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# Large Motion Assessment in Soils Under Dynamic Loading

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## ABSTRACT

This paper presents the mathematical formulation of the nonlinear multiphase dynamic model meant for porous media, obtained by applying the finite transformation assumption. This assumption is appropriate when large motions take place either during mass wasting processes, such as large slumps and earthflows, or during earthquake events when site liquefaction occurs and results for instance in large irrecoverable settlements or lateral spreads. The weak formulation and numerical implementation of the dynamic model uses the mesh-free h-p clouds method, which is based on the more general *Partition of Unity Method.* The mesh-free numerical methods seem indeed to be more appropriate for large transformation problems, where geometry may change in an important manner during simulation, as usual mesh constraints no longer exist. The numerical simulations of observed liquefaction-induced lateral spreads, performed with the proposed model are not presented in this paper.

## INTRODUCTION

The analysis of large or finite transformations has become a major field of investigation in Continuum Mechanics and in many Engineering fields. Nonetheless, the small perturbation assumption still prevails in Geomechanics and especially in the slope stability or anti-seismic design analyses. This hypothesis is suitable as long as the soil under any loading condition remains always close to its original configuration. However, it is no longer appropriate when large motions take place either during mass wasting processes, such as large slumps and earthflows, or during earthquake events when large irrecoverable displacements occur resulting from site liquefaction. During past earthquakes, widespread ground areas, as large as a few square kilometers were observed to shift laterally and permanently due to soil liquefaction, either during or after earthquake shaking. The amplitudes of these induced-lateral displacements could range from a few centimeters to tens of meters. Liquefaction-induced permanent deformations usually take place within the upper 20 meters of soil deposits, in gently sloping conditions (0.1% to 6%) and along river dikes, quaywalls or embankments. As they are generally very destructive, these large motions represent a major concern for the engineering practice, which should be able to assess the extent of these large-scale lateral deformations when designing new specific constructions or analyzing existing ones.

In the past decade, the physical mechanisms of liquefaction-induced lateral spreads have been investigated by means of various experimental tests (laboratory, shaking table, centrifuge). Several empirical (Bartlett and Youd 1992) or simple analytical models (Towata *et al.* 1997) have been proposed as well for modeling liquefaction-induced ground deformations. There exist also some other models (Aubry *et al.* 1982) based on effective stress constitutive modeling using the finite difference or finite element methods, but they generally lack to describe post-liquefaction behavior.

Either dealing with theory or with numerical modelling, large transformations in soils raises problems of strong non-linearities at the geometry level, as well as at the soil behaviour one. Indeed, little information on constitutive laws with finite transformation assumption is available, mainly because most of the physical and mechanical laboratory characterizations are limited to small strain tests. As geometry may change in an important manner during simulation, the use of standard numerical methods based on mesh discretization, such as the finite element method for instance, means to re-mesh the problem many times while computing, so that the mesh remains suitable. However, when re-meshing, the internal variables have also to be "transported" from the original mesh to the new one, which is a time-consuming process. On the contrary, the mesh-free numerical methods seem to be more appropriate for large transformation problems, as the partial differential equations are discretized using a set of unconnected points or nodes, so that mesh constraints no longer exist. When needed, re-meshing becomes just a matter of increasing the number or rearranging nodes.

The purpose of this paper is first to present the mathematical model for multiphase deformable porous media under dynamic loading, a model using the effective stress concept and based on the finite strain formulation in an Updated Lagrangian (UL) frame. Then, the weak formulation of the problem used to determine the numerical approach, namely the mesh-free h-p clouds method (Duarte & Oden 1995, Aubert 1997) is also presented. The constitutive model itself and the numerical simulations of observed liquefaction-induced lateral spreads, performed with the proposed model will be presented later in another paper.

## FINITE STRAIN FORMULATION FOR MULTIPHASE DEFORMABLE POROUS MEDIA

### General Assumptions

At the macroscopic level, the porous medium is represented as the superposition of three continuous media in space and time (Coussy 1991), namely one solid phase, one wetting (e.g. water) and one non-wetting (e.g. air, oil, etc.) fluid phases. Both fluids are free to flow through the medium and are assumed to be viscous, Newtonian and immiscible. The solid skeleton is assumed to undergo finite strain and is supposed to be initially homogeneous, isotropic and incompressible. Physico-chemical interactions as well as thermal effects are neglected. A Cartesian referential is assumed as well.

The porous medium initially occupies an open, regular volume $\Omega^n$ (reference configuration at time $t_n$), bounded by a surface $\Gamma^n$. Meanwhile, in the current configuration (at time $t_{n+1}$), the medium resides in the volume $\Omega^{n+1}$, bounded by $\Gamma^{n+1}$.

The present formulation utilizes a UL approach based on the transformation of the skeleton, assuming that at any time, a material point of the soil skeleton is constituted simultaneously of one solid and two fluid particles. Therefore, the current state for the skeleton containing the three components is given by the spatial coordinate vector $\underline{x} = \underline{x}^{n+1}$ and the reference state, by the coordinate vector $\underline{X} = \underline{x}^n$.

In the following formulation, vectors, second- and fourth-order tensors are underlined once, twice and fourth respectively. Variables with upper-script or subscript "$n$" refer to quantities expressed in the reference state (at time $t_n$), and variables with upper-script or subscript "$n+1$", to quantities expressed in the current configuration (at time $t_{n+1}$).

Kinematics. The mapping between reference and current configurations is defined as:

$$\underline{x} = \underline{\Phi}(\underline{X}, t) \tag{1}$$

The solid material displacement field $\underline{u}_s^{n+1}$ and the corresponding deformation gradient $\underline{\underline{F}}_{n+1}$ and Jacobian $J^{n+1}$ of the transformation are expressed at time $t = t_{n+1}$ as:

$$\underline{u}_s^{n+1} = \underline{u}_s(\underline{x}^n, t_{n+1}) = \underline{x} - \underline{X} \tag{2}$$

$$\begin{cases} \underline{\underline{F}}_{n+1} = \underline{\underline{1}} + \underline{\underline{GRAD\underline{u}}}_s^{n+1} \\ J^{n+1} = det\underline{\underline{F}}_{n+1} \end{cases} \tag{3}$$

with $\underline{\underline{1}}$, the identity tensor and $GRAD$, the gradient operator with respect to reference coordinates $\underline{x}_n$.

Fluid Phases. Assuming the connected porous space to be completely filled by fluid components, and denoting by $\theta_\alpha^{n+1}$, the volumetric fraction occupied by any phase $\alpha$ and $n^{n+1}$, the soil porosity at time $t_{n+1}$, then the following relationships are verified:

$$\theta_s^{n+1} = 1 - n^{n+1} \tag{4}$$

$$\theta_w^{n+1} = n^{n+1} S_w^{n+1} \tag{5}$$

$$\theta_{nw}^{n+1} = n^{n+1} S_{nw}^{n+1} = n^{n+1}(1 - S_w^{n+1}) \tag{6}$$

with subscripts $s$, $w$ and $nw$ referring to the solid, wetting and non-wetting components respectively, and $S_\alpha^{n+1}$ representing the degree of saturation at time $t_{n+1}$ for fluid phase $\alpha$.

A generalized Darcy's law is adopted to describe each fluid phase flow through the porous medium and coupling terms that may exist due to flowing of both fluids are neglected. Therefore, the velocity field at time $t_{n+1}$, $\underline{v}_{r\alpha}^{n+1}$, representing the velocity of fluid $\alpha$ relative to the solid skeleton is given by:

$$\forall \alpha \neq s, \theta_\alpha^{n+1} \underline{v}_{r\alpha}^{n+1} = \frac{k_\alpha}{\mu_\alpha}\left[-\underline{grad}\,p_\alpha^{n+1} + \rho_\alpha^{n+1}(\underline{g} - \underline{\gamma}_\alpha^{n+1})\right] \tag{7}$$

with $grad$, the gradient operator with respect to current coordinates $\underline{x}_{n+1}$, $p_\alpha$, the pore-pressure, $\rho_\alpha$, the unit mass, $k_\alpha$, the absolute (isotropic) permeability, $\mu_\alpha$, the constant dynamic viscosity, $\gamma_\alpha$, the absolute acceleration vector and $g$, the gravity acceleration vector. The absolute permeability is defined as the product of an intrinsic (or geometric) permeability which depends on the geometry of the pores, and a relative permeability which is a function of the degree of saturation (Aubert 1997).

