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## Liquefaction Analysis of Sands: Some Interpretation of Seed's $K_\alpha$ (Sloping Ground) and $K_\sigma$ (Depth) Correction Factors Using Steady State Concept

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# Liquefaction Analysis of Sands: Some Interpretation of Seed's $K_\alpha$ (Sloping Ground) and $K_\sigma$ (Depth) Correction Factors Using Steady State Concept

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**SYNOPSIