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Roberto Cairo

University of Calabria, Italy

Enrico Conte

University of Calabria, Italy

Giovanni Dente

University of Calabria, Italy

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Roberto Cairo
University of Calabria
Cosenza, ITALY

Enrico Conte
University of Calabria
Cosenza, ITALY

Giovanni Dente
University of Calabria
Cosenza, ITALY

ABSTRACT

In this paper, two simplified methods are used to calculate the impedance function of an axially loaded pile embedded in layered soils. The methods are: a semi-analytical procedure which uses the discrete layer stiffness matrices derived by Kausel and Roësset (1981), and the cone model which was developed by Wolf et al. (1992). A number of comparisons with more rigorous solutions are shown in order to assess the accuracy of the methods used.

INTRODUCTION

The response of piles to dynamic excitation has been the subject of many researches over the past decades, and a variety of methods has been developed to solve this complicated problem. Novak (1991) presented an extensive critical review of the more widely accepted procedures of analysis.

In the simplest method, soil-pile system is represented as a Winkler foundation with springs and dashpots that are distributed along depth or concentrated at a finite number of nodes (Penzien, 1970; Novak, 1974; Matlock and Foo, 1979; Dobry et al., 1982; Gazetas and Dobry, 1984; Conte and Dente, 1988). The stiffness of the springs and the damping of the dashpots are generally assumed to be constant or frequency dependent; they are derived from theoretical studies or from experimental data. The major advantage of the method is that nonlinearity, inhomogeneity and hysteretic behaviour of soil can be simulated without requiring considerable computational efforts, but by simply changing the spring and dashpot parameters. However, this approach is not conceptually suitable to describe the behaviour of pile groups, as Winkler's model ignores continuity between piles through the surrounding soil. Gazetas and Makris (1991) presented an analytical approximate procedure based on dynamic Winkler foundation model and a symplified wave propagation analysis to account for pile-soil-pile interaction.

The finite element method is certainly the most comprehensive approach to analyse the dynamic response of piles and pile groups. This is because the method is in principle completely general with respect to the geometry of the problem, boundary conditions, variations in material properties, stress-strain relationships and pile-soil interface modelling. In practice, however, the solution usually requires high numerical costs, a large amount of data preparation and accurate measurements of the material properties. Finite element analyses of piles and pile groups under dynamic loading conditions have been carried out by many authors (Blaney et al., 1976; Wolf and Von Arx, 1978; Kuhlemeyer, 1979; Krishnan et al., 1983; Roësset, 1984).

More recently, the boundary element method has also been employed to deal with the piles subjected to dynamic loading. As known, the method is very suitable to analyse dynamic problems involving infinite domain because the radiation condition at the far field is directly accounted for. Furthermore, this method partly reduces the computational costs in comparison with the finite element method. Boundary element formulations of the dynamic problem of piles embedded in homogeneous soils as well as inhomogeneous soils were developed by Kaynia and Kausel (1982), Davies et al. (1985), Banerjee and Sen (1987), and El-Marsafawi et al. (1992).

On the other hand, few studies have been conducted on the dynamic response of piles in multi-layer soil profiles. This problem was dealt with by Kaynia and Kausel (1991) that developed a general formulation, in which Green's functions for layered media along with analytical solutions for the dynamic response of piles were used. In this procedure, Green's functions are evaluated numerically by the application of integral transform techniques.

In many circumstances it is desirable to have a method available that allows the dynamic behaviour of piles in layered soils to be readily analysed. Recently, Mylonakis and Gazetas (1998) have presented an attractive procedure which is based on the repeated use of a closed-form expression derived by the same authors to calculate the dynamic impedance of pile embedded in a homogeneous layer resting on a deformable base.

In this paper, two simplified methods are used to determine the steady-state dynamic response of axially loaded piles embedded in a layered soil deposit. The methods used are: a semi-analytical procedure which uses the discrete layer stiffness matrices derived earlier by Kausel and Roësset (1981), and the cone model which was developed by Wolf et al. (1992). The two methods are briefly described and comparisons to more rigorous solutions are presented.

STATEMENT OF PROBLEM

Figure 1 shows the problem under consideration. A vertical pile of diameter d and length L is embedded in a layered viscoelastic soil medium resting on a half-space. In Fig. 1, ν_s indicates Poisson's ratio, E_s is Young's modulus, ρ_s is the mass density, and β_s is the damping ratio of the generic soil layer.

