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29 Mar 2001, 4:00 pm - 6:00 pm

Global Methodology for Soil Behavior Identification and its Application to the Study of Site Effects

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GLOBAL METHODOLOGY FOR SOIL BEHAVIOR IDENTIFICATION AND ITS APPLICATION TO THE STUDY OF SITE EFFECTS

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ABSTRACT

Though it is now admitted that non-linear modeling of soil behavior is necessary to represent some important aspects of the soil response under strong ground motion (for example, irreversible settlements and pore-pressure build-up), the elastoplastic models are not yet used in the everyday design processes. One of the obstacles is the difficulty to identify the models' parameters. A methodology to identify the soil mechanical parameters is presented and applied to an elastoplastic model. The strategy is based on the use of minimum physical and easily measurable parameters. The key parameter necessary for characterizing the clay is its Liquidity Limit, while for the sand, the grain size distribution plays an important role. Once the methodology presented and validated comparing the response of the model's response with the available data from the literature, the methodology is used to study the seismic response of the Mexico site.

INTRODUCTION

Many earthquakes such as the 1985 Mexico City, 1995 Hyogoken Nambu and 1999 Chi-chi Taiwan have shown the local site effects due to the non-linear soil behavior, resulting in irreversible settlements and pore pressure build-up leading to liquefaction. Studying such effects necessitates a good knowledge of the site geometry as well as the characteristics of the soil. Once, the site's profile is estimated, the soil response can be analyzed using numerical methods.

Classically two approaches have been used. The first one consists of the equivalent-linear method, which is largely appreciated because of its simplicity and rapidity of calculations. Though theoretically elastoplastic models offer more realistic simulation of soil behavior, in practice they are not yet widely used. One of the obstacles is the difficulty to which one is confronted to identifying such models' parameters.

In practice, the lack of geotechnical data is common at the moment of making seismic studies and often, one ends up using data, which are not coherent between them. In this paper, we present a methodology to identify the soil behavior parameters with a minimum laboratory data. The elastoplastic one-dimensional model implemented in the program CyberQuake (H.Modaressi *et al.*, 1997), which is a derivation of Hujieux's model (Hujieux, 1985), has been used.

The strategy is based on the use of easily measurable parameters. For example, the most important parameters, which influence clay's behavior, are its Liquidity Limit w_L and its Plasticity Index

I_p (Lambe and Whitman, 1979; Biarez and Hicher, 1994; Bardet, 1997). For sands, the Density Index (I_D) or Relative Density (D_r) and the ratio d_{60}/d_{10} , play such a role (Lambe and Whitman, 1979; Biarez and Hicher, 1994).

The parameter identification methodology is developed for both remolded clays and sands. As the $G-\gamma$ and $D-\gamma$ curves are largely used for the material identification in seismic analyses, we focus our work on such results. Thus, the objective of the soil identification is to obtain the elastoplastic model parameters resulting in a given set of $G-\gamma$ and $D-\gamma$ curves in a shear test. Several authors according to the material type (i.e. Vucetic and Dobry, 1991 for the cohesive soils and Seed *et al.*, 1986 for the cohesionless soils) summarized such curves.

ELASTOPLASTIC MODEL

Experimental results show that only when cyclic shear amplitude is less than 10^{-6} , the soil response is reversible and non-linear elasticity caused by the effect of confinement may describe rather well the stress-strain relationship. Between 10^{-6} and 10^{-4} irreversible deformations take place but the cycles are rather stable and liquefaction is rare under undrained conditions. For this range of deformations the use of linear equivalent modulus with hysteretic damping may be applicable. For larger shear strains, the stress-strain loops get strongly modified due to either densification/dilatancy of the material or the pore pressure increase/decrease. In this range of deformation incremental constitutive equations in the framework of elastoplasticity theory taking into account the evolution of internal variables such as the

porosity of the material can be a good solution.

In what follows, we will only recall a brief overview of the type of constitutive model that is currently used in the CyberQuake program without any detailed description. The model is implemented for seismic analyses of one-dimensional soil geometries. It should be mentioned that under cyclic loading conditions, soil properties vary still more than under monotonic conditions. So special attention is focused on the shear strain amplitude on which, the shear modulus and damping ratio are strongly dependant.

