Ground Motion Occurrence Rates for Scenario Spectra

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ABSTRACT

It is common practice for probabilistic seismic hazard analysis (PSHA) to use the uniform hazard spectra (UHS) to describe the ground motion. A short-coming of UHS is that it represents an envelope of many different earthquakes that control the hazard at different spectral periods. As an alternative, the UHS can be broken into a suite was developed by Baker and Cornell (2006), in which conditional mean spectra (CMS) are developed that represent realistic earthquake scenarios given a specified spectral acceleration at a single period. Using only the CMS as scenarios is too restrictive and does not provide enough scenarios to reproduce the hazard. The concept of the CMS is expanded to produce three scenario spectra for each CMS. The three scenarios represent the mean (the CMS) and two lower fractiles of the conditional spectra. Using these three scenario spectra for three difference spectral periods (0.2, 0.5, and 2.0 sec) and four different return periods (250, 500, 1000, and 2500 years) results in 36 scenario spectra. Rates for these 36 representative scenarios can be derived that approximate the hazard curves at all three spectral periods simultaneously. The advantage of the new approach over the CMS approach is that it provides rates of occurrence of the scenario spectra in addition to providing realistic scenario spectra. These scenario spectra with their associated rates of occurrence can be used in seismic risk calculations for estimating the probability of structural performance.

INTRODUCTION

The results of a probabilistic seismic hazard analysis (PSHA) can be used to compute the rates of occurrence of different levels of the ground motion. The most common approach is to consider the rate of occurrence of a single ground motion parameter, such as the spectra acceleration at the fundamental period of the structure. In this case, the rate of occurrence can be computed easily from the probabilistic hazard results as shown below.

The hazard curve, Haz(Sa(T)), gives the rate of exceeding the spectral acceleration, Sa, for a given spectral period, T. Therefore, the rate of occurrence of the spectral acceleration between Sa_1 and Sa_2 is given by the difference in the hazard:

$$Rate(Sa_1 < Sa < Sa_2) = Haz(Sa_1) - Haz(Sa_2) \qquad (1)$$

For closely spaced values of Sa_1 and Sa_2, we can approximate this as the rate of the center Sa value:

$$Rate\left(Sa = \frac{Sa_1 + Sa_2}{2} \right) \approx Haz(Sa_1) - Haz(Sa_2) \qquad (2)$$

This approach only provides the rate of occurrence at a single spectral period. If a full spectrum is needed to quantify the frequency content of the ground motion, then the common approach has been to use the uniform hazard spectra (UHS).

In a typical PSHA, a set of UHS is developed covering a range of return periods. By subtracting the probability of exceedance of neighboring UHS, the rate of the spectral values between the two UHS spectra with return periods RP_1 and RP_2 can be computed. For closely spaced UHS, the rate can be associated with the average of the two UHS:

$$Rate\left(Sa(T) = \frac{UHS(T,RP_1) + UHS(T,RP_2)}{2} \right) = \frac{1}{RP_1} - \frac{1}{RP_2} \qquad (3)$$

This approach leads to a set of response spectra and rates of occurrence that represent the hazard curves at all periods. At any spectral periods, the sum of the rates of the spectra that exceed a target value z will be equal the hazard curve for that spectral period.

This approach is simple and straight-forward. The problem with this approach is that it creates a set of spectra that, in general, do not represent individual earthquakes because the UHS represents an envelope of multiple earthquakes. In many cases, the deaggregation of the hazard will show that different earthquakes will control different period ranges of the UHS. For example, a large magnitude distant earthquake may control the long period part of the UHS, whereas, a moderate magnitude nearby earthquake may control the short period part of the spectrum. While the rates of occurrence of the individual spectral periods is correct, they will not occur in a single earthquake. Therefore, using the UHS leads to a ground motion that is too rich in its frequency content, simultaneously exciting a greater number of modes of the structure than would occur in any single earthquake.

An alternative to using the UHS was developed by Baker and Cornell (2006). They define a conditional mean spectrum (CMS) that results in response spectra that are consistent with real earthquakes and avoids the problems of the enveloping multiple earthquakes that occurs with the UHS.