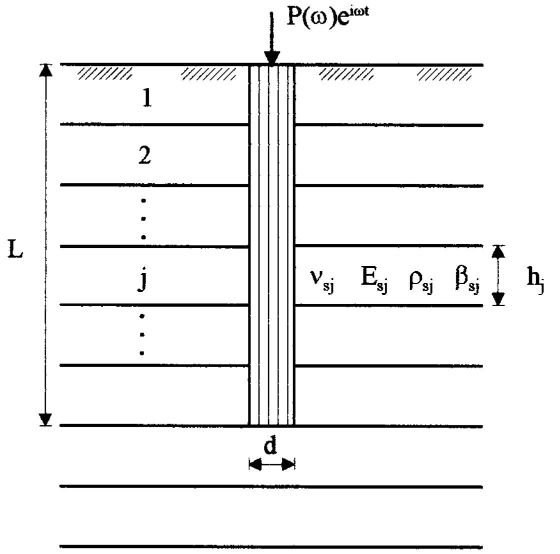

Fig. 1. Pile embedded in a layered soil.

In order to model the effect of internal energy dissipation within the soil, complex moduli are introduced. Therefore, Young's modulus of soil is replaced by its complex counterpart as

$$E_s^* = E_s\,(1+2i\beta_s) \tag{1}$$

where $i=\sqrt{-1}$.

The pile is assumed to be a linear elastic beam with Young's modulus E_p, cross section A_p, and mass density ρ_p. Since the soil-pile system is under steady-state vibrations, any time dependent variable is expressed as a complex quantity multiplied by the factor $e^{i\omega t}$, where ω is the frequency of harmonic excitation.

DISCRETE STIFFNESS MATRIX METHOD

The analysis of a pile embedded in layered soils may be carried out considering the pile and the soil as two components (Fig. 2). The pile component is discretized by a finite number of simple one-dimensional cylindrical elements (Fig. 2a), and the soil is represented by a viscoelastic layered continuum (Fig. 2b).

For the pile component, the equilibrium equation can be expressed in matrix form as

$$([K_p]+\omega^2[M_p])\{w_p\}=\{P\}+\{P_p\} \tag{2}$$

where $[K_p]$=pile stiffness matrix; $[M_p]$=pile mass matrix; $\{P_p\}$=vector containing the pile-soil interaction forces; $\{P\}$=external load vector; $\{w_p\}$=pile displacements. Forces $\{P_p\}$ are applied at the interfaces of the pile elements (nodes). These forces are given by ring loads acting on the circumferential area of each pile shaft element, and by a uniform pressure acting at the pile base. In the case considered, vector $\{P\}$ contains only the axial load applied at the pile top.

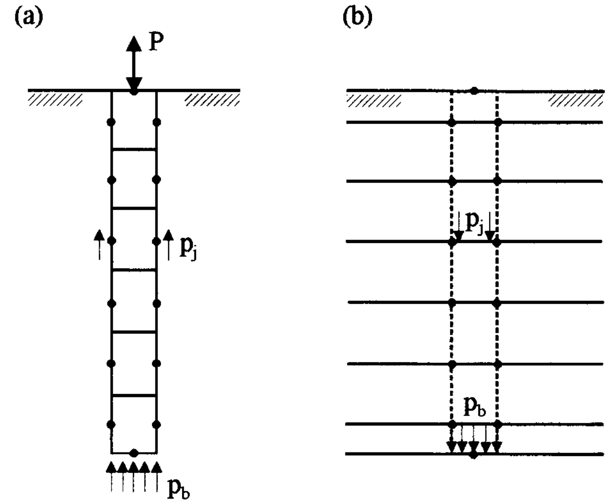

Fig. 2. Analysis of the pile. (a) Forces acting on pile. (b) Forces acting on soil adjacent to pile.

For the soil component, we can write the following equation:

$$[K_s]\{w_s\}=\{P_s\} \tag{3}$$

where $[K_s]$=soil stiffness matrix; $\{P_s\}$=vector containing the forces acting on the soil due to the loaded pile; $\{w_s\}$=soil displacement vector.

Owing to equilibrium, we have

$$\{P_p\}=-\{P_s\} \tag{4}$$

In addition, under the assumption that there is no loss of bondage between the pile and the soil, compatibility condition requires that

$$\{w_p\}=\{w_s\} \tag{5}$$

Substituting these relationships into Eq. (2) yields

$$([K_p]+\omega^2[M_p]+[K_s])\{w_p\}=\{P\} \tag{6}$$

Solving Eq. (6), the vector of the displacement amplitudes for the pile are obtained. The impedance function of the pile is given by the inverse of the displacement at the pile top when P=1.

Matrix $[K_s]$ can be determined as the inverse of a flexibility matrix, the columns of which consists of the nodal displacements due to unit ring loads applied at the soil-pile interface elements and a uniform load acting at the pile base.

In order to compute the response of the soil system to these loads, the stiffness matrix approach proposed by Kausel and Roësset (1981) is used. In particular, the discrete formulation is adopted due to its simplicity. This technique is in principle restricted to layered soils over rigid bedrock. However, analysis of soils over elastic half-spaces can be accomplished with a hybrid formulation that involves the exact solution for the half-space only. Following Kausel and Roësset (1981), the layer stiffness matrix $[K_L]$ can be obtained for the discrete case as

$$[K_L] = \alpha^2[A] + \alpha[B] + [G] - \omega^2[M] \quad (7)$$

where α is the wave number that describes the variation of the variables in radial direction; $[A]$, $[B]$, $[G]$ and $[M]$ are matrices the terms of which can be found in the paper of Kausel and Roësset (1981). These terms depend on the elastic constants and mass density of soil, and the layer thickness. The stiffness matrix for the soil deposit can be assembled by overlapping the contribution of $[K_L]$ matrices of each layer.

However, it should be noted that solution is a function of the wave number. Therefore, a numerical procedure has to be used. Such a procedure requires first that the unit load be expanded by a Hankel transform, and then that solution be derived for each value of wave number α, in order to obtain the transformed displacements as a discrete function of α. Finally, once these quantities are found the actual displacements can be calculated by inversion of the Hankel transform. In the present paper, this integration has been done using a Gaussian quadrature technique.

CONE MODEL

Cone model is a simple physical elastic model representing the unbounded soil in a dynamic soil-structure interaction analysis (Meek and Wolf, 1992). For each degree of freedom of the foundation, an equivalent rigid massless disk on the surface of a homogeneous half-space is considered. The half-space below the disk is modeled as a truncated semi-infinite cone with the same material properties: mass density ρ_s, Young's modulus E_s, Poisson's ratio ν_s, and damping ratio β_s (Fig. 3).

A load applied to the disk induces stresses on an area that increases with depth. The three-dimensional pattern of body and surface wave propagation in the half-space is replaced by a one-dimensional wave propagation scheme (Meek and Wolf, 1993). As a consequence, the displacements are assumed to be constant over the cross section of the cone.

The cone model has been applied for the analysis of pile foundations in a homogeneous half-space by Wolf et al. (1992). The cylindrical soil region, which will be occupied by the pile, is viewed as a series of rigid disks equally spaced with soil between them. Using approximate Green's function (Meek and Wolf, 1994) permits to calculate the displacement of a receiver disk caused by a unit harmonic force applied to another disk. Recently, cone model has been used to determine the dynamic response of piles embedded in a soil layer by Cairo et al. (1999).

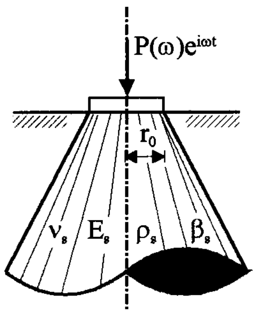

Fig. 3. Cone model.

In order to analyse a pile embedded in a layered half-space, the so-called *backbone cone* has to be constructed (Wolf and Meek, 1994). For each embedded disk, with radius r_o, the backbone cone determines the radius of the disks at all interfaces of soil (Fig. 4). Using these cone frustums, i. e. the disks at the upper and lower interfaces of each layer, the complex dynamic stiffness matrices of the layers are determined.

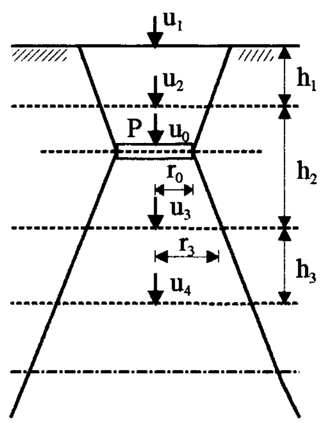

Fig. 4. Backbone cone.