As in the simplified formulation of Biot's macroscopic model (Zienkiewicz *et al.* 1980), the permeability of fluid $\alpha$ is assumed to be independent of frequency. Moreover, fluid relative accelerations are neglected before the solid one ($\gamma_s$):

$$\underline{\gamma}_\alpha = \underline{\gamma}_{r\alpha} + \underline{\gamma}_s \approx \underline{\gamma}_s \tag{8}$$

Then, in the UL description, Eq. (7) becomes ($\forall \alpha \neq s$):

$$\theta_\alpha^n \frac{d_t^\alpha J^{n+1} - d_t^s J^{n+1}}{J^{n+1}} = DIV(K_\alpha[\underline{GRAD}p_\alpha^{n+1} - J^{n+1}\rho_\alpha^{n+1}(\underline{g} - \underline{\gamma}_s^{n+1})]) \tag{9}$$

with $DIV$, the divergence operator with respect to reference coordinates $\underline{x}_n$, $d_t^\alpha$ and $d_t^s$, the material derivatives following fluid $\alpha$ and solid phases respectively, and $K_\alpha = \dfrac{k_\alpha}{\mu_\alpha}$.

Stresses. The concept of effective stress as postulated by Terzaghi for saturated soils can be generalized to multiphase porous media by partitioning the Cauchy total stress tensor $\underline{\underline{\sigma}}$ into an effective stress tensor $\underline{\underline{\sigma}}'$ and a spherical tensor:

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}' - \hat{p}\underline{\underline{1}} \tag{10}$$

For some authors (Schreffler *et al.* 1990), $\hat{p}$ corresponds to the mean fluid pore-pressure:

$$\hat{p} = S_w p_w + S_{nw} p_{nw} \tag{11}$$

In Eq. (10), replacing the Cauchy total and effective stress tensors $\underline{\underline{\sigma}}$ and $\underline{\underline{\sigma}}'$ by $\underline{\underline{\Pi}}$ and $\underline{\underline{\Pi}}'$, the corresponding first Piola-Kirchoff stress tensors, yields at time $t_{n+1}$:

$$\underline{\underline{\Pi}}_{n+1} = \underline{\underline{\Pi}}'_{n+1} - J^{n+1} \underline{\underline{F}}^{-T}_{n+1} \hat{p}^{n+1} \tag{12}$$

with $\underline{\underline{F}}^{T}$, the transpose of $\underline{\underline{F}}^{1}$.

## Coupled-field Equations

Mass Conservation. The mass conservation equation for phase $\alpha$ in the UL formulation is simply given by:

$$J^{n+1}\theta_\alpha^{n+1}\rho_\alpha^{n+1} = \theta_\alpha^n \rho_\alpha^n \tag{13}$$

Assuming that all phases are homogenous and applying the material derivative $d_t^\alpha$ to Eq. (13), yields:

$$\forall \alpha \in \{s, w, nw\}, \frac{d_t^\alpha J^{n+1}}{J^{n+1}} + \frac{\partial_t \rho_\alpha^{n+1}}{\rho_\alpha^{n+1}} + \frac{\partial_t \theta_\alpha^{n+1}}{\theta_\alpha^{n+1}} = 0 \tag{14}$$

with $\partial_t$, the partial derivative with respect to time.

Assuming incompressibility of the solid grains, i.e. $\rho_s^{n+1} = \rho_s^n$, Eq. (13) gives also:

$$n^{n+1} = 1 - \frac{\theta_s^n}{J^{n+1}} \tag{15}$$

We introduce the (constant) compressibility coefficient $\beta_\alpha$ for fluid phase $\alpha$ as:

$$\partial_t \rho_\alpha^{n+1} = \rho_\alpha^{n+1} \beta_\alpha \partial_t p_\alpha^{n+1} \tag{16}$$

The capillary component of the matricial suction, $p_c$, often called the capillary pressure, is the pressure discontinuity between both fluid phases:

$$p_c = p_{nw} - p_w \tag{17}$$

A strong dependency between $S_w$ and $p_c$ exists and some explicit empirical relationships are given in literature (Van Genuchten 1980). Hence, time derivative of $S_\alpha$ can be written as:

$$\partial_t S_\alpha^{n+1} = d_{p_c} S_\alpha^{n+1} \partial_t p_c^{n+1} \tag{18}$$

where $d_{p_c}$ is the derivative with respect to $p_c$.

