Damping in Torsional Vibrations of Embedded Footings

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Proceedings: First International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, April 26 - May 3, 1981, St. Louis, Missouri

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SYNOPSIS The existing theoretical models to explain the dynamic behaviour of embedded footings, overestimate the real response by neglecting damping forces which are inevitable as a result of slip at the interface of the embedded footing and soil. Many researchers in the field of Soil Dynamics have suggested that the inclusion of friction damping and internal damping in the mathematical model is necessary to improve the reliability of theoretical predictions.

In this paper, results of the experimental investigations on full scale model embedded footings subjected to torsional mode of vibration have been presented. The results have been analysed making use of three theoretical models, as developed by, Novak and Sachs (1973); Sankaran et al (1978) and Sankaran et al (1980). The importance of damping in predicting the dynamic response is brought out by a comparison of field vibratory test data with the corresponding values predicted by each of the above mentioned theoretical models.

INTRODUCTION

Torsional vibrations are excited in foundations of structures and machinery when there exist eccentric forces in a horizontal plane. Foundations of radar and communication towers and certain reciprocating machinery are some examples. Torsional excitation is also possible in the foundations of structures during earthquake shaking. These foundations are essentially embedded. Hence it is important to develop a rational theoretical method to design and analyse embedded foundations subjected to torsional vibrations.

One of the important factors influencing the response of the foundation-soil system is the damping associated with the system. Three types of damping are to be accounted in developing any theoretical model to describe the dynamics of the system. They are the radiation damping, friction damping and the internal damping. The radiation damping is usually represented as an equivalent viscous damping for mathematical convenience. The essential feature of the viscous damping is that the damping is proportional to the first power of velocity of motion at any instant of time.

During vibration a certain amount of friction is mobilised at the interfaces between the base of the footing and the soil beneath as well as between the sides of the footing and the surrounding soil. Some energy is lost due to this slip friction and this is known as friction damping. This is independent of the frequency of vibration and is of a constant nature. The damping caused by the absorption of energy by the system itself is called the internal damping. The hysteretic effects, in other words, the imperfect elasticity of the soil is responsible for the internal damping in soils. The energy dissipated by internal damping is independent of the frequency of vibration, for torsional mode of vibration, internal damping is of considerable significance as its omission can result in unrealistically low values of total damping.

PREVIOUS INVESTIGATIONS

Studies on embedded footings have been concerned mainly with vertical vibrations and to some lesser extent with rocking and sliding vibrations, while the least investigated has been the torsional motion.

In this paper, an attempt is made to make use of the three existing theoretical models, as developed by Novak and Sachs (1973), and Sankaran et al (1978) and (1980), to bring out the importance of damping on the torsional response of embedded footings. These theoretical models are not reviewed in detail here for want of space. However, the theoretical behaviour of each of these models is presented in graphical form.

The model developed by Novak and Sachs (1973) takes into consideration radiating damping only in their analysis of torsional response of an embedded footing. The lumped parameter analogue model developed by Sankaran et al (1978) takes into consideration radiation as well as friction damping due to slip at the interface of footing-soil in regions of high shear. The recent theoretical model developed by Sankaran et al (1980) is an improvement over their earlier model and takes into consideration radiation, friction and internal dampings.

The importance of damping in predicting the dynamic response is evident from an examination of typical theoretical response curves generated by the three mathematical models illustrated in Figs. 1 - 3. The mathematical treatment of these theoretical models are discussed in detail by Novak and Sachs (1973) and Sankaran et al (1978, 1980) and hence not repeated here.

EXPERIMENTAL INVESTIGATION

Field vibratory tests on several precast and cast-in-place footings of reinforced concrete, of various sizes and shapes were conducted to obtain data to check the validity of the theoretical approaches mentioned above.

The soil at the site at Indian Institute of Technology, Madras, was silty sand with some clay binder (SM). The unit weight and the moisture content of the soil were found to be $19.31 kN/m^3$ and 11 per cent respectively. The angle of internal friction and cohesion of the soil were 32^0 and $23.52\ kN/m^2$. The insitu dynamic shear modulus of the soil at various depths was determined by a procedure reported by Sankaran et al (1979). The variation of shear modulus with depth is reported by Sankaran et al (1980 a,b).

Seven precast footings and six cast-in-place footings were used in this experimental investigation. Precast footings have been designated as Base 1, Base 2,... Base 7, whereas cast-in-place footings have been designated as Base 8, Base 9,... Base 13 respectively. The dimensions of the footings are given in Table I. Bases 4 - 7 had the same weight, and base area, but had different base shapes, Base 4 was circular, Base 5 was square, and Base 6 and 7 were rectangular with L/B ratios of 1.19 and 1.44 respectively.

The experimental program and the test procedure have been described in detail by Sankaran et al (1980 a,b) and hence not reported here. The results of tests on cast-in-place and precast footings for various depths of embedment are presented in Table 2.

COMPARISON OF TEST RESULTS WITH THEORY

For analysing the results, the value of D_θ is obtained from the formula,

$$D_\theta = 0.5/(1 + 2B_\theta^-) \qquad (1)$$

in which inertia ratio, $B_\theta = I_\theta / r_o^5 . \rho$ (2)

As recommended by Weissmann (1971) a value of $D_1 = 0.05$ is taken in calculations. Since shear modulus varies with depth, the value of G at a depth of one radius below the bottom of the footing is taken for the purpose of analysis.

The value of $M_{F\theta}$ is obtained from the expression developed by Sankaran et al (1978). For a footing embedded in a C-Ø soil,

$$M_{F\theta} = [(2/3)\ \pi\ \mu_{fb}\ p\ r_o^3 + 0.5\ K_o\ \gamma_s\ H^2 \cdot \mu_{fs}\ P\ r_o] + [(2/3)\ \pi\ C_{ab}\ r_o^3 + C_{as}\ H\ P\ r_o] \qquad (3)$$

in which p = static pressure at the base, P=perimeter of footing, H= height of embedment, K_o=coefficient of earth pressure at rest, γ_s=unit weight of side soil, μ_{fb}=coefficient of kinematic friction at the sides, C_{ab}=wall adhesion at the base, and C_{as}=wall adhesion at the sides.

The natural frequency, w_n, is obtained from

$$w_n = \sqrt{(K_\theta / I_\theta^-)} \qquad (4)$$

in which, $K_\theta = (16/3)\ G\ r_o^3$ (5)

The theoretically predicted and experimentally observed values are given in Table 2.

SUMMARY AND CONCLUSIONS

Table 2 indicates that the resonant amplitudes predicted by the theory of Sankaran et al(1980) are in good agreement with the observed amplitudes. It is observed that the predicted resonant frequencies are in fair agreement with the experimental observations. The single-degree-of-freedon analogue model developed by Sankaran et al(1980) is quite satisfactory to predict the response of embedded footings subjected to steady-state torsional vibrations. The experimental results confirm the earlier findings by Anandakrishnan and Krishnaswamy (1973), Krishnaswamy (1972,1976). The analysis of the experimental data using the three theoretical models illustrates that the inclusion of radiation damping, friction damping and internal damping, improves the reliability of theoretical predictions.

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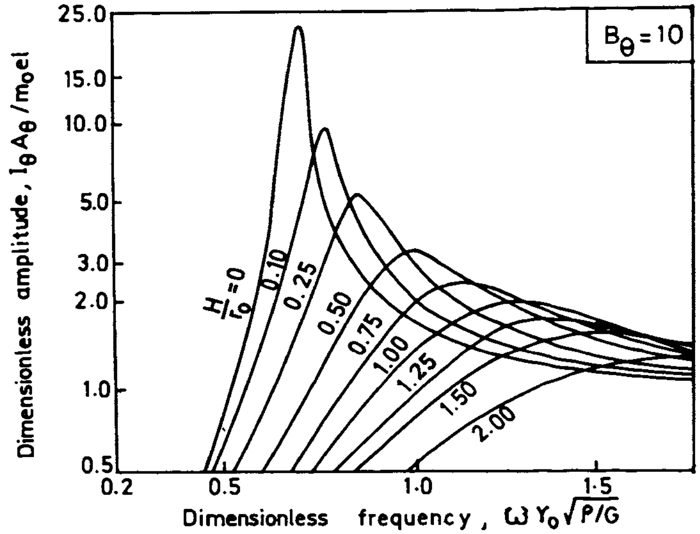

Fig.1 Typical Response Curves(after Novak and Sachs, 1973).

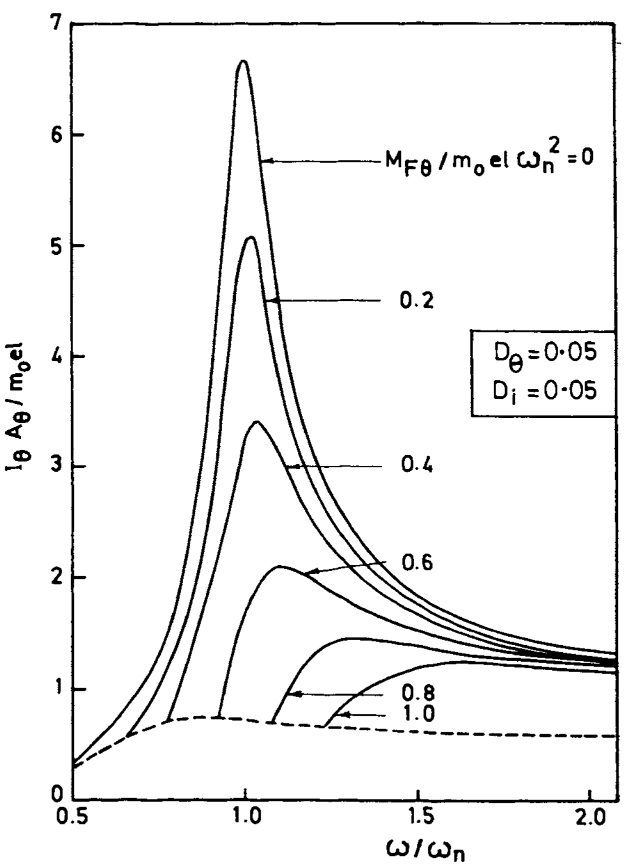

Fig.2 Typical Response Curves(after Sankaran et al, 1980).

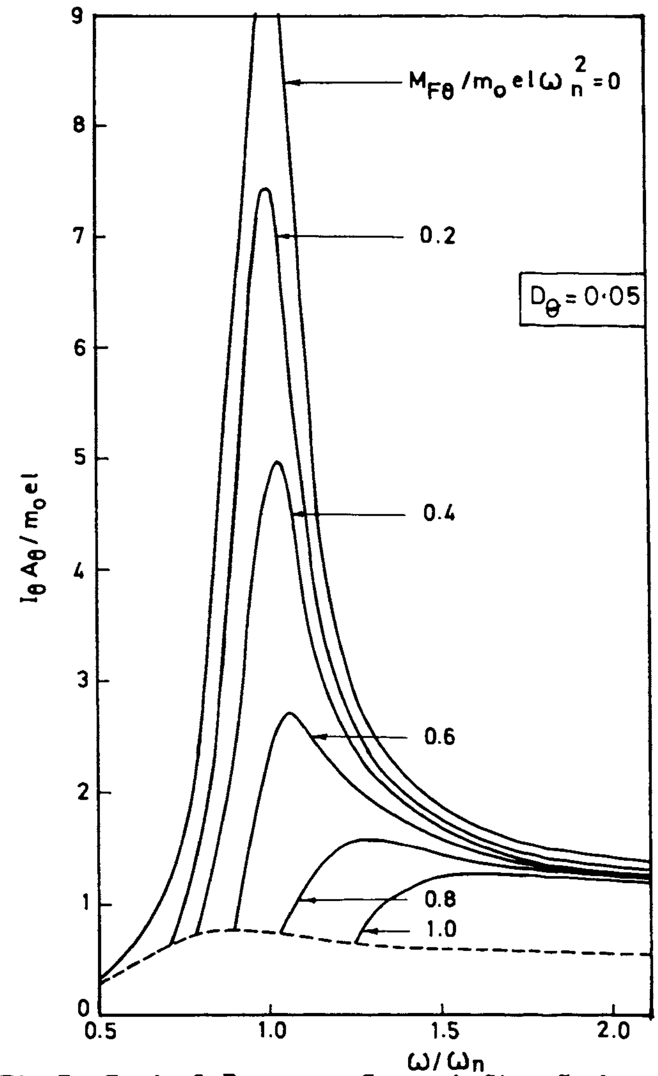

Fig.3 Typical Response Curves(after Sankaran et al, 1978).

TABLE I. TEST CONDITIONS

Base No.	Dimensions (m)	r_o (m)	D_θ	m_e el (N.m.s²)
1	0.50 x 0.50 x 0.50	0.2854	0.016	0.0096
2	0.60 x 0.60 x 0.45	0.3424	0.025	0.0096
3	0.70 x 0.70 x 0.50	0.3996	0.050	0.0096
4	0.67 dia x 1.20	0.3385	0.020	0.0096
5	0.60 x 0.60 x 1.20	0.3424	0.021	0.0096
6	0.65 x 0.55 x 1.20	0.3440	0.022	0.0096

... Contd.

TABLE I. (Contd.)

