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Permanent Displacements and Tilting Angle of Small Footings on Sand

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SYNOPSIS This paper presents the comparison between the proposed theoretical prediction and experimental results of horizontal and vertical permanent displacements, and tilting angle of small rigid square and rectangular footings on relatively uniform sand. Old and new lumped parameters for different modes of vibrations, i.e. horizontal, vertical and rocking vibrations are used in the analysis. Some good agreement is found between the theory with the new lumped parameters and experiments, but the relevant dynamic soil properties are still the most critical parameters that need to be measured. Finally, effects of frequency and rocking heights are also investigated.

INTRODUCTION

The design problem of shallow foundations for structures subjected to cyclic loading has carefully been investigated in recent years. Besides bearing capacity and settlement, which are normally the two major requirements to be satisfied in the design of foundations under static condition, permanent displacement is an important factor to be considered under dynamic condition. The failure modes can be in the form of sliding, sinking and tilting depending on loading conditions. These modes are always coupled, and can rarely be categorised simply as bearing and sliding.

To study the behaviour of foundations of structures subjected to cyclic loading, it was decided to carry out tests on small-size timber footings in a laboratory controlled situation. The load applied to the footing consists of an initial constant (static) load and a cyclic load. The footing sizes, the properties of sand and equipment used were described by Truong (1991b and d, and 1992). The differential equations and new expressions of stiffness and damping to calculate the horizontal and vertical permanent displacements and tilting angle were mentioned in the paper by Truong (1991a, 1995). Note that old lumped parameters can be found in normal textbooks of soil dynamics (Richart et al., 1970). The horizontal permanent displacement could only when the combination of the constant load and the amplitude of cyclic load exceeded the maximum static horizontal load irrespective of the value of the constant load or the amplitude of the cyclic load (Truong 1991b & 1992). The condition of the horizontal permanent displacement can be represented as a horizontal displacement slider in a proposed lumped parameter model for horizontal permanent displacement (Truong 1992).

In this paper, calculated time-dependent horizontal vibration amplitude, horizontal and vertical permanent displacements, and tilting angle of the footings are compared with those of the experiment for different shapes of footings, frequencies and different modes of vibrations, e.g. pure sliding, and coupled sliding and rocking condition. Sensitivity analysis has been performed on a spreadsheet, which has been written to calculate footing properties, e.g. moment of inertia; resilient displacement, the horizontal and vertical permanent displacements, tilting angle of the footing (Truong 1991c or 1992). Different combinations of different values of stiffness and damping for different modes of vibrations, which are all required to calculate the above permanent displacements, are defined as whether the method **LTH** (Luco et al. 1971, Timoshenko et al. 1951 and Hall 1967) or the method **BTH** (Bycroft 1956, Timoshenko et al. 1951 and Hall 1967) by using the first letters of the names of the authors.

EQUIPMENT AND EXPERIMENTAL LAYOUT

The timber footings were located initially on the surface of the sand mass and the effect of small degree of embedment of the footing in the sand examined. Block footings were made of particle board 150 mm thick of different shapes, square (406 mm x 406 mm and 450 mm x 450 mm) and rectangular (288 mm x 576 mm) but all having the same area of 1648 cm^2 , except the larger square which was 2025 cm^2 (Truong 1991d or 1992). To obtain a rough texture on the contact surfaces, the base and the sides of footings were coated with the test sand, using ARALDITE K80 kit with a Resin/Hardener ratio 5/1 as recommended by the supplier. The wind load or any cyclic load was simulated by using a servo-controlled hydraulic system to apply static and cyclic loads varying in intensity and frequency. The applied loads were recorded by using a load cell; the horizontal and vertical displacements were recorded by using a Linear Variable Differential Transformer (LVDT) and two potentiometer-type transducers, respectively. Tilting angles were calculated based on the differences between the two vertical potentiometer-type transducer over a known distance. The distance between the two-potentiometer type transducers are about 200 mm to 300 mm, which depends on the sizes of the footings. The load cell, LVDT and the two transducers were connected to a four channel Watanabe heat pen recorder (Fig.1).