The CMS approach leads to a set of realistic spectra; however, if these are to be used in a risk analysis, then the rates of occurrence of the realistic scenario spectra need to be estimated. In this paper, we expand on the concept of the CMS to develop a set of realistic spectra and provide a method for estimating the rates of each scenario that are compatible with the original hazard curves.

The next section provides a more in-depth discussion of the UHS and scenario spectra, followed by the development of a method to estimate the rates of occurrence of the scenario spectra.

CONDITIONAL MEAN SPECTRA

The CMS is based on the scenario spectrum concept (Baker and Cornell, 2006). The main feature of the realistic scenario spectrum is that it matches the UHS level only at the period of interest, which is typically the expected fundamental period of the structure (T_0). At other periods, the CMS is given by the mean value of the log spectral values given (conditioned on) the UHS value at the period of interest.

Basic steps in constructing a CMS begin with a deaggregation of the hazard for the period of interest (T_0) at a specified return period. The median spectrum, $\hat{Sa}(M,R)$, is computed for the dominant magnitude-distance based on the deaggregation. Next, the number of standard deviations, $\varepsilon(T_0,RP)$, that the UHS is above the median spectrum at spectral period T_0 is found. The mean value of epsilon at the other periods is then found taking into account the correlation of the variability of the ground motion between different spectral periods:

$$\bar{\varepsilon}(T,RP) = \rho(T,T_0)\,\varepsilon(T_0,RP) \tag{4}$$

The correlation of the variability between two spectral periods, $\rho(T,T_0)$, is becoming a standard parameter reported in modern ground motion models. The CMS is then given by:

$$CMS(T,RP) = \hat{Sa}(M,R)\exp\left(\bar{\varepsilon}(T,RP)\sigma(T,M)\right) \tag{5}$$

where $\sigma(T,M)$ is the standard deviation from the ground motion model.

Figure 1 shows a sample uniform hazard spectrum at RP= 2500 years, and the corresponding CMS developed for three distinct periods of T_0: 0.2 s, 0.5 s and 2.0 s. This figure shows how the CMS fall below the UHS for periods away from T_0.

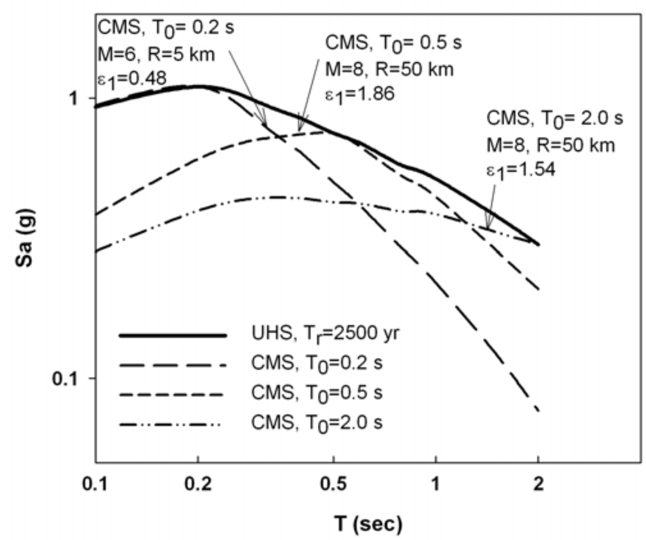

Fig. 1. Sample UHS and associated CMS for three different spectral periods of T_o.

The rates of occurrence of the individual CMS are not as straightforward to estimate as for the UHS. Because the CMS cross each other, the simple form of Eq. (3) cannot be applied. In the example shown in Figure 1, there are three CMS for one UHS. We can't simply assign the UHS rate from Eq. (3) to all three CMS. Doing so would overestimate the total hazard because the individual CMS contribute the hazard at all spectral periods, not just at the period of interest for the CMS. For example, the CMS based on the 1/2500 year UHS at T=0.5 spectral period also contributes to the hazard at T=0.2 sec for a lower return period.

The objective of this paper is to find a set of scenario spectra rates that, when taken together, reproduce the hazard at all three spectral periods simultaneously. We find that using only the CMS to define the scenario spectra, there is no set of rates that reproduces the hazard curves. Additional scenario spectra are needed that represent a range of the epsilon values, rather than just the mean value shown in Eq. (4). For each CMS, we develop two additional scenario spectra that represent lower fractiles of the epsilon values, conditioned on $\varepsilon(T_0,RP)$, as shown in Eq. (6):

$$\varepsilon(T,RP,N) = \rho(T,T_0)\,\varepsilon(T_0,RP) + N\sqrt{1-\rho^2(T,T_0)} \tag{6}$$

where N is the number of standard deviations. The scenario spectra are defined similar to the CMS but allowing for different values of N:

$$Scenario(T,RP,N) = \widehat{Sa}(M,R)\exp\left(\varepsilon(T,RP,N)\sigma(T,M)\right) \qquad (7)$$

For N=0, the scenario spectrum is the CMS. We find that using values of N= 0, N= -1, and N= -2 to define the scenario spectra provides enough flexibility in the scenarios to allow a reasonable approximation to the hazard. Figure 2 shows examples of the three scenario spectra (N= 0, N= -1 and N= -2) for the 2500 year return period with T=0.2 and T=0.5.

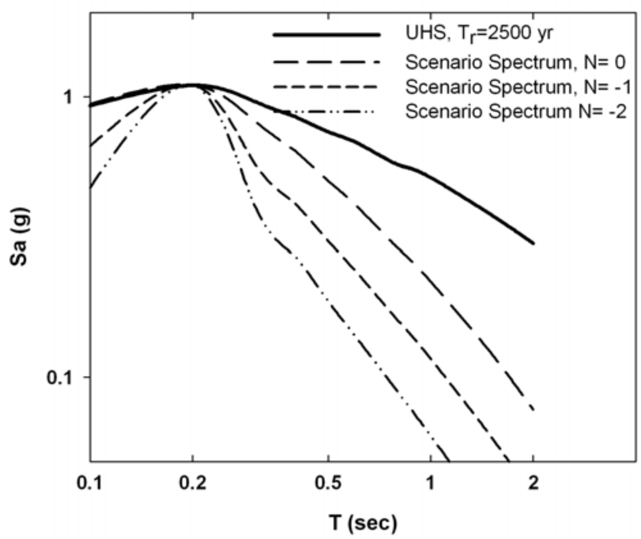

Fig. 2. CMS broken down to scenario spectra for T_0= 0.2 s.

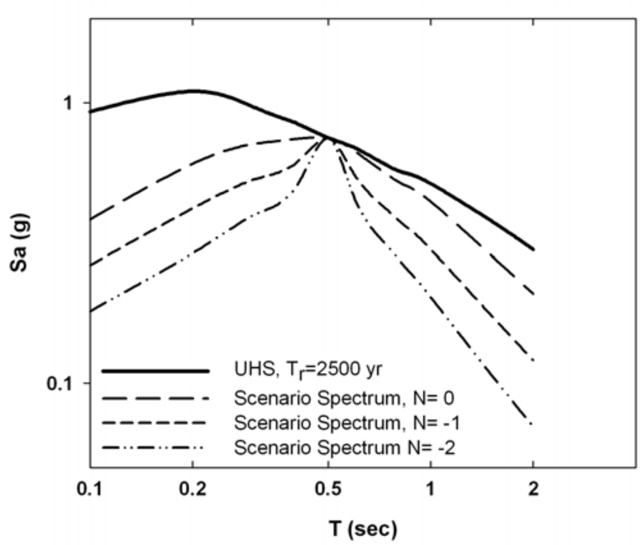

Fig. 3. CMS broken down to scenario spectra for T_0= 0.5 s.

The procedure for determining the rates for the scenario is given below, followed by an example application.

PROCEDURE FOR OBTAINING THE GROUND MOTION RATES ASSOCIATED WITH SCENARIO SPECTRA

We define a step by step procedure to calculate the occurrence rates for the scenario spectra.

1) Construct the uniform hazard spectra (UHS) for various levels of return periods using the conventional probabilistic seismic hazard analysis workflow.