Base No.	Dimensions (m)	r_o (m)	D_θ	m_e el (N.m.s²)
7	0.72 x 0.50 x 1.20	0.3480	0.023	0.0096
8	0.40 x 0.40 x 0.50	0.2283	0.012	0.0070
9	0.45 x 0.45 x 0.50	0.2568	0.019	0.0070
10	0.50 x 0.50 x 0.50	0.2854	0.030	0.0070
11	0.50 x 0.50 x 0.50	0.2854	0.016	0.0096
12	0.55 x 0.55 x 0.40	0.3139	0.030	0.0096
13	0.60 x 0.60 x 0.45	0.3424	0.025	0.0096

TABLE II. COMPARISON OF EXPERIMENTAL RESULTS WITH THEORETICAL PREDICTIONS

Base No.	I_θ N.m.s²	H/r_o	$\frac{M_{F\theta}}{m_o e l w_n^2}$	Resonant Amplitude x (0.00001 rad)				Resonant Frequency (rad/sec)			
				Novak and Sachs (1973)	Sankaran et al (1978)	Sankaran et al (1980)	Observed	Novak and Sachs (1973)	Sankaran et al (1978)	Sankaran et al (1980)	Observed
1	55.42	0.000	0.42	19.81	28.25	10.23	10.05	188	209	211	192
		0.584	0.44	9.67	26.60	9.62	8.80	226	209	211	198
		1.168	0.47	5.87	24.35	8.84	7.45	289	209	213	207
		1.752	0.54	4.50	17.33	7.19	5.90	358	210	213	220
2	66.30	0.000	0.39	11.53	15.14	7.46	7.80	226	251	256	220
		0.438	0.40	6.48	14.75	7.39	6.65	264	251	256	236
		0.876	0.43	4.08	13.62	6.74	5.45	320	251	256	245
		1.314	0.48	3.02	11.70	5.94	4.20	390	254	259	254
3	90.70	0.000	0.44	6.68	4.78	3.37	3.75	257	289	297	236
		0.417	0.45	3.92	4.65	3.28	3.30	295	289	297	242
		0.834	0.49	2.50	4.16	2.97	2.75	364	291	300	251
		1.251	0.55	1.87	3.47	2.54	1.55	446	294	303	264
4	100.96	0.000	0.56	9.30	6.71	3.27	4.75	232	264	269	233
		0.886	0.60	4.34	5.55	2.80	3.42	295	266	274	242
		1.772	0.69	2.60	3.39	2.05	1.75	383	271	284	258
5	104.333	0.000	0.56	9.13	6.43	3.20	4.05	239	269	272	239
		0.876	0.60	4.22	5.32	2.76	3.38	295	269	277	245
		1.752	0.70	2.53	3.03	1.98	1.70	383	275	288	261
6	104.90	0.000	0.56	9.04	6.17	3.11	4.05	239	270	273	242
		0.872	0.60	4.13	5.13	2.74	3.27	302	270	277	251
		1.163	0.62	3.40	4.69	2.56	2.60	327	273	278	258
		1.744	0.70	2.50	2.92	1.97	1.55	390	276	289	267
7	106.33	0.000	0.55	8.59	6.05	3.07	4.00	299	273	276	245
		0.862	0.59	4.04	4.90	2.71	3.30	302	273	279	251
		1.149	0.62	3.30	4.52	2.53	2.50	327	276	281	261
		1.724	0.70	2.43	2.64	1.94	1.40	390	279	292	273
8	25.09	0.000	0.55	37.34	37.44	11.93	11.35	182	203	207	151
		0.745	0.69	10.16	15.98	6.66	8.25	295	207	219	170
		1.490	0.87	7.11	4.88	4.16	4.90	377	242	262	195
9	27.20	0.000	0.52	26.29	23.85	10.86	10.20	207	232	239	163
		0.584	0.62	8.30	15.40	7.42	7.70	314	237	246	182
		1.363	0.83	5.39	4.71	4.09	4.80	427	274	290	226
10	30.06	0.000	0.47	19.51	15.49	8.91	7.50	239	269	274	188
		0.455	0.56	7.08	11.48	6.83	6.25	333	271	279	201
11	55.42	0.000	0.86	19.81	3.05	2.69	2.95	188	253	265	226
		0.438	0.95	7.70	2.50	2.34	2.35	251	284	298	264
		0.584	0.99	6.48	2.37	2.25	2.20	270	297	314	273
12	46.92	0.000	0.56	17.02	10.11	5.93	5.30	226	256	259	226
		0.478	0.65	6.14	6.56	4.60	3.05	320	261	271	257
13	66.30	0.000	0.73	11.53	3.82	2.75	2.60	226	271	284	232
		0.438	0.85	4.51	2.42	2.10	2.00	308	311	326	257