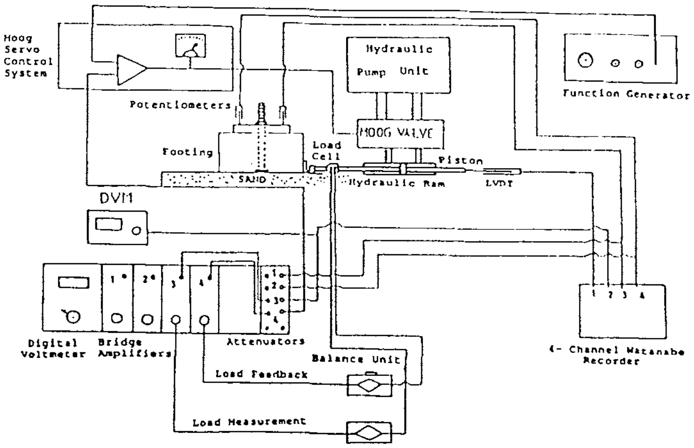


Fig. 1 Schematic Diagram of The Apparatus.

The footings were placed on a large pit of dry sand which was prepared by pluvial deposition (Kolbuszewski, 1948). Some relevant soil properties:

Dry Density .... 1510 Kg/m^3
Max. Dry Density .... 1675 Kg/m^3
Min. Dry Density .... 1465 Kg/m^3
(According to SAA AS1289 E5.1)

Specific Gravity 2.65
Mean Grain Size .... 1.2 mm
Uniformity Coefficient 1.87

Details of results of static tests, different modes of horizontal permanent displacements and three criteria of failure for the shallow foundations under cyclic loading were presented in the paper by Truong (1991d or 1992).

COUPLED ROCKING AND SLIDING OF THE RIGID FOOTING ON THE ELASTIC HALF SPACE

The rocking and sliding modes are frequently encountered as coupled motions. The analytical treatment of this coupled mode of vibration can be handled using the equations of motion of the centre of gravity (C.G.) of the footing given by Ratay (1971), Kuppusamy (1977), Moore (1985), Richart et al. (1970) and also including the Coulomb friction forces for horizontal and vertical directions denoted as F_x , and F_y , respectively. Truong (1991a or 1992) has presented the following equations :

$$m\frac{d^2z}{dt^2}+C_z\frac{dz}{dt}+K_zz+e_f\left(C_z\frac{d\theta}{dt}+K_z\theta\right)+F_z=mg \quad \textbf{(1)}$$

$$m\frac{d^2x}{dt^2}+C_x\frac{dx}{dt}+K_xx-h\left(K_x\theta+C_x\frac{d\theta}{dt}\right)+F_x=P_oT+P_o\sin\omega t \quad \textbf{(2)}$$

$$I_c\frac{d^2\theta}{dt^2}+\frac{d\theta}{dt}\left[C_\theta+e_f^2C_x+h^2C_x\right]+\theta\left[K_\theta+e_f^2K_x+h^2K_x-Wh\right]$$
$$+e_fK_xz+e_fC_x\frac{dz}{dt}-hK_xx-hC_x\frac{dx}{dt}=(h_r-h)\,(P_oT+P_o\sin\omega t) \quad \textbf{(3)}$$

where

m is the mass of the footing system.

$P_o + P_o \sin \omega t =$ P(t) is the dynamic horizontal force applied to the footing.

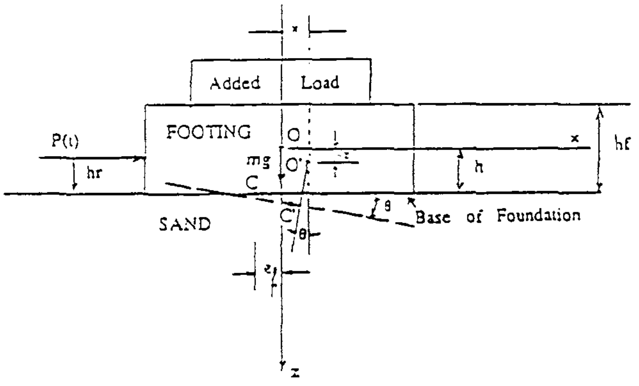
d^2z/dt^2, dz/dt, d^2x/dt^2, dx/dt, $d^2\theta/dt^2$ and $d\theta/dt$ are the second and first derivatives of z, x and θ with time t, respectively.