The plastic yield surface follows a hardening regime depending on the plastic shear strain γ^p and the influence of volumetric strain is taken into account through the critical stress σ_c as in the Cam-Clay model:

$$f(\sigma', \tau, \varepsilon^p, \gamma^p) = |\tau| + \sigma' F r(\gamma^p) \quad (1)$$

With:

$$F = 1 - b \ln(\sigma'/\sigma_c) \quad (2)$$

$$\sigma_c = \sigma_{c0} \exp(-\beta \varepsilon^p) \quad (3)$$

Where (σ', τ) and $(\varepsilon^p, \gamma^p)$ are normal and shear stress and plastic strains on a surface parallel to the natural slope. The parameter b controls the form of the yield surface and varies from $b=0$ to 1 passing from a Coulomb type surface to a Cam-Clay type one. The internal variable $r(\gamma^p)$, called degree of mobilized friction, introduces the effect of shear hardening of soil and permits the decomposition of the behavior domain into elastic, hysteretic and mobilized domains, it is given by:

$$r(\gamma^p) = \tan\phi (\gamma^p / (\tan\phi/E_p + \gamma^p))^{nr} \quad (4)$$

The definition of all parameters is given in the Table I.

ELASTOPLASTIC MODEL PARAMETERS

The parameters of the model concern both the elastic and plastic behavior of the soil (Table I). The model parameters are classified according to their estimation method. In this optic, the parameters used in the elastoplastic model are separated in two categories, those that can be directly measured either *in-situ* or in the laboratory and those which, cannot be directly measured.

DETERMINATION OF DIRECTLY MEASURABLE PARAMETERS

Clays

Atterberg limits, though very easy to obtain, have great significance and most behavior parameters can be classified by them (Biarez and Favre, 1972; Bardet, 1997). Therefore, we will try to find the value of different physical parameters of the model using the Liquidity Limit w_L and the Plasticity Index I_p of clay.

Table I. Parameters of the model

Measurable Parameters	
V_s	Shear Wave Velocity
V_p	Compression Wave Velocity
ϕ'	Friction angle at critical state
ψ	Dilatancy angle of the characteristic state line
β	Plasticity compression modulus
Non measurable parameters	
E_p	Plasticity modulus of rigidity
b	Yields function form parameter
n_r	Parameter related to hardening
γ_{ela}	Elasticity domain limit
γ_{hvs}	Hysteretic domain limit
γ_{mob}	Mobilized domain limit
α_w	Parameter representing the amplitude of dilatancy
Initial State	
σ'_{c0}/σ'	Compaction ratio
ρ	Soil unit mass

Determination of V_s and V_p . The isotropic elasticity assumption imposes the following relation between the shear and compression wave velocities and the Poisson's ratio ν : $(V_p/V_s)^2 = 2(1-\nu)/(1-2\nu)$. It shows that only two of the above three parameters have to be determined. When shear wave velocity measurement is not available, it can be estimated by: $V_s^2 = G_{max}/\rho$.

Determination of G_{max} . Laboratory test data suggest that the maximum shear modulus is a function of the void ratio e , the overconsolidation ratio OCR and the mean effective stress σ'_m (Dobry and Vucetic, 1987; Vucetic and Dobry, 1991; Kramer, 1996). Empirical relations can be used to determine this parameter according to the soil type. For example, Hardin (1978) suggested that:

$$G_{max} = 625 \text{ OCR}^k / (0.3 + 0.7e^2) (Pa \sigma'_m)^{0.5} \quad (5)$$

Where Pa is the atmospheric pressure and k is a factor depending on the I_p , such that for I_p between 0-100% k varies from 0 to 0.5. Kallioglou *et al.* (1999) proposed the following relation for the undisturbed normally to lightly over consolidated Greece clays:

$$G_{max} = 1421 e^{-1.505} \sigma'_m^{0.623} \quad (\text{kPa}) \quad (6)$$

Figure 1 shows a comparison between the modulus obtained with the relations suggested by Anastasiadis and Ptilakis (1996), Kokusho *et al.* (1982), Jamiolkowski *et al.* (1991) and Eqs. (5)-(6) as a function of voids ratio e for $\sigma'_m = 200 \text{ kPa}$.

Determination of e . For a normally consolidated clay, the following relation exists between the voids ratio e and the vertical effective stress σ' : $e = e_0 - C_c \log(\sigma')$. Where C_c is the compression index. Different authors propose correlation

between I_p and the C_c (Terzaghi and Peck, 1967; Biarez and Favre, 1972; Bardet, 1997). In this paper we use the correlation given by Biarez and Favre (1972) where: $C_c = 0.009(w_L - 13)$. The strategy for the determination of e knowing the effective vertical stress is gathered in the Fig. 2, where as it can be seen that the e is equal to $G_S w_L / 100$ for $\sigma'_v = 7 \text{ kPa}$ and $G_S w_L / 100$ for $\sigma'_v = 1 \text{ MPa}$, where G_S is the soil specific gravity.

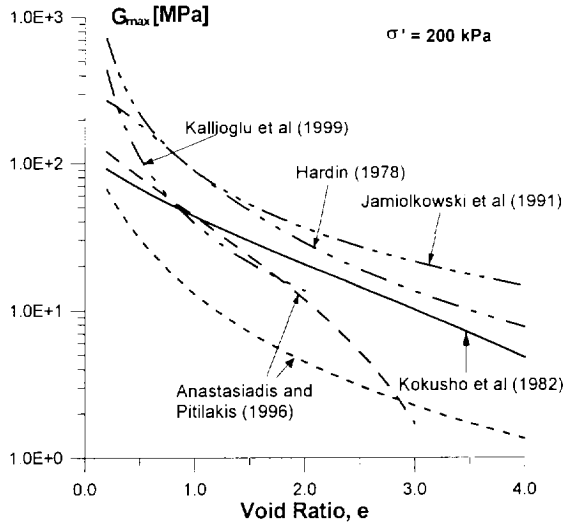


Fig. 1. Comparison of different relationships gives for the maximum shear modulus.

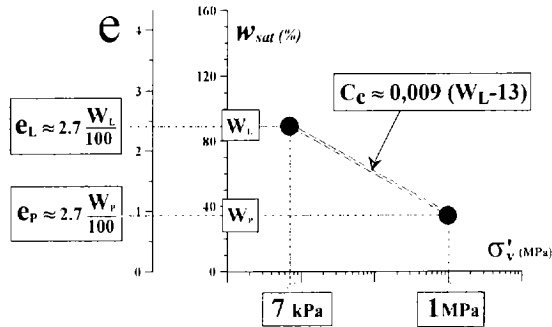


Fig. 2. The slope and the position of the oedometric compressibility curve for normally consolidated clays. (After Biarez and Favre, 1972)

Determination of β . The plastic compressibility modulus β can be expressed in terms of λ and κ parameters of Cam-Clay model using the following relation:

$$\beta = (1+e)/(\lambda-\kappa) \quad (7)$$

Where λ represents the slope of the virgin consolidation line and κ is the swelling slope of an isotropic compression test expressed in the $(e - \ln \sigma'_m)$ plane. These parameters are related to the compression indices C_c and C_s through: $C_c = 2.3\lambda$ and $C_s = 2.3\kappa$. Finally, the values of κ are generally about 4 to 5 times smaller than λ (Biarez and Favre, 1972; Bardet, 1997).

Determination of ϕ' . Biarez and Favre (1972) give a correlation

between the friction angle ϕ' and the Liquidity Limit w_L . In this correlation ϕ' decreases from 32° to 20° when w_L varies from 20% to 100%.

The whole methodology for the determination of elastoplastic model parameters for clays is summarized in Fig 3.

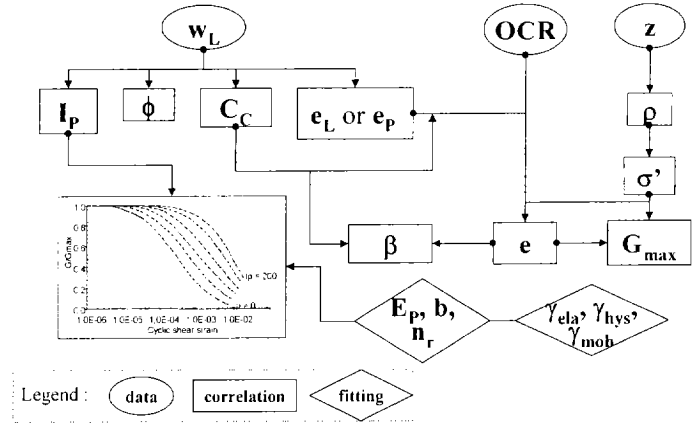


Fig. 3. Methodology for elastoplastic model parameters identification of clays.

Sands

We have decided to work with the Hostun RF sand whose behavior is largely studied and for which lots of experimental data exist. Once the methodology developed, it can be extended to other types of sands.

Determination of G_{max} . After Rivera (1988), the maximum shear modulus G of Hostun RF sand obtained using the cyclic triaxial test can be expressed as a function of the void ratio e and the mean effective stress σ'_m by the relation:

$$G_{max} = 1680 (1.6 - e)^2 / (1 + e) (Pa \sigma'_m)^{0.5} \quad (8)$$

Where Pa is the atmospheric pressure.

Determination of β . Gresillon *et al.* (1974) referred by Saïm (1997), propose a correlation between C_c and the minimum and maximum void ratios e_{min} and e_{max} for the sands, where as it can be seen that D_R is equal to 0.0 for $\sigma'_v = 100 \text{ kPa}$ and 1.0 for $\sigma'_v = 5 \text{ MPa}$ at critical state. Saïm (1997) has gathered all available results on Hostun R.F. sand and proposes $e_{max} = 0.96$, $e_{min} = 0.62$ and $C_c = 0.177$.

Determination of ϕ' . Due to test results obtained by several authors, the friction angle ϕ' of Hostun RF sand can be estimated as 30° . Favre (1980) gives the following relation for the friction angle of sands:

$$\phi'_{pp} = 31.5^\circ + \phi_D + \phi_F + \phi_{Uc} \quad (9)$$

Where ϕ_D , ϕ_F and ϕ_{Uc} are the influence of shape parameters such as grain size, angularity and granulometry distribution ($Uc =$

d60/d10) respectively. Using this correlation the value of $\phi=32$ is obtained for Hostun RF sand.

DETERMINATION OF NOT DIRECTLY MEASURABLE PARAMETERS

As the Cam-Clay model represents better clay behavior while the Mohr-Coulomb is more adapted for sands, the value of b is determined with respect to this consideration. The parameters γ_{ela} , γ_{hys} and γ_{mob} permit the decomposition of the behavior domain into elastic, hysteretic and mobilized domains, so they are important in the liquefaction studies. The parameter n_r has been chosen equal to 0.5 for all cases.

Finally, The parameter E_p governs the evolution of the yield surface toward the total plastic mobilization. It will be determined in order to match the $G-\gamma$ and $D-\gamma$ curves for each type of soil. The compiled curves of Vucetic and Dobry (1991) have been used for clays, while those of Seed and Idriss (1970) and Seed *et al.* (1986) have been used for sands.

Determination of σ'_{CO}/σ' . This parameter is the compaction ratio of soil and represents the position of the critical state pressure σ'_{CO} with respect to the initial state σ' . In the case of clays, it can be determined by the following relation:

$$\sigma'_{CO}/\sigma' = OCR \exp(-b) \quad (10)$$

For the sands, the compaction ratio can be determined using the relations given above to obtain C_c .

APPLICATIONS

A homogeneous 35m deep layer of either clayey or sandy soil has been considered and the degradation of the G and D values with γ in shear tests performed at the mid height of the layer has been studied. Different clays with $I_p = 15, 30$ and 200% and two different densities of Hostun sand (loose and dense) have been chosen in order to study the effect of plasticity of clays and the density of sands on their cyclic behavior respectively. In addition, the role of the ratio of overconsolidation (OCR) on the clay and the model parameters has been evaluated. The equation (6) has been used to calculate the G_{max} value. For the overconsolidated clays an additional parameter multiplying the right hand side of equation (6) of the type OCR^k has been included.

Clayey soil layer

As mentioned above, clays with four different Plasticity Indexes have been studied. We have supposed that the soil has no cohesion. As it can be seen in Table II only the Plasticity Index and the overconsolidation ratio of the soil have been assumed. Other parameters are computed either by using correlations or by curve fitting.

The figure 4 gives the comparison of the computed normalized modulus, G/G_{max} and $D-\gamma$ curves for $I_p = 30\%$ with those given by Vucetic and Dobry (1991). As it can be noticed, the $G-\gamma$ curves match relatively good for all OCRs and I_p values. For strains less than 0.01%, the D is under estimated while for large strains it is over estimated.

