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Uniform Cycles in Earthquakes: A Statistical Study

A. Haldar
Georgia Institute of Technology, Atlanta, Georgia

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A. Haldar

Assistant Professor, School of Civil Engineering, Georgia Institute of Technology, Atlanta, Georgia

SYNOPSIS In the evaluation of soil behavior due to earthquake motions, uniform intensity load cycles are frequently used instead of irregular-patterned loadings generated during actual earthquakes. A statistical conversion procedure is discussed in this paper based on results available in the literature. The actual irregular stress-time history produced by an earthquake can be represented by uniform amplitude cyclic stresses, although there may be considerable uncertainty associated with them. The statistical relationship proposed here between the earthquake magnitude and the equivalent uniform stress cycles is somewhat different from the relationship commonly used in practice. It is observed in this study that this discrepancy may not yield significant differences in estimating the soil strength in a liquefaction study. However, this variation should not be overlooked in other soil dynamics problems involving earthquake loading.

INTRODUCTION

The behavior of soils during and immediately following an earthquake must be understood properly in order to design a safe and reliable structure. To achieve this objective, representative soil specimens can be tested in a laboratory under dynamic loading conditions similar to those produced by an earthquake. An earthquake loading pattern is extremely irregular. Moreover, the exact future acceleration time history of an earthquake at a particulat site is unknown. Even if a time history of earthquake acceleration is assumed, different shear stress-time histories may be induced depending on the site characteristics. Furthermore, it is very difficult and expensive to duplicate assumed shear stress-time loading conditions on a soil specimen in the laboratory. To avoid these difficulties, a considerable number of laboratory investigations were carried out under uniform cyclic loading conditions in the past, and are still being carried out at present. This vast source of information can be used to predict the behavior of soil under earthquake loading conditions when a reliable correlation between the earthquake loading and the uniform intensity cyclic loading conditions can be found. A reliable correlation between the two loading conditions is necessary, particularly in a liquefaction study.

The problem will be greatly simplified if a unique, deterministic relationship between the two loading conditions can be found. Unfortunately, the process of converting the earthquake loading to equivalent uniform cycles involves quite a few parameters, and a considerable amount of uncertainty is expected in each parameter. The objective of this paper is to identify the factors involved in converting the earthquake motions to equivalent uniform stress cycles, to estimate the uncertainty associated with each of these factors, to express the available information on past recorded earthquake motions in a usable form, and to compare the results thus obtained with other existing methods such as that suggested by Seed and Idriss (1971).

PROBLEM DESCRIPTION

The underlying principles of converting the irregular-patterned earthquake motions to the equivalent number of uniform cycles N_{eq} were well explained by Lee and Chan (1972). According to Lee and Chan, "N_{eq} refers to that number of uniform cycles of stress intensity τ_{av}, which if applied to an element of soil in the field or a sample of the same soil in the laboratory, would have the same effect in terms of the soil strength or deformation as if the actual train of irregular cyclic shear stresses were applied." This concept of N_{eq} is essentially based on Miner's (1945) damage rule. In an earthquake engineering problem, it may be convenient to base N_{eq} calculations on acceleration rather than on stress-time histories because of the direct proportionality between acceleration, force and stress.

To estimate N_{eq} corresponding to an earthquake time history motion, the value of stress level S_L and a soil strength curve must be available. The value of stress level S_L is usually referred to as a percentage of the maximum stress. A soil strength curve can be described as a failure curve representing the relationship of the applied uniform stress and the number of cycles required to cause a soil specimen to fail. The steps involved in the estimation of N_{eq} for a given earthquake motion have been discussed in detail in the literature by Annaki and Lee (1977) and Lee and Chan (1972). The details of the steps involved will not be discussed here due to space limitations. However, the uncertainties associated with each parameter will be quantified in the following sections.

Assumptions in N_{eq} Concepts

When the uniform cycles of stress intensity concept was used in solving problems related to earthquake excitation, some implicit assumptions were made. They include (i) the ground motion is uniform at all sites in the same general area; (ii) the stress-time history at the depth of interest is directly proportional to the acceleration recorded at or near the ground surface; and (iii) for all soils, the laboratory liquefaction test data results can be represented by a single normalized curve relating stress ratio or stress level S_L to the number of cycles causing liquefaction.

These assumptions have been studied extensively by Annaki and Lee (1977), Lee and Chan (1972) and Haldar and Tang (1981). Due to the tectonic nature of earthquake loads, the first assumption seems reasonable. The validity of the second assumption depends on the soil profile through which the acceleration has been propagated. To study these aspects related to N_{eq} evaluation, Lee and Chan (1972) considered six different soil deposits. They concluded that "The value of N_{eq} computed from the surface or near surface time history is appropriate for all other depths within the soil profile." They added that for routine work, it should not be necessary to determine N_{eq} rigorously for more than one location in a soil profile being analyzed. Regarding the third assumption, although a general trend can be established, a wide spread in the soil strength curve has been observed (Fig. 1). The reasons for this spread have been explored elsewhere by Haldar and Tang (1981).

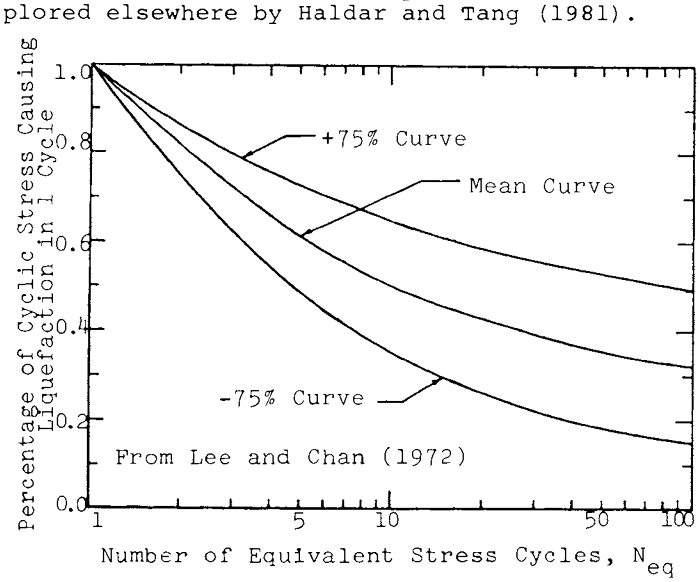

Fig. 1 - Normalized Soil Strength Curves

The general procedure for converting earthquake motions to uniform cyclic motion appears simple with the aforementioned assumptions. Yet, the study would be incomplete without answering the following questions: (i) would the value of N_{eq} be sensitive to the shape of the soil strength curve?, (ii) what stress level S_L should be used?, (iii) how would N_{eq} vary with the magnitude of the earthquake?, and (iv) if past earthquake records are used to find a relationship between N_{eq} and the earthquake magnitude, which one or both of the two horizontal records should be considered? In the following sections, attempts will be made to answer all these questions.

STATISTICAL EVALUATION

Uncertainties in the Soil Strength Curves

Haldar and Tang (1981) explored the area of uncertainty associated with the soil strength curve extensively. It was observed that the soil strength curve depends on N_{eq}, the initial ambient pressure under which the sample was consolidated, relative density of the sample, and the mean grain size of the specimen D_{50}. When the soil strength curves are normalized properly (for details refer to Haldar and Tang (1981)), they can be presented like the curves shown in Fig. 1. However, there is considerable scatter in the data. Observing this, Lee and Chan (1972) proposed upper and lower 75 percentage ranges of the available data along with the mean curve as shown in Fig. 1. As discussed previously, for a given earthquake motion a different value of N_{eq} will be obtained for each soil strength curve. This variation in N_{eq} values due to soil strength is not negligible and will be discussed in the subsequent section.

Earthquake Magnitude and N_{eq} Relationship

For a given S_L and soil strength curve, N_{eq} values can be calculated for past recorded earthquake motions. The values of N_{eq} thus obtained should be correlated with some other characteristic of the corresponding earthquake motions. It has been shown that larger magnitude earthquakes are associated with a longer duration of earth shaking (Bolt, 1973). Since the number of equivalent uniform cucles varies with the duration of earth shaking, it is expected that some type of correlation would exist between N_{eq} and the earthquake magnitude. Earthquake magnitudes expressed in Richter's scale are considered for this discussion.

Lee and Chan (1972) reported values of N_{eq} for 57 earthquakes recorded at or near the ground surface. They also included 12 artificially generated earthquakes in their study. Only earthquakes of magnitudes greater than 5.0 were considered. These data on N_{eq} and earthquake magnitude are used in the subsequent statistical studies.

Observing the trend of the data mentioned above, the relationship between N_{eq} (for a given S_L and soil strength curve) and magnitude M may be represented by the following regression equation:

$$E\,(N_{eq}|M = m) = A + B\,m + C\,m^2 \qquad (1)$$

in which A, B and C are regression coefficients. Eq. 1 will give an expected or mean value of N_{eq} for a given earthquake magnitude m. The scatter of the data about the mean curve is observed to be approximately constant. Thus, the variance of N_{eq} or Var $(N_{eq}|M{=}m)$ is assumed to be constant. The regression coefficients and variance are estimated later in this paper.