2) At each return period, deaggregate seismic hazard to find the dominant magnitude and distance for several spectral periods (T_0) spanning the short and long period range. The selected suite of T_0 values are not limited to the fundamental modes of the structure. They are selected so that the resulting scenario spectra will cover the entire period range of interest.

3) For each spectral period and return period, develop three scenario spectra using N=0, N= -1, and N= -2 in Eq. (6). For the smallest return period, there is no need to develop the scenario spectra for N values other than N=0.

4) For return periods with positive epsilon values, the scenario spectra will remain lower than the UHS at periods away from T_0; however the case will be reversed for very short return periods with negative epsilon values. In cases in which the CMS (N=0) is greater than the UHS, select the UHS as the scenario spectrum. This only occurs for very short return periods.

5) For the longest return period, set the combined rate of the three scenario spectra (N= 0, N= -1, and N= -2) to be equal to the UHS hazard at T_0. The relative rates for the N= 0, N= -1, and N= -2 are set to 0.6, 0.3, and 0.1, respectively. This is repeated for each spectral period. Note that the sum of rates for the three scenario spectra at T_0 is equal to the rate of UHS since the spectral values at T_0 are the same.

6) Move to the next longest return period, RP_i. At each spectral period, sum the rates of the scenario spectra that exceed the UHS for RP_i. Subtract this summed rate from the UHS hazard (Eq. 8). Continue until the UHS level for the smallest return period is reached.

$$Rate(T,RP_k,N_j) = wt_j\left(\frac{1}{RP_k} - \sum_{i=1}^{k-1}\sum_{l=1}^{3} H\left(Sa_{il} - UHS_k\right)Rate_{il}\right) \qquad (8a)$$

where

$$Sa_{il} = Sa(T,RP_i,N_l) \qquad (8b)$$

$$UHS_k = UHS(T,RP_k) \qquad (8c)$$

$$Rate_{il} = Rate(T,RP_i,N_l) \qquad (8d)$$

and H(x) is the Heaviside function (H(x) is 1 for x>0 and 0 otherwise).

Sample Problem

We demonstrate the solution to the problem through a simple example with just two fault sources that only generate characteristic magnitude earthquakes: Fault A, located 5 km from the site, generates ***M***= 6 earthquakes at a rate of 0.001 events per year; and Fault B, located 50 km from the site, generates ***M***= 8 earthquakes at a rate of 0.005 events per year. First step is to conduct a conventional probabilistic hazard analysis using the given source characteristics. Note that the only source of variability comes from the ground motion model itself. Uniform hazard spectra for various exceedance rates (return periods) are plotted in Fig. 4. The deaggregation for three spectral periods (T=0.2, T=0.5, and T=2.0 sec) is shown in Table 1 for return periods of 250, 500, 1000, and 2500 years.

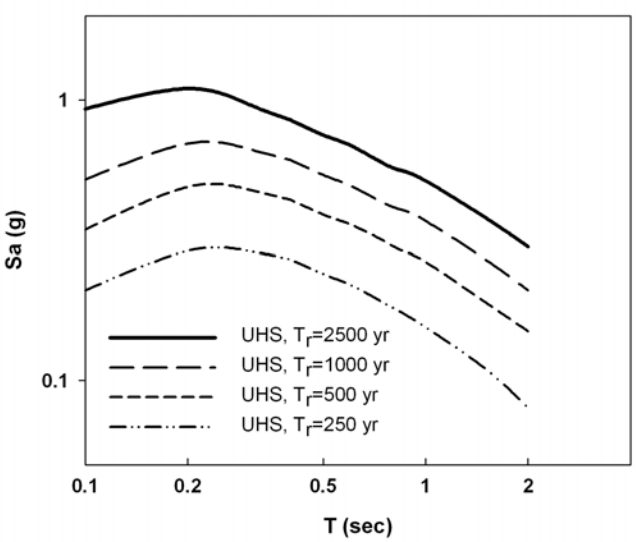

Fig. 4. UHS for the sample problem for a range of return periods.