C_z, K_z, C_x, K_x, C_θ and K_θ are the new values of damping and stiffness of vertical, horizontal and rocking vibrations as discussed above or the old values of damping and stiffness of the above discussed authors.

h_r is the rocking height, which is the vertical distance between the horizontal force and the base of the footing (See Figs.2 and 3)

h is the vertical distance between the base of the footing and the centre of gravity of footing system which is composed of the footing and the known added dead loads on the footing (See Figs. 2 and 3).

e_f, which is the horizontal distance between the centroid of the base and the centre of gravity of the footing, can have any positive or negative value (See Figs. 2 and 3).



O is the original centre of gravity (C.G.)
O' is the later centre of gravity
C is the original centroid of base
C' is the later centroid of base

Fig.2 Notation For Rocking- Sliding Vibration (After Moore, 1985)

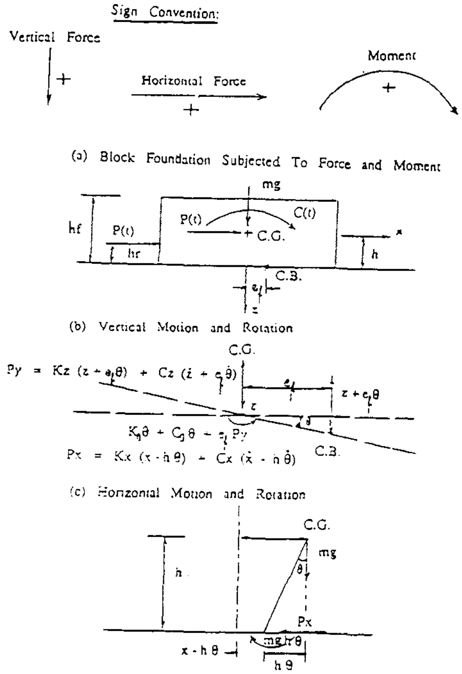


Fig. 3 (a) Block Foundation Subjected to Force and Movement
(b) Vertical Motion and Rotation
(c) Horizontal Motion and Rotation
(After Kuppusamy, 1977)

I_c is the mass moment of inertia of the footing, with respect to the y-axis through the centre of gravity.

These three equations show that the three modes of vibration, vertical, horizontal and rocking, are coupled.

A Lotus spreadsheet, which has been created to solve the above differential equations, was presented in the paper by Truong (1991c or 1992).

PURE SLIDING VIBRATION

Pure sliding vibration occurs only in the theory of vibration of a half-space, because the requirements which can not be achieved in reality are (i) the horizontal force only applies at the centre of gravity of the footing system, (ii) no moment is applied at the centre of gravity of the footing system, and (iii) no moment is applied at the interface between the footing and the sand. These requirements can only be theoretically met if the vertical distance from the centre of gravity of the footing to the sand/footing interface is zero. The calculated horizontal and vertical permanent displacements for pure sliding using the old and new values of stiffness and damping are presented, and of course, no comparison with experimental data is carried out.

Footing R10, which has the dimensions - width of 288 mm and length of 576 mm - and the weight of 257 N, has been used to compared results from using different values of stiffness and damping with the initial frequency of 0.1 Hz. Theoretical results of pure sliding of the footing R10 are presented in four figures (Figs.4, 5, 6 and 7) by using the old formulas of stiffness and damping for different modes of vibrations (Richart et al. 1970). These four figures has been used extensively for sensitivity analysis (Truong 1992), by comparing different values of the horizontal and vertical permanent displacements by changing some important parameters, such as frequency, the constant load and the magnitude of the cyclic load, the modified radius (Chae 1969) and the internal damping constant of 0.05 (Richart and Whitman 1967). In Figs. 5 and 7, lines join points which have the Poisson's ratios of 0.25, 0.275, 0.3 and 0.35 for each value of Young's modulus; the last point with Poisson's ratio of 0.350 is connected to the point with Poisson's ratio of 0.25 for the next value of Young's modulus and so on.