Stress Level S_L Selection

The choice of S_L is primarily subjective, based on literature review. Seed and Idriss (1971) used $S_L = 0.65$ in their study. Intuitively, the choice of S_L could be based on the degree of sensitivity of the N_{eq} versus magnitude relationship to different soil strength curves. To find a suitable value for S_L, the data reported by Lee and Chan (1972) and Lee (1975) are considered here. For S_L = 65%, 75% and 85% and considering the mean and ± 75% of data soil strength curves (Fig. 1), 9 sets of data on pairs of N_{eq} and M values are generated for each of the 69 earthquake time histories. Regression analysis is then performed on each set of data, thus obtaining 9 separate regression equations. The corresponding regression coefficients A, B and C and Var $(N_{eq}|M)$ of each regression equation are tabulated in Table 1. The results for S_L = 65% and 75% are plotted in Figs. 2 and 3. No figure is given for S_L = 85% due to lack of space. For comparison, the N_{eq}

TABLE I. Regression Equations Between N_{eq} and M, $M \geq 5.0$

		$E(N_{eq}\|M{=}m) = A{+}Bm{+}Cm^2$				
Eq. No.	S_L	Soil Strength Curve	A	B	C	Var $(N_{eq}\|M)$
1	0.65	+75% Curve	219.5	-74.4	6.8	162.8
2	0.65	Mean	174.6	-59.9	5.4	78.7
3	0.65	-75% Curve	169.6	-58.5	5.3	66.1
4	0.75	+75% Curve	92.0	-31.2	2.9	31.9
5	0.75	Mean	106.1	-36.4	3.3	29.1
6	0.75	-75% Curve	120.0	-41.4	3.8	33.1
7	0.85	+75% Curve	48.8	-16.5	1.5	8.1
8	0.85	Mean	66.4	-22.9	2.1	14.4
9	0.85	-75% Curve	92.0	-31.7	2.9	19.3

versus M relationships proposed by Seed and Idriss (1971) are also plotted in these figures after appropriate modification. Seed and Idriss (1971) suggested that when $S_L = 0.65$, the number of equivalent cycles be 10, 20 and 30 cycles, corresponding to the earthquake magnitudes of 7.0, 7.5 and 8.0, respectively. The following observations can be made from these figures: (i) the effect of variation of the soil strength

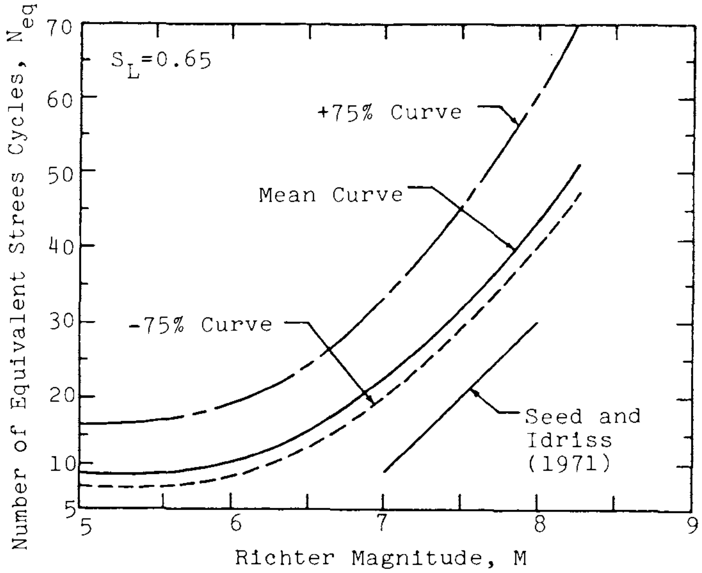

Fig. 2 - Relationship Between Number of Equivalent Stress Cycles and Magnitude for $S_L = 0.65$

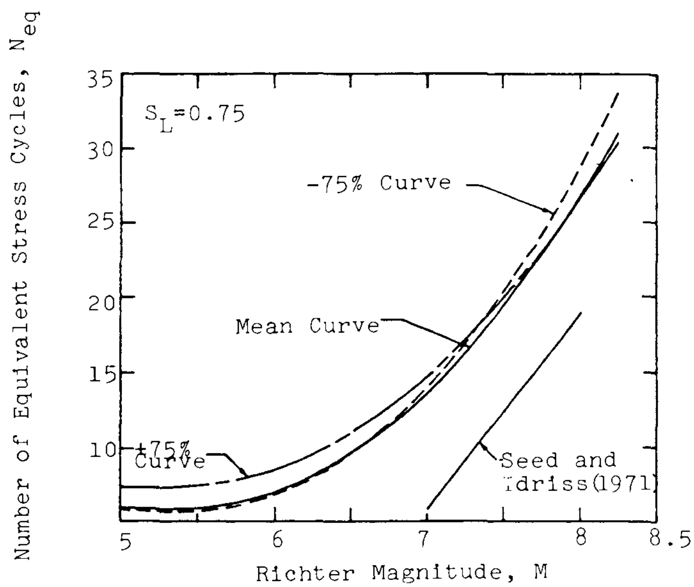

Fig. 3 - Relationship Between Number of Equivalent Stress Cycles and Magnitude for $S_L = 0.75$

curves on the N_{eq} versus M relationship is minimum when $S_L = 0.75$, and (ii) Seed and Idriss' suggested relationship underestimates the value of N_{eq} for a given magnitude earthquake.