Table 1. Deaggregation Summary for the Sample Problem

Return Period (yr)	T=0.2 s		T=0.5 s		T=2.0 s	
	M, R	ε_1	M, R	ε_1	M, R	ε_1
2500	M6, R5	0.48	M8, R50	1.86	M8, R50	1.54
1000	M6, R5	-0.23	M8, R50	1.25	M8, R50	0.98
500	M8, R50	0.83	M8, R50	0.65	M8, R50	0.46
250	M8, R50	-0.21	M8, R50	-0.25	M8, R50	-0.52

Step 3 is the development of the suite of scenario spectra for three spectral periods (T=0.2 s, T=0.5 s and T=2.0 s) using Eq. (7) with N= 0, N= -1, and N= -2 for the three spectral periods and the four return periods. This leads to a total of 36 scenario spectra. Because the scenario spectra ordinates for RP=250 years exceeds the UHS level at periods other than T_0 due to the negative epsilons, we use the UHS as the single target scenario for RP=250 years.

We assign the relative weights of 0.6, 0.3, and 0.1 for the N=0, N=-1, and N=-2 scenario spectra, respectively. We found that these weights will allow us to approximate the hazard curves. Tables 2 – 4 list the scenarios in descending order, contributing to ground motion rates at each T, with dominating M, R pair and T_0 value. The first letter in the abbreviation of the rates are as following: ***"S"*** stands for spectra corresponding to short period range (0.2 s), ***"M"*** in short for medium periods (0.5 s) and finally ***"L"*** for long period range (2.0 s). The first subscript is the return period of the ground motion at the period where the realistic scenario spectrum meets the target UHS level from PSHA.

Table 2. Sorted spectral values for scenarios used in T=0.2 s

Scenario Name	T_0 (sec)	M, R Pair	N (Eq. 6)	RP (years)	Sa (g)
S_{2500}	0.2	M6, R5	0	2500	1.100
S_{1000}	0.2	M6, R5	0	1000	0.700
$M_{2500\ A}$	0.5	M8, R50	0	2500	0.606
$M_{1000\ A}$	0.5	M8, R50	0	1000	0.493
S_{500}	0.2	M8, R50	0	500	0.49
$M_{500\ A}$	0.5	M8, R50	0	500	0.402
$L_{2500\ A}$	2.0	M8, R50	0	2500	0.396
$M_{2500\ B}$	0.5	M8, R50	-1	2500	0.380
$L_{1000\ A}$	2.0	M8, R50	0	1000	0.368
$L_{500\ A}$	2.0	M8, R50	0	500	0.343
$M_{1000\ B}$	0.5	M8, R50	-1	1000	0.341
S_{250}	0.2	M8, R50	0	250	0.290

Table 3. Sorted spectral values for scenarios used in T=0.5 s

Scenario Name	T_0 (sec)	M, R Pair	N (Eq. 6)	RP (years)	Sa (g)
M_{2500}	0.5	M8, R50	0	2500	0.750
M_{1000}	0.5	M8, R50	0	1000	0.540
$S_{2500\ A}$	0.2	M6, R5	0	2500	0.502
$S_{1000\ A}$	0.2	M6, R5	0	1000	0.485
$L_{2500\ A}$	2.0	M8, R50	0	2500	0.425
M_{500}	0.5	M8, R50	0	500	0.390
$S_{500\ A}$	0.2	M8, R50	0	500	0.372
$L_{1000\ A}$	2.0	M8, R50	0	1000	0.363
$L_{500\ A}$	2.0	M8, R50	0	500	0.313
$S_{2500\ B}$	0.2	M6, R5	-1	2500	0.307
$S_{1000\ B}$	0.2	M6, R5	-1	1000	0.296
$L_{2500\ B}$	2.0	M8, R50	-1	2500	0.268
$S_{500\ B}$	0.2	M8, R50	-1	500	0.250
M_{250}	0.5	M8, R50	0	250	0.240