Calculated time-dependent horizontal vibration amplitudes for a pair of the methods of **LTH** and Barkan, 1962 and a pair of the methods of **BTH** and Barkan, 1962 are shown in Figs. 4 and 6, respectively. The type of motion is the motion in damped oscillation case. The calculated horizontal amplitude of vibration of the method **LTH** is slightly higher than that of the method **BTH**, so the horizontal permanent displacement of the method **LTH** is higher than that of the method **BTH** up to about 4%. The amplitude/time curves for both methods of **LTH** and **BTH** are truly sinusoidal curves compared with the time varying maximum amplitude curve of the Barkan's method. The maximum value of the single amplitude of vibration on Figs. 4 and 5 is about 0.00002 mm, which is very small to measure. The methods of **LTH** and Barkan (1962) will be used mainly in the following sections because of minimal difference between the methods of **LTH** and **BTH**.

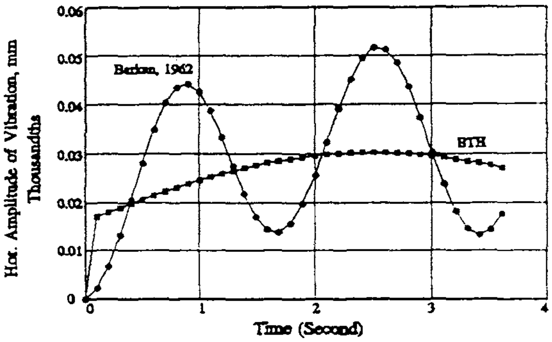


Fig. 4 Calculated Time-Dependent Horizontal Vibration Amplitude Pure Sliding - R10 (288x576) (0.1 Hz.) - LTH & Barkan, 1962

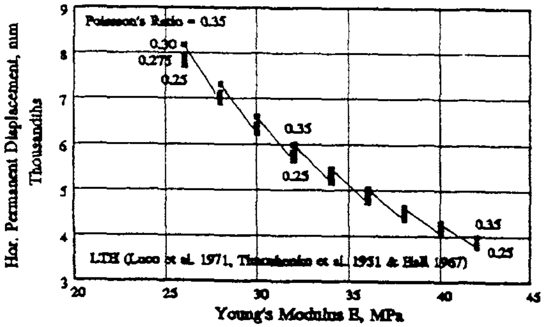


Fig. 5 Calculated Horizontal Permanent Displacement vs E Pure Sliding - R10 (288mm x 576 mm) (0.1 Hz.) - Method LTH

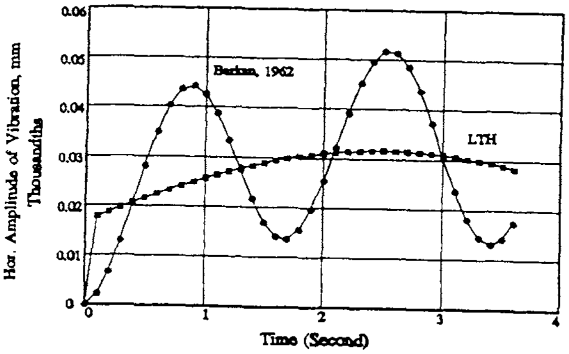


Fig. 6 Calculated Time-Dependent Horizontal Vibration Amplitude Pure Sliding - R10 (288x576) (0.1 Hz.) - BTH & Barkan, 1962

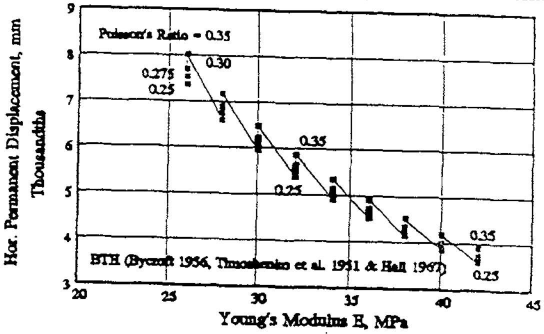


Fig. 7 Calculated Horizontal Permanent Displacement vs E Pure Sliding - R10 (288mm x 576 mm) (0.1 Hz.) - Method BTH