Finally, combining Eq. (9), (13) to (18), we obtain the continuity equations for the multiphase porous medium:

$$\begin{cases} Q_{w-w}^{n+1} \partial_t p_w^{n+1} + Q_{w-nw}^{n+1} \partial_t p_{nw}^{n+1} + S_w^{n+1} DIV\, \underline{v}_s^{n+1} \\ - DIV\left(K_w \left[J_w^{n+1} \underline{GRAD p_w^{n+1}} - \rho_w^n (\underline{g} - \underline{\gamma}_s^{n+1})\right]\right) = 0 \\ Q_{nw-w}^{n+1} \partial_t p_w^{n+1} + Q_{nw-nw}^{n+1} \partial_t p_{nw}^{n+1} + (1 - S_w^{n+1}) DIV\, \underline{v}_s^{n+1} \\ - DIV\left(K_{nw} \left[J_{nw}^{n+1} \underline{GRAD p_{nw}^{n+1}} - \rho_{nw}^n (\underline{g} - \underline{\gamma}_s^{n+1})\right]\right) = 0 \end{cases} \tag{19}$$

with:

$$\begin{aligned} Q_{w-w}^{n+1} &= \theta_w^{n+1} \beta_w - \left(1 - \frac{\theta_s^n}{J^{n+1}}\right) d_{p_c} S_w^{n+1} \\ Q_{w-nw}^{n+1} &= Q_{nw-w}^{n+1} = \left(1 - \frac{\theta_s^n}{J^{n+1}}\right) d_{p_c} S_w^{n+1} \\ Q_{nw-nw}^{n+1} &= \theta_{nw}^{n+1} \beta_{nw} - \left(1 - \frac{\theta_s^n}{J^{n+1}}\right) d_{p_c} S_w^{n+1} \\ J_\alpha^{n+1} &= \frac{\theta_\alpha^{n+1}}{\theta_\alpha^n} \end{aligned} \tag{20}$$

Momentum Conservation. In the current configuration, the porous medium is subjected to body forces $\rho^{n+1} \underline{b}_{n+1}$ (mainly gravity) applying to all solid and fluid particles present in the volume $\Omega^{n+1}$ and to surface tractions $\underline{T}_{n+1}$ over the boundary $\Gamma^{n+1}$:

$$\int_{\Omega^{n+1}} \rho^{n+1} \underline{b}_{n+1} d\Omega^{n+1} = \sum_{\alpha \in \{s,w,nw\}} \int_{\Omega^n} \theta_\alpha^n \rho_\alpha^n \underline{g}\, d\Omega^n \tag{21}$$

$$\int_{\Gamma^{n+1}} \underline{T}_{n+1} d\Gamma^{n+1} = \int_{\Gamma^n} \underline{\underline{\Pi}}_{n+1} . \underline{N} d\Gamma^n \tag{22}$$

with $\underline{N}$, the unit normal outward to reference surface $\Gamma^n$.

Using Gauss' theorem, the surface integral in Eq. (22) can be replaced by a volume integral over $\Omega^n$:

$$\begin{aligned} \int_{\Gamma^{n+1}} \underline{T}_{n+1} d\Gamma^{n+1} &= \int_{\Omega^n} \underline{DIV(\underline{\underline{\Pi}}'_{n+1})} d\Omega^n \\ &- \int_{\Omega^n} J^{n+1} \underline{\underline{F}}^{-T}_{n+1} . \underline{GRAD \hat{p}^{n+1}} d\Omega^n \end{aligned} \tag{23}$$

with $\hat{p}$ expressed using expression (11).

The total linear momentum is expressed as:

$$\int_{\Omega^{n+1}} \rho^{n+1} \underline{\gamma}^{n+1} d\Omega^{n+1} = \sum_{\alpha \in \{s,w,nw\}} \int_{\Omega^n} \theta_\alpha^n \rho_\alpha^n \underline{\gamma}_\alpha^{n+1} d\Omega^n \tag{24}$$

The principle of linear momentum conservation for the multiphase porous medium in the UL formulation gives:

$$\rho^n \underline{\gamma}_s^{n+1} = \underline{DIV(\underline{\underline{\Pi}}'_{n+1})} - J^{n+1} \underline{\underline{F}}^{-T}_{n+1} . \underline{GRAD \hat{p}^{n+1}} + \rho^n \underline{g} \tag{25}$$

If choosing $p_{nw}$ as a main unknown in this formulation, instead of $p_c$ as often seen in literature, Eq. (25) becomes:

$$\begin{aligned} &\rho^n \underline{\gamma}_s^{n+1} = \underline{DIV(\underline{\underline{\Pi}}'_{n+1})} + \rho^n \underline{g} \\ &- J^{n+1} \underline{\underline{F}}^{-T}_{n+1} . \left[S_w^{n+1} \underline{GRAD p_w^{n+1}} + (1 - S_w^{n+1}) \underline{GRAD p_{nw}^{n+1}}\right] \end{aligned} \tag{26}$$

