Moment of Different Reliability Index for the Nonlinear Retaining Wall Considering the Random of Structural Parameters

			Vertical Resultant Force on the Base / KN	Horizontal Resultant Force on the Base / KN	Moment Based on the Heel of Wall / (KN·m)
Kanai model	Reliability index (β)	1.5	886.36	2489.81	2447.35
		2.0	1134.30	3208.79	3147.65
		2.5	1430.30	4067.15	3983.72
		3.0	1783.67	5091.88	4981.83
		4.0	2512.22	7204.57	7039.66
Ou-jinping model	Reliability index (β)	1.5	554.96	1569.29	1538.5
		2.0	710.80	2024.25	1980.48
		2.5	896.85	2567.40	2508.03
		3.0	1118.97	3215.84	3137.83
		4.0	1576.92	4552.72	4436.29

COMPARATIVE ANSLYSIS OF THE DYNAMIC RELIABILITY RESULT

Compare With the Richard-Elms Displacement Method

In order to compare with the Richard-Elms displacement method, we choose the same material parameters as used in this article. First, the retaining wall is designed by the Richard-Elms displacement method. Assuming the seismic intensity is 9, the ground peak acceleration and the ground peak velocity are individual 0.4. The allowable displacement is 120mm according to Eurocode-8. A vertical reference wall, 6m high and 0.5m wide at the top, has been selected for a detailed study with the foundation and backfill properties shown in Table 1.

According to the dynamic reliability analysis based on the maximum response, using the Kanai model as the input power spectrum , we obtained the horizontal displacement value of different reliability index for the Nonlinear retaining wall considering the random of structural parameters, as shown in Table 14. Contrasting with the designed value 120mm, the reliability index is 1.76 by considering the uncertainty of the nonlinear model. So, we can see that the Eurocode-8 can satisfy the reliability request very well.

Table 14. Horizontal Displacement Value at Different Reliability Index at the Top of Nonlinear Retaining Wall

Reliability index (β)	Certainty Parameters (cm)	Random Parameters (cm)
1.5	8.52	10.33
2.0	10.48	13.67
2.5	12.81	17.66
3.0	15.59	22.42
4.0	21.34	32.24

Compare With the Field Actual Measurements

In order to contrast with the field result, we choose the earth-retaining wall located in Woodland Hills , which fell during the Northridge earthquake. Structural dimensions of retaining wall in the area of Woodland Hills are shown in Fig. 8, the wall is 5m height, 0.5m width at the top and 2.99m width at the base. On account of lacking the field data, we choose the same material parameters as this article. the horizontal ground peak acceleration is 0.4g.

Dynamic reliability analysis of the horizontal displacement value at the top of the wall is done based on the method of maximum response , which including certainty and random analysis of structural parameters, the input power spectrum is Kanai model. Horizontal displacement value with different reliability index at the top of nonlinear retaining wall is shown in Table 15.

According to the Eurocode-8, the allowable displacement is 180mm (300×0.6), the retaining wall's reliability index considering nonlinear and random is 1.25. So, the simulated results are approximately in accordance with the practical case.

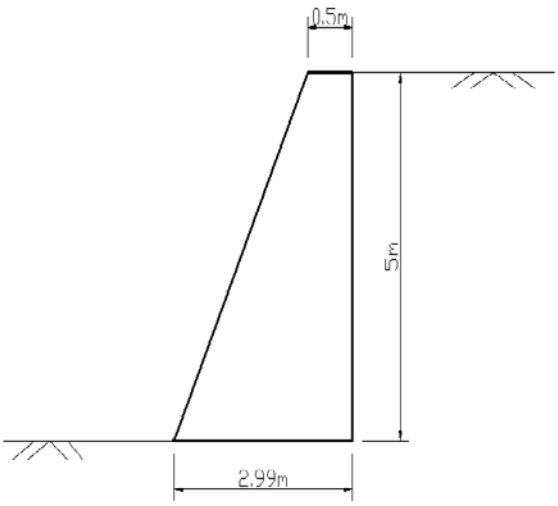

Fig.8. Structural dimension of retaining wall in the area of Woodland Hills，CA.

Table 15. Horizontal Displacement Value with Different Reliability Index at the Top of Nonlinear Retaining Wall

Reliability index (β)	Certainty Parameters (cm)	Random Parameters (cm)
1.5	14.26	21.36
2.0	17.80	29.07
2.5	22.02	38.28
3.0	27.05	49.27
4.0	37.44	71.93

CONCLUSIONS

The following conclusion can be obtained according to the analysis:

(1) The proposed method is suitable to the random response analysis and the dynamic reliability calculation of earth-retaining wall.

(2) It is equally important both structural parameters and seismic excitations, and more reasonable that considering the nonlinear characteristic of earth-retaining wall.

(3) On the basis of the proposed method, the nonlinear random seismic response of retaining wall is analyzed. From the results, we can see that considering the nonlinearity of foundation and backfill soil materials , the mean square roots response of horizontal displacement has larger increase.

(4) The total coefficient of the variation with three perturbation step size is different, and it is also important that choosing the perturbation step size.

(5) According to the Perturbation law, we can see the influence degree of all the factors. Take the horizontal displacement root-mean-square response as an example, we can see that the foundation soil's young’s modulus, density and damp ratio are the primary effect factor.

(6) The maximum value response including the displacement, moment and resultant force will increase with the heightening of reliability index, and the aseismic design by linear model will result risk than non-linear model.

(7) The simulation analysis verifies that the proposed method is applicable to the reliability analysis of uncertain nonlinear structural system under the random seismic excitations by contrast with the Richard-Elms displacement method and field actual measurements.

(8) It is too conservative to deal with the seismic excitations as stationary process. We should consider the non-stationary characteristic of earthquake loadings in the future.

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