The horizontal permanent displacement calculated based on the combination of the methods of Luco et al. (1971), Timoshenko et al. (1951) and Hall (1967) (**the method LTH**) was found to be in the range of 0.0038 mm to 0.0083 mm (Fig. 5) compared with the values varying from -8.0 mm to +5.5 mm by the Barkan's method for the range of Young's modulus of 26 MPa to 42 MPa. This range of Young's modulus was chosen, based on the typical range of values for the static stress-strain modulus for the normal medium sand between loose and dense condition. Note that the shear moduli measured by Lokuratna (1983) are about from 37 MPa to 135 MPa for the very dense sand and for rigid circular footings with diameters of 100 mm and 150 mm, respectively; Lim (1985) found that the elastic Young's modulus, which is about 81.4 MPa for the dense sand with the density of 1.75 t/m3 and for the rigid rectangular footing with the width of 188 mm and the length of 376 mm, increases with the increase in the average normal static pressure for the rigid footing but there is no significant variation with footing shape. Note that even for different sands, Lambe and Whitman (1979) have presented that the Young's modulus for initial loading for the typical angular and breakable particle sand varies from 14 MPa and 35 MPa for loose and dense states; but the Young's modulus for repeated loadings for the screened medium subangular sand varies from 138 MPa to 241 MPa for loose and dense conditions. The decrease in the calculated horizontal permanent displacements for both methods of **LTH** and **BTH** with the assumed increase in the Young's modulus are shown in Figs. 5 and 7, respectively. The calculated horizontal permanent displacement of Barkan's method can have either positive or negative even at the same Young's modulus (Truong 1992). The slight increase in the calculated horizontal permanent displacement of the method **LTH** is found for Poisson's ratio range from 0.250 to 0.350, but the change of the calculated horizontal permanent displacement becomes noticeable with the method of Barkan (1962).

Truong (1992) has found that the slight increase in the frequency from 0.10 Hz to 0.15 Hz caused a slight increase in the calculated horizontal amplitude of vibration with time for both methods of **LTH** and Barkan (1962), and the horizontal permanent displacement of the method **LTH**. The calculated horizontal permanent displacement always increases with the assumed increase in Poisson's ratio, and the decrease in Young's modulus for the method **LTH**. While the horizontal permanent displacement for the Barkan's method can have positive or negative value depending on the value of Poisson's ratio. The vertical permanent displacement of Barkan's method, showing no trend with the value of Poisson's ratio, can also be negative or positive with different values of Young's moduli. The horizontal permanent displacement, which always increases with the increase in Poisson's ratio with the method LTH, surprisingly also increases with the increase in Poisson's ratio for the Barkan's method only at the frequency of 0.15 Hz. Note that the horizontal and vertical amplitudes of vibration which have been found increase with the frequency up to 15 Hz by Kuppusamy (1977) for the method **BTH** with zero eccentricity.

COMPARISON THEORETICAL RESULTS USING OLD AND NEW VALUES OF STIFFNESS AND DAMPING

The calculated horizontal permanent displacement for old or new values of stiffness and damping always decreases with the assumed increase in Young's modulus and slightly increases with Poisson's ratio up to 0.35, except the method of Barkan (1962). Generally, the calculated horizontal permanent displacement is very small, using the old values of stiffness and damping (Richart et al. 1970), while the vertical permanent displacement is rather large when calculated by using the new values of stiffness and damping. Few typical cases are presented for coupled rocking and sliding vibration in the following sections, e.g. initially for small rocking height of 5 mm and then large rocking heights of 75 mm and 135 mm for cases without and with embedment.