Assembling the stiffness matrices of the layers and the stiffness coefficient of the disk on the underlying half-space, leads to the dynamic stiffness matrix of the soil deposit $[S]$. By subdividing the layers accordingly, the displacements can be calculated in any point on the axis of the backbone cone, i. e. at the location of all embedded disks, by solving the dynamic equilibrium equation

$$[S]\{u\} = \{Q\} \quad (8)$$

where $\{u\}$ is the displacement vector, and $\{Q\}$ is the vector of the external loads. This latter contains only a single non-zero element, that is the load applied to the source disk with radius r_o.

This procedure provides the columns of the complex dynamic flexibility matrix $[G]$ of the free field, that is discretized by the nodes corresponding to the disks. The dynamic stiffness matrix $[S_s]$ is determined by inverting $[G]$. Moreover, replacing the cylindrical soil region by the pile gives a dynamic stiffness matrix $[\Delta S]$ defined as

$$[\Delta S] = [\Delta K] - \omega^2[\Delta M] \quad (9)$$

where $[\Delta K]$ and $[\Delta M]$ are the static stiffness and mass matrices of the pile, respectively, that can be calculated using beam theory after subtracting the stiffness and mass of the (excavated) soil cylinder. Finally, assembling $[S_s]$ and $[\Delta S]$ leads to the complex dynamic stiffness matrix $[S_p]$ of the soil-pile system.

For a vertical harmonic load with unit amplitude applied at the head of the pile, the vector of the external load is

$$\{F\} = [1, 0, 0, \ldots 0]^T \quad (10)$$

and the displacement vector follows as

$$\{v\} = [S_p]^{-1}\{F\} \quad (11)$$

COMPARISONS

In this section, the results of some dynamic problems involving piles embedded in non-homogeneous soils are presented. The purpose is to compare the feasibility and accuracy of the simplified procedures described above against other more rigorous methods. It should be noted that in all the cases examined the soil-pile system has been discretized by means of 9 elements only, when the discrete stiffness matrix approach is used.

The results are presented in terms of the dynamic pile impedance versus the non-dimensional frequency a_0 that is defined as

$$a_0 = \frac{\omega d}{V_s} \quad (12)$$

where V_s is the shear wave velocity at a prescribed depth. Pile impedance is a complex quantity usually defined as

$$S = k + i a_0 c \quad (13)$$

where k and c are the dynamic stiffness and damping, respectively.

The first problem concerns a pile embedded in a soil deposit whose elastic modulus increases linearly from zero at the ground surface to $E_s=10^{-2}E_p$ at the pile tip, and remains constant throughout the underlying half-space. The parameters for the soil and pile are: $\nu_s=0.4$, $\rho_s/\rho_p=0.7$, $\beta_s=0.05$, and $L/d=20$. This problem was examined by Kaynia and Kausel (1991) using a general procedure based on Green's functions for layered media along with analytical solutions for the dynamic response of pile. The comparisons are shown in Fig. 5, where k_0 indicates the static stiffness of the pile.

As can be seen, there is a good agreement between the results obtained by the discrete stiffness matrix method (DSMM) and those presented by Kaynia and Kausel (1991), both in terms of the stiffness and damping of the pile. The maximum difference is of the order of 10%. Cone model generally provides results that are more different. Anyhow, the values of pile stiffness predicted by this model compare well with the results of the other methods. On the contrary, differences in excess of about 30% can be observed for the imaginary part of the impedance function. This occurs when the value of a_o is low.

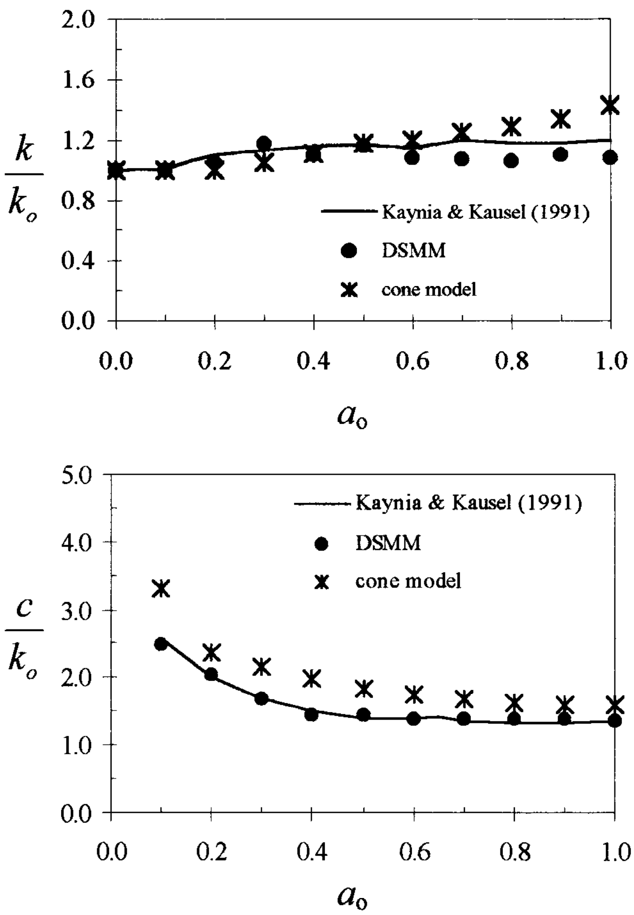

Fig. 5. Impedance function of a pile in a linear soil profile. (Adapted from Kaynia and Kausel, 1991).

A second comparative study refers to a pile in soil with parabolically increasing Young's modulus from the ground surface to the pile tip. Soil modulus is assumed to be constant below the pile base. The parameters selected for this case are: $E_p/E_s=10^2$, $\nu_s=0.4$, $\rho_s/\rho_p=0.8$, $\beta_s=0.05$, and $L/d=20$. Figure 6 shows the stiffness and damping of the pile obtained by El-Marsafawi et al. (1992) using a three-dimensional boundary integral formulation that allows the dynamic response of pile groups to be also analysed.

The results obtained using the cone model and the discrete stiffness matrix approach are also shown in Fig. 6. The same trend previously found examining the first example may be observed in the comparison shown in this figure.

Another example considered is that of a pile embedded in the two-layer soil shown in Fig. 7.

The solution to this problem has been obtained by Mylonakis and Gazetas (1998) using both the rigorous formulation of Kaynia (1982) and a closed-form expression derived by themselves. The comparison is presented in Fig.

8. As can be seen, the values of pile impedance calculated using the discrete stiffness matrix method are in close agreement with those obtained by Mylonakis and Gazetas (1998). Cone model provides again values of the dynamic stiffness in good agreement with those obtained using the other methods. The differences in terms of damping are of the same order of magnitude as that found in the cases previously examined.

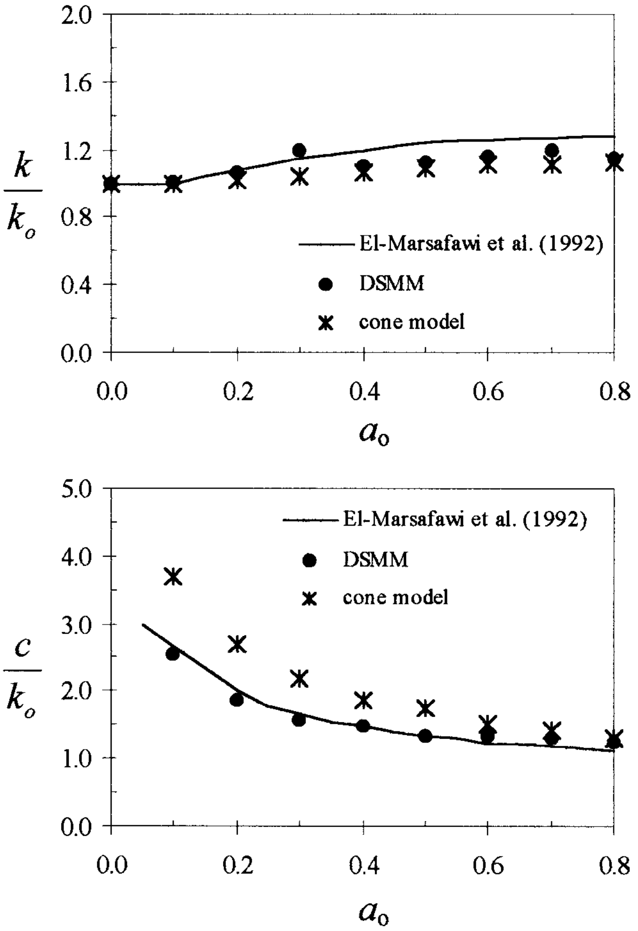

Fig. 6. Impedance function of a pile in a parabolic soil profile. (Adapted from El-Marsafawi et al., 1992).

CONCLUDING REMARKS

The comparisons carried out show that both the discrete stiffness matrix method and the cone model are suitable alternatives to complicated numerical solutions for calculating the harmonic steady-state axial stiffness and damping of piles embedded in non-homogeneous or layered soils. The results obtained using the discrete stiffness matrix method are found to be in good agreement with those derived from more rigorous methods. The maximum difference among the results is of the order of 10%. From a practical point of view, the major drawback of the method is that it works in the wave-number domain, and consequently a repeated use of Hankel transform is required.

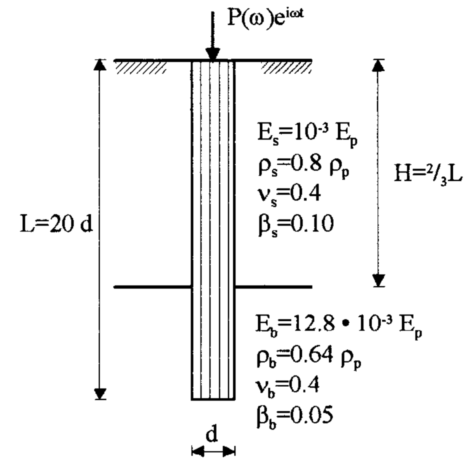

Fig. 7. Pile in two-layer soil.

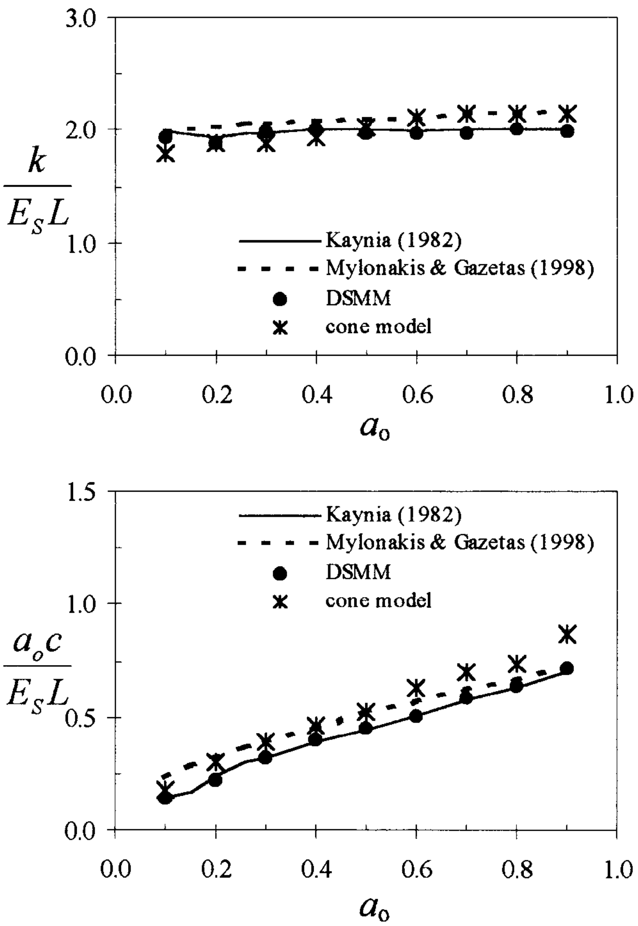

Fig. 8. Impedance function of a pile in two-layer soil. (Adapted from Mylonakis and Gazetas, 1998).

Although the cone model is of more approximate nature, it provides values of the pile dynamic stiffness in reasonable agreement with more rigorous solutions. On the contrary, differences in excess of about 30% have been found for the imaginary part of pile impedance, especially when the value of non-dimensional frequency is low. However, it should be noted that application of this method requires that no transformation to the wave-number domain is performed. This makes the cone model be very suitable for practical applications.

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