Table II. Physical and elastoplastic model parameters for different clays.

I_p [%]	15	30	30	200
OCR	1.0	1.0	2.7	1.0
Physical parameters				
w_L [%]	34	54	54	287
C_c	0.18	0.37	0.37	2.47
e	0.59	0.84	0.72	4.01
ρ [kg/m ³]	2070	1920	1990	1340
σ' [kPa]	355	330	342	230
G_{max} [MPa]	122	68	115	5
Elastoplastic model parameters				
V_s [m/s]	240	188	240	62
V_p [m/s]	450	350	450	115
$\phi=\psi$ [°]	30	26	26	21
β	26	15	14	6
σ'_{CO}/σ'	0.6	0.6	1.0	0.6
E_p	500	500	300	70
γ_{ela}	1.E-10	1.E-10	1.E-10	1.E-10
γ_{hys}	1.E-7	1.E-7	1.E-7	1.E-7
γ_{mob}	1.E-3	1.E-3	1.E-3	1.E-3
b	1.0	1.0	1.0	1.0

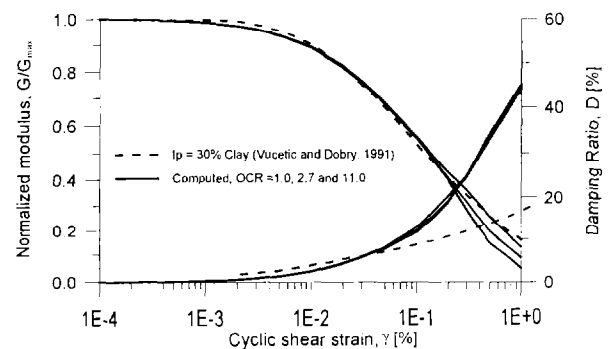


Fig. 4. $G/G_{max} - \gamma$ and $D-\gamma$ curves for $I_p=30\%$ clay with 3 different OCRs (1.0, 2.7 and 11).

As it can be noticed in Table II, the values of the parameters γ_{ela} , γ_{hys} , γ_{mob} , b and n_r remain unchanged for all types of clays at all ratios of overconsolidation. So, though it is signaled that they are evaluated by curve fitting, we have judged not necessary to change them and in general the given values can be used. In this way, there is a significant reduction in the number of parameters to identify.

Sandy soil layer

A unique type of sand with two different densities has been studied. Starting with e_{min} and e_{max} , other parameters have been estimated either by using correlations or by curve fitting. The normalized modulus, G/G_{max} and $D-\gamma$ curves are compared with those given by Seed *et al.* (1986) for the same D_R (Fig. 5). The set of parameters used is given in Table III. It should be noticed that in the case of sands the γ_{hys} and γ_{mob} depend on the D_R , due to the liquefaction potential. As it can be noticed, the $D-\gamma$ curves are much better for sands than clays.

Table III. Physical and elastoplastic model parameters for Hostun RF sand with different D_R .

D_R [%]	40	80
Physical parameters		
e_0	0.82	0.69
C_c	0.17	0.17
ρ [kg/m ³]	1910	1980
σ'_v [kPa]	328	340
G_{max} [MPa]	115	170
Elastoplastic model parameters		
V_s [m/s]	245	295
V_p [m/s]	460	550
$\phi=\psi$ [°]	30	30
β	25	23
σ'_{CO}/σ'	0.5	4.0
E_p	50	220
γ_{ela}	1.E-10	1.E-10
γ_{hys}	1.E-4	1.E-7
γ_{mob}	1.E-1	1.E-3
b	0.1	0.1

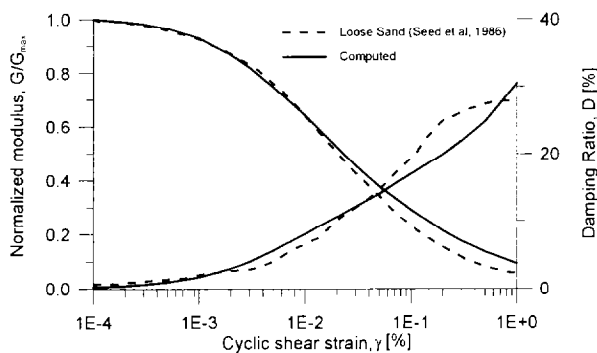


Fig. 5. $G/G_{max}-\gamma$ and $D-\gamma$ curves for loose Hostun RF sand.

APPLICATIONS TO STUDIED SITES

In order to examine the validity of the proposed methodology of parameter identification we have applied it to one site; the seismic response of Mexico City to the September 1985

earthquake.

After the 1985 Mexican earthquake, several authors have studied the response of the SCT site in the city of Mexico because of the registered amplification. Several soil profiles have been proposed to model such amplification. We can cite the papers by Dobry and Vucetic (1987), Seed *et al.* (1988), Vucetic and Dobry (1991) and Romo (1995). In all these works the equivalent linear method is used. We will use CyberQuake to perform both equivalent linear method and the elastoplastic approach. This will enable us to evaluate the potential advantage of such an approach and the strategy for the soil parameter identification to define the model parameters.

The seismic input is the accelerogramme registered on the UNAM site with a maximum acceleration of 0.032g and the dominant periods of 1 and 2 seconds (Seed *et al.* 1988). This signal has been measured on the bedrock near the city. The model of soil profile proposed by Seed *et al.* (1988) has been used (Fig. 6).

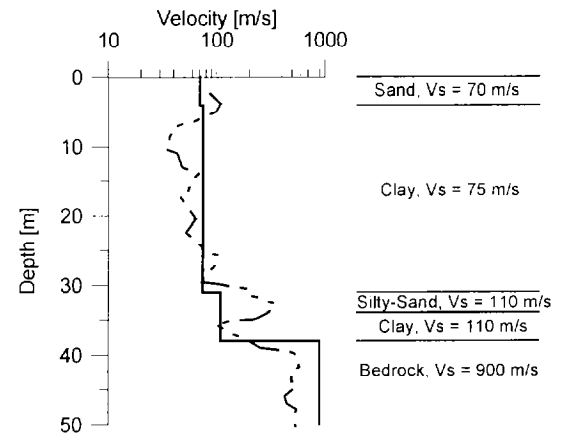


Fig. 6. Profile used (Seed *et al.*, 1988).

We note that the spectral response of this model is also similar to the observed one but the calculated acceleration at the surface is less than the observed one (Fig. 7). The comparison of the response obtained with the two approaches shows that the equivalent linear model does not take into account the degradation of the fundamental period of the profile subjected to a strong motion. On the contrary, the elastoplastic model changes this period from 1.92 seconds (elastic) to 2.2 seconds.

CONCLUSIONS

A consistent and coherent methodology to determine clayey and sandy soil parameters has been proposed. For clays only Atterberg limits and overconsolidation ratio are necessary to identify the mechanical parameters while for sands the relative density or the void ratio is the dominant parameter.

This methodology has two aims. First, give a handy, easy to obtain and coherent set of parameters to use when no experimental data is available. Second, to be used as the starting

point for cases where geotechnical measurements are available. The next step is to generalize the methodology to natural soil and to validate it by evaluating the response of more sites subjected to natural accelerogrammes.

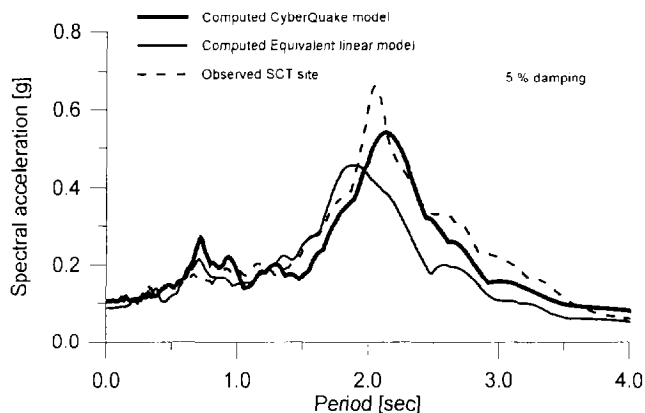


Fig. 7. Comparison of observed and computed response spectra at SCT site in Mexico City.

ACKNOWLEDGMENTS

This study has been done in the framework of the European Community Contract No ENV-CT97-0392.

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