COUPLED ROCKING AND SLIDING CONDITION WITHOUT EMBEDMENT WITH SMALL ROCKING HEIGHT OF 5 MM

If the horizontal force is applied below the centre of gravity of the footing system (footing and added known dead loads), negative rocking moment at the centre of gravity of the footing system will occur. In real situations, this type of vibration occurs when the centre of gravity of any structures, such as buildings or offshore gravity structures under earthquake conditions or dynamic loads applied at or below the bases or footings of structures. The rocking moment of the footing system will be zero, if the horizontal force applied to the centre of gravity of footing. This case cannot be considered as pure sliding, because of soil reaction moment at the interface between the footing and the sand surface. The positive rocking moment will occur if the horizontal load is applied above the centre of gravity of the footing. This type of vibration occurs with all machine foundations and high rise buildings subjected to high fluctuating wind loading.

Footing R10, having the height of the centre of gravity of 75 mm and no added dead load is applied by the same value of cyclic load as in pure sliding but this time the small rocking height of 5 mm is used. Negative rocking moment occurs for this case as discussed above (see Fig.3 for sign convention).

RESULTS BASED ON OLD EXPRESSIONS OF STIFFNESS AND DAMPING

Truong (1992) showed that the calculated time-dependent horizontal vibration amplitudes, the horizontal and vertical permanent displacements for both methods of LTH and Barkan (1962), which have the similar curves as for pure sliding, decrease their values slightly with the small rocking height of 5 mm compared with pure sliding (see also Figs. 4 and 5). The calculated vertical permanent displacement for the method LTH is negative and very small and approximately equal to 0 which is the measured value. Tilting angles (or rocking angles), which can be negative or positive for the same Young's ratio when calculated for both methods LTH and Barkan (1962), are measured as zero. The calculated horizontal permanent displacement for the method LTH increases with the assumed increase in the Poisson's ratio, but this is not happened with Barkan's method. The calculated horizontal permanent displacements versus the number of cycles for different methods tend to underestimate the test results due to the high values of stiffness, as expected, even though the Young's modulus or shear modulus selected is the secant modulus and very small (G = 13 MPa).

RESULTS BASED ON NEW EXPRESSIONS OF STIFFNESS AND DAMPING

Generally, the calculated horizontal amplitude of vibration, the horizontal and vertical permanent displacements slightly decrease with the increase in the rocking height of 5 mm compared with the results in pure sliding (zero rocking height). The calculated time-dependent horizontal vibration amplitude shows that the motion in overdamped cases, as expected due to the high values of damping for low frequencies, e.g. 0.1 Hz, as also the zero horizontal accelerations observed for most cases, e.g. even with the rocking height of 75 mm. Calculated tilting angles always decrease with the assumed increase in Young's modulus for both methods of new LTH (Truong, 1991a) and Truong (1992). The calculated horizontal permanent displacements for both new methods, increasing with the assumed increase in Poisson's ratio, agree well within the range of the observed values of test results (Fig.8). There is absolutely no difference of the horizontal and vertical permanent displacements between the two new methods in this case.

EFFECT OF FREQUENCY

Normandeau and Zimmie (1991) found that there is a definite effect of the frequency of the cyclic loading on the accumulated deformation of the San Fernando sandy silt with an initial constant load. The effect of frequency measured in their investigation is

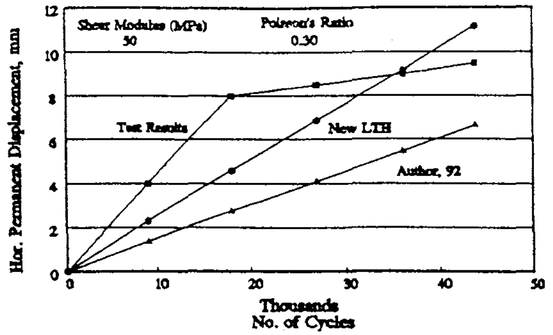


Fig. 8 Horizontal Permanent Displacement Vs No. of Cycles h_r = 5 mm, R13 (0.1 Hz.), New LTH, Author 92 & Tests

substantially less than the inverse-square effect obtained from Newmark's sliding block analogy. They suggested that Newmark's method can be considered as an upper bound of the frequency effect.