Table 4. Sorted spectral values for scenarios used in T=2.0 s

Scenario Name	T_0 (sec)	M, R Pair	N (Eq. 6)	RP (years)	Sa (g)
L_{2500}	2.0	M8, R50	0	2500	0.300
L_{1000}	2.0	M8, R50	0	1000	0.210
$M_{2500\ A}$	0.5	M8, R50	0	2500	0.209
$M_{1000\ A}$	0.5	M8, R50	0	1000	0.170
L_{500}	2.0	M8, R50	0	500	0.150
$M_{500\ A}$	0.5	M8, R50	0	500	0.139
$S_{500\ A}$	0.2	M8, R50	0	500	0.129
$M_{2500\ B}$	0.5	M8, R50	-1	2500	0.111
$M_{1000\ B}$	0.5	M8, R50	-1	1000	0.099
L_{250}	2.0	M8, R50	0	250	0.080

Tables 5 through 7 summarize the ground motion occurrence rates and hazard curve ordinates for the scenario spectra obtained once Steps 5 and 6 are followed. Approximated weights are not calculated and listed for ground motion values falling below the RP=250 year UHS level. Lines in bold characters represent the ground motion occurrence rates for the scenarios corresponding to UHS. The right column is the reconstructed form of the approximate hazard curve using the ground motion occurrence rates from the suite of scenario spectra.

Using the rates for the scenario spectra given in Tables 5 through 7, the reconstructed hazard curves are compared to the original hazard curves in Fig. 5. Ground motion occurrence rates for the scenario spectra and UHS do not differ for high return period ground motions; however, they begin to deviate from the original hazard curve with the influence of scenario spectra for N= 0, N= -1 and N= -2 crossing UHS at periods away from T_0. The algorithm succeeds to recover the original hazard curve at the RP=250 year ground motion level for each T_0.

To demonstrate the need for considering multiple values of N (not jut the CMS with N=0), we attempt to reproduce the hazard using only the CMS scenario spectra. Figure 6 shows the resulting hazard curves for this case. Using only the CMS fails to recover the hazard curve, at T=0.5 s and T= 2.0 s.

The robustness of the algorithm will be sensitive to distribution of controlling earthquake scenarios for the site-specific hazard results (M and R pairs at each RP and T), and how the scenario spectra decay at periods away from T_0; mainly dependent on the correlation function between the periods as well as the aleatory variability term of the ground motion model. The proposed set of weighting factors may need adjustments and validation through detailed testing.

Table 5. Ground motion occurrence rates for scenario spectra, for determining the hazard at T=0.2 s

Scenario Name	Sa (g)	Ground Motion Occurrence Rate	Hazard Curve Ordinates
S_{2500}	**1.100**	**0.0004**	**0.0004**
S_{1000}	**0.700**	**0.0006**	**0.001**
$M_{2500\ A}$	0.606	0.00024	0.00124
$M_{1000\ A}$	0.493	0.00036	0.0016
S_{500}	**0.49**	**0.0004**	**0.002**
$M_{500\ A}$	0.402	0.00010	0.002096
$L_{2500\ A}$	0.396	0.00024	0.002336
$M_{2500\ B}$	0.380	0.00012	0.002456
$L_{1000\ A}$	0.368	0.00036	0.002816
$L_{500\ A}$	0.343	0.00024	0.003056
$M_{1000\ B}$	0.341	0.00018	0.003236
S_{250}	**0.290**	**0.00076**	**0.004**

Table 6. Ground motion occurrence rates for scenario spectra, for determining the hazard at T=0.5 s

Scenario Name	Sa (g)	Ground Motion Occurrence Rate	Hazard Curve Ordinates
M_{2500}	**0.750**	**0.0004**	**0.0004**
M_{1000}	**0.540**	**0.0006**	**0.001**
$S_{2500\ A}$	0.502	0.00024	0.00124
$S_{1000\ A}$	0.485	0.00036	0.0016
$L_{2500\ A}$	0.425	0.00024	0.00184
M_{500}	**0.390**	**0.00016**	**0.002**
$S_{500\ A}$	0.372	0.00024	0.00224
$L_{1000\ A}$	0.363	0.00036	0.0026
$L_{500\ A}$	0.313	0.00024	0.00284
$S_{2500\ B}$	0.307	0.00012	0.00296
$S_{1000\ B}$	0.296	0.00018	0.00314
$L_{2500\ B}$	0.268	0.00012	0.00326
$S_{500\ B}$	0.250	0.00012	0.00338
M_{250}	**0.240**	**0.00062**	**0.004**

Table 7. Ground motion occurrence rates for scenario spectra, for determining the hazard at T=2.0 s

Scenario Name	Sa (g)	Ground Motion Occurrence Rate	Hazard Curve Ordinates
L_{2500}	**0.300**	**0.0004**	**0.0004**
L_{1000}	**0.210**	**0.0006**	**0.001**
$M_{2500\ A}$	0.209	0.00024	0.00124
$M_{1000\ A}$	0.170	0.00036	0.0016
L_{500}	**0.150**	**0.0004**	**0.002**
$M_{500\ A}$	0.139	0.000096	0.002096
$S_{500\ A}$	0.129	0.00024	0.002336
$M_{2500\ B}$	0.111	0.00012	0.002456
$M_{1000\ B}$	0.099	0.00018	0.002636
L_{250}	**0.080**	**0.001364**	**0.004**

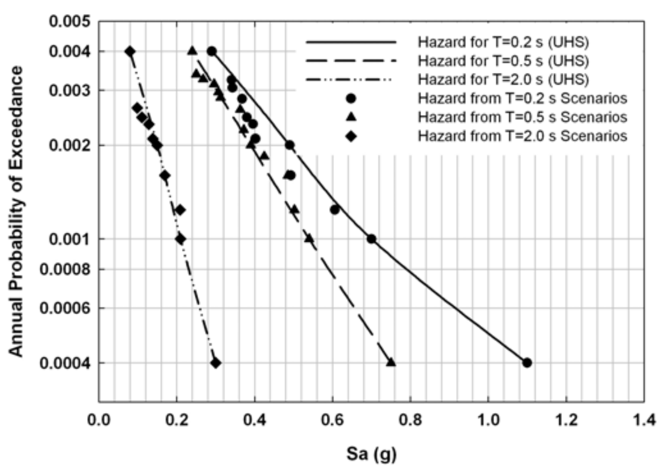

Fig. 5. Comparison of reconstructed hazard curve ordinates from scenario spectra, with the original hazard curve.

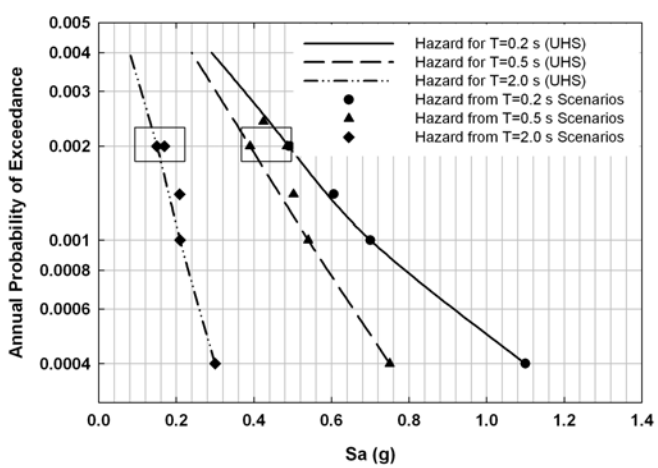

Fig. 6. Case representing the failure to reconstruct the hazard curve when only the CMS were used (N=0 case).

CONCLUSION

Design of critical structures require spectrum compatible time histories that represent the nature of the seismic demand close to reality as much as possible. Emerging alternatives to the uniform hazard spectrum should exhibit a clear compatibility with the probabilistic hazard assessment workflow. The conditional mean spectra address the key shortcomings of the uniform hazard spectrum, but they are not adequate to recover the hazard. By expanding the CMS concept to define a wider range of scenario using three spectra for each CMS, the hazard can be approximately recovered. These scenario spectra with their associated rates of occurrence can be used in seismic risk calculations for estimating the probability of structural performance (e.g. probability of collapse).

For the example application, using three scenario spectra for each CMS and relative weights of 0.6, 0.3, and 0.1 was adequate. For other cases, another set of weighting factors may be needed. The approach described here for developing rates of scenario earthquakes should be evaluated for a wide variety of cases to determine if it is robust.

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