## Constitutive Model for Solid Skeleton

An incremental elastoplastic relationship is assumed between effective stresses and strains, using the Jaumann co-rotational rate and appropriate strain measures.

$$\overset{\circ}{\underline{\underline{\pi}}}'(\underline{u}_s) = \underline{\underline{\underline{\underline{D}}}}^{ep} : \dot{\underline{\underline{E}}} \tag{27}$$

with $\underline{\underline{\pi}}'$, the second Piola-Kirchoff effective stress tensor, $\underline{\underline{E}}$, the Green-Lagrange strain tensor and $\underline{\underline{\underline{\underline{D}}}}^{ep}$, the elastoplastic stiffness tensor.

*Remark.* It should be noted that spurious stress oscillations may result from the use of Jaumann rates in conjunction with kinematic hardening plasticity models (Lee and Mallett 1983).

## Boundary and Initial Conditions

Let $\bar{\Omega} = \Omega \cup \Gamma$ and $\underline{n}$ be the unit normal on boundary $\Gamma$, with particles labeled by $\underline{x} \in \bar{\Omega}$ in the current configuration (time t). The problem is to find the unknown fields $\underline{u}_s$, $p_w$ and $p_{nw}$ at any time $t$ in $[0,T]$, satisfying Eq. (19), (26), (27) and the following boundary conditions:

$$
\begin{aligned}
&\underline{u}_s(\underline{x},t) = \underline{\bar{u}}_s(t) \text{ on } \Gamma_u \times ]0,T] \\
&\underline{\underline{\sigma}}(\underline{x},t).\underline{n} = \underline{\bar{T}}(t) \text{ on } \Gamma_\sigma \times ]0,T] \\
&p_\alpha(\underline{x},t) = \bar{p}_\alpha(t) \text{ on } \Gamma_{p_\alpha} \times ]0,T] \\
&K_\alpha \partial_{\underline{n}}\left[p_\alpha(\underline{x},t) - \rho_\alpha(\underline{x},t)(\underline{g} - \underline{\gamma}_s).\underline{x}\right] = \bar{\varphi}_\alpha(t) \text{ on } \Gamma_{\varphi_\alpha} \times ]0,T]
\end{aligned}
\tag{28}
$$

$\Gamma_v, \Gamma_\sigma, \Gamma_{p_\alpha}, \Gamma_{\varphi_\alpha}$ are parts of the boundary where the displacement of the skeleton, the total stress, the pore pressures and fluxes of both fluid phases are prescribed. The following relations are also verified:

$$
\begin{aligned}
&\Gamma = \Gamma_u \cup \Gamma_\sigma \quad and \quad \Gamma_u \cap \Gamma_\sigma = \varnothing \\
&\forall \alpha \in \{w,a\}, \Gamma = \Gamma_{p_\alpha} \cup \Gamma_{\varphi_\alpha} \quad and \quad \Gamma_{p_\alpha} \cap \Gamma_{\varphi_\alpha} = \varnothing
\end{aligned}
\tag{29}
$$

The solution is sought given the initial conditions, which specify velocity, stresses and pore fluid pressures, i.e.:

$$
\begin{aligned}
&\forall \underline{x} \in \bar{\Omega}\Big|_{t=0}, \ \underline{v}_s(\underline{x},0) = \underline{v}_s^0(\underline{x}) \\
&\underline{\underline{\Pi}}(\underline{x},0) = \underline{\underline{\sigma}}(\underline{x},0) = \underline{\underline{\Pi}}_0(\underline{x}) \\
&p_\alpha(\underline{x},0) = p_\alpha^0(\underline{x})
\end{aligned}
\tag{30}
$$

Note that in the initial configuration, the Cauchy and first Piola-Kirchoff stress tensors are identical.

## WEAK FORMULATION

The previous equations are discretized in space using the mesh-free h-p clouds technique (HPC) for all degrees of freedom. To simplify notations, the upper-script "$n+1$" will be dropped for current configuration. The variational (weak) formulation is obtained from the principle of virtual work, stated on the reference configuration as:

$$
\begin{aligned}
&\left(\rho^n \underline{g}, \delta\underline{u}\right)_{\Omega^n} + \left\langle \underline{\bar{T}}, \delta\underline{u}\right\rangle_{\Gamma^n} = \left(\underline{\underline{F}} \cdot \underline{\underline{\pi}}'(\underline{u}_s), \underline{\underline{E}}(\delta\underline{u})\right)_{\Omega^n} + \left(\rho^n \underline{\gamma}_s, \delta\underline{u}\right)_{\Omega^n} \\
&-\left(J\left[S_w p_w + (1-S_w) p_{nw}\right], DIV(\underline{\underline{F}} \cdot \delta\underline{u})\right)_{\Omega^n} \\
&\left(Q_{w-w}\partial_t p_w, q_w\right)_{\Omega^n} + \left(Q_{w-nw}\partial_t p_{nw}, q_w\right)_{\Omega^n} + \left(S_w DIV\, \underline{v}_s, q_w\right)_{\Omega^n} \\
&+\left(K_w J_w \underline{GRAD} p_w, \underline{GRAD} q_w\right)_{\Omega^n} + \left(K_w \rho_w^n \underline{\gamma}_s, \underline{GRAD} q_w\right)_{\Omega^n} \\
&-\left\langle \underline{\lambda}_w, q_w\right\rangle_{\Gamma^n_{p_w}} = \left(K_w \rho_w^n \underline{g}, \underline{GRAD} q_w\right)_{\Omega^n} + \left\langle \underline{\bar{\varphi}}_w, q_w\right\rangle_{\Gamma^n_{\varphi_w}} \\
&\left(Q_{nw-w}\partial_t p_w, q_{nw}\right)_{\Omega^n} + \left(Q_{nw-nw}\partial_t p_{nw}, q_{nw}\right)_{\Omega^n} \\
&+\left((1-S_w) DIV\, \underline{v}_s, q_{nw}\right)_{\Omega^n} + \left(K_{nw} J_{nw} \underline{GRAD} p_{nw}, \underline{GRAD} q_{nw}\right)_{\Omega^n} \\
&+\left(K_{nw} \underline{\gamma}_s, \underline{GRAD} q_{nw}\right)_{\Omega^n} - \left\langle \underline{\lambda}_{nw}, q_{nw}\right\rangle_{\Gamma^n_{p_{nw}}} \\
&= \left(K_{nw} \rho_{nw}^n \underline{g}, \underline{GRAD} q_{nw}\right)_{\Omega^n} + \left\langle \underline{\bar{\varphi}}_{nw}, q_{nw}\right\rangle_{\Gamma^n_{\varphi_{nw}}} \\
&\left\langle p_w, l_w\right\rangle_{\Gamma^n_{p_w}} = \left\langle \bar{p}_w, l_w\right\rangle_{\Gamma^n_{p_w}} \\
&\left\langle p_{nw}, l_{nw}\right\rangle_{\Gamma^n_{p_{nw}}} = \left\langle \bar{p}_{nw}, l_{nw}\right\rangle_{\Gamma^n_{p_{nw}}}
\end{aligned}
\tag{31}
$$

where $(.,.)_\Omega$ and $<.,.>_\Gamma$ are sums over $\Omega$ and $\Gamma$ of internal products of tensors, vectors and scalars. $\delta\underline{u}$ and $q_\alpha$ represent respectively any admissible virtual displacement vector and pore fluid pressures, and $\delta l_\alpha$, any virtual admissible Lagrange multipliers. Lagrange multipliers $\lambda_\alpha$ are introduced to impose essential boundary conditions in a weak form. They correspond to fluxes through these boundaries (Aubert 1997). The unknowns of the problem are approximated in the HPC method:

$$
\begin{aligned}
&\underline{u}_s(\underline{X},t) = \sum_I \phi_I(\underline{X}) \underline{u}_{sI}(t) \\
&p_\alpha(\underline{X},t) = \sum_I \phi_{\alpha I}(\underline{X}) p_{\alpha I}(t) \\
&\lambda_\alpha(\underline{X},t) = \sum_I \phi'_{\alpha I}(\underline{X}) \lambda_{\alpha I}(t)
\end{aligned}
\tag{32}
$$

$I$ stands for a node number and $\phi_I(\underline{X})$, $\phi_{\alpha I}(\underline{X})$, $\phi'_{\alpha I}(\underline{X})$ designate the shape functions for the HPC method associated with variables $\underline{u}_s$, $p_\alpha$ and $\lambda_\alpha$ at node $I$ in the reference configuration.

## CONCLUSIONS

The numerical implementation of the non linear dynamic model presented in the previous sections, is still in progress. It will be tested on observed liquefaction-induced lateral spreads during recent strong motion events, such as the 1995 Hyogoken-Nambu (Kobe, Japan) earthquake.

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