When considering the effect of frequency in the pure sliding, the slight increase of frequency from 0.1 Hz to 0.15 Hz. was considered because of the above reason. The other frequencies used in this investigation are 1.0 Hz and 10 Hz. After an extensive curve fitting exercise using the spreadsheet developed to match the horizontal permanent displacement obtained from the theory with that of the test results for different cyclic loads, size and shape of footings, Truong (1992) found that no fixed expression of the shear modulus with the circular frequency has been found even though there is definite trend in which the horizontal permanent displacement is approximately square-inversely proportional to the frequency based on the results obtained in this study. Note also that the test results are limited.

COUPLED ROCKING AND SLIDING CONDITION WITHOUT EMBEDMENT WITH LARGE ROCKING HEIGHTS OF 75 MM AND 135 MM

For the cases with the rocking heights of 75 mm and 135 mm, typical results of the square footing S11 (406 mm by 406 mm) for both methods of new LTH and Truong (1992 or 1995) are considered for the frequency of 1.0 Hz. Two figures 9 and 10 showed the typical tilting angle and the horizontal permanent displacement, respectively. The predicted tilting angle, decreasing with the assumed increase in the shear modulus, is very close to the value of the test result (Fig.9). At the rocking height of 135 mm and the frequency of 1.0 Hz. for the same size of the square footing S11, the predicted horizontal permanent displacement agrees well with that of the test results for the first two thousand cycles (Fig. 10). Note that the prediction did not take into account (i) the sand building at the other end of the footing and (ii) the interface between the footing and the sand might be disturbed during handling the footing and preparation by screeding (Taylor et al. 1981) and (iii) the dynamic shear modulus might change or slightly increase after a number of cycles.

CONCLUSIONS

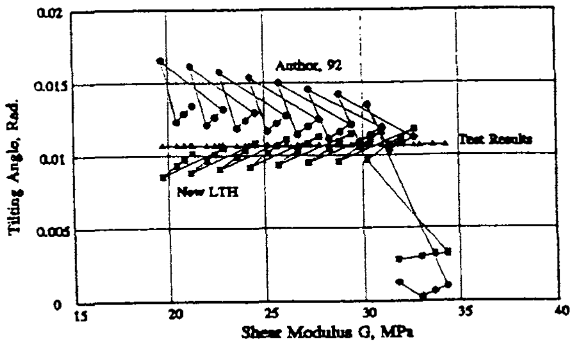


Fig. 9 Tilting Angle vs Shear Modulus
h_r = 75 mm, S11 (406x406) 3A (1 Hz.), New LTH & Truong, 92

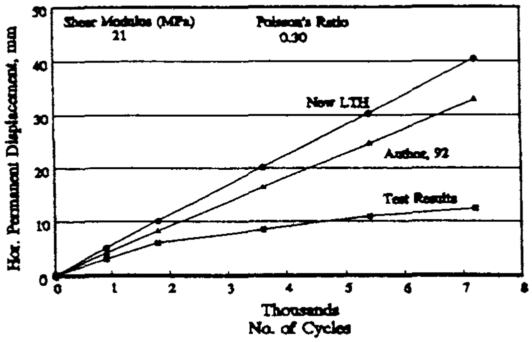


Fig. 10 Horizontal Permanent Displacement Vs No. of Cycles h_r=135 mm, S11(406x406) 2A(1 Hz.), New LTH, Truong,92 & Tests

In general, the prediction of the horizontal permanent displacement using the new dynamic values of stiffness and damping agrees very well with the experimental data even though the linear relationship between the horizontal permanent displacement and the number of cycles was assumed, because the final horizontal permanent displacement is the product of the initial horizontal permanent displacement in the first cycle and the number of cycles.

The old values of stiffness for different modes of vibrations (Richart et al. 1970) tend to underestimate the horizontal permanent displacement because of very high values of stiffness, in which the calculated time-dependent horizontal amplitude of vibration at the frequency of 0.1 Hz is the amplitude of motion for the underdamped cases. While the motion of vibration with the new values of stiffness and damping is overdamped cases because of the higher value of damping, especially at the frequency of 0.1 Hz.

The effect of frequency on permanent displacement should be taken into account in order to get the best agreement with the experimental results. Finally, the higher the rocking height, the higher are the vertical permanent displacement and tilting angles.

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