The first observation is very interesting. The closeness of the three curves in Fig. 3 indicates that the uncertainty in the soil strength curve would not have a significant effect on the N_{eq} versus M relationship, if the stress level chosen is 0.75. Thus, it is proposed here that Eq. 5 of Table 1 be considered as an acceptable relationship between N_{eq} and M. The relationship can be presented as

$$E\ (N_{eq}|M = m) = 106.1 - 36.4\ m + 3.3\ m^2;\quad m \geq 5.0 \qquad (2)$$

and the corresponding Var $(N_{eq}|M) = 29.1$.

$N_{eq_{max}}$ versus M Relationship

The N_{eq} value for the two horizontal accelerograms can be calculated in two ways: (i) by considering the accelerogram containing the maximum acceleration a_{max}, or (ii) by considering the accelerogram giving the maximum value of N_{eq}. All discussions made so far were based on alternative (i). To study alternative (ii), the larger of the two values is designated $N_{eq_{max}}$, and a regression analysis (Eq. 1) is performed. The relationship between $N_{eq_{max}}$ and M based on the mean soil strength curve and $S_L = 0.75$ is plotted in Fig. 4, along with two other curves for comparison. The difference is not significant, especially in view of the large values of Var $(N_{eq}|M=m)$. Since a_{max} is the main design input parameter in the evaluation of liquefaction potential, it is perhaps satisfactory to compute N_{eq} considering the accelerogram containing a_{max}.

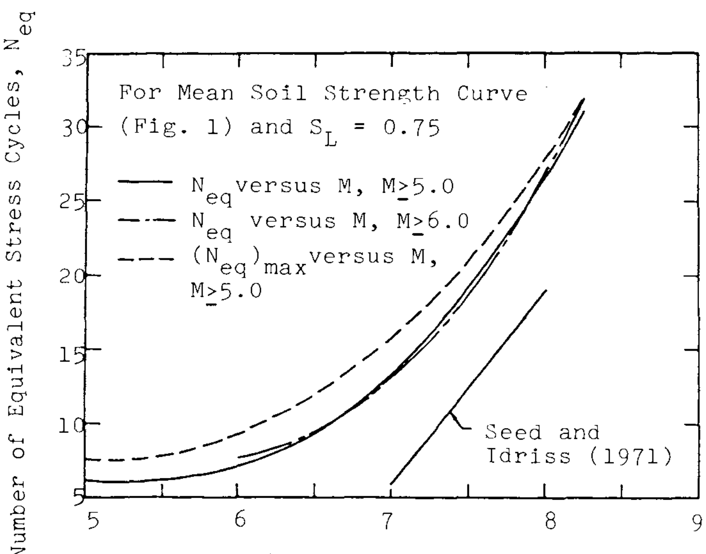

Fig. 4 - Comparison of Number of Equivalent Stress Cycles Versus Magnitude Relationships for Various Criteria

Comparison of N_{eq} versus M Relationships

The N_{eq} versus M relationship suggested by Seed and Idriss(1971) and the mean statistical relationship proposed here have considerable differences, which may create great concern among practicing engineers. It is emphasized that the relationship proposed here has a considerable amount of uncertainty. Moreover, Haldar and Tang (1981) showed that a large variation in N_{eq} values may not change the soil strength significantly. The increase in uncertainty in the soil strength curve may not be as significant as the uncertainty in some other parameters in a liquefaction study, e.g., relative density or maximum acceleration (Haldar and Tang, 1979). The discrepancy in N_{eq} values should be considered carefully for other problems.

CONCLUSION

The actual irregular time histories of shear stresses produced by an earthquake can be represented by uniform amplitude cyclic shear stresses, although there may be considerable uncertainty associated with them. It is suggested here that the intensity of the uniform stress should be taken as 75% of the maximum stress. It is also shown that N_{eq} could be estimated adequately by considering the component of excitation containing the peak acceleration. The relationship proposed here is somewhat different from that of Seed and Idriss. This discrepancy in N_{eq} values may not be significant in a liquefaction study, but should be considered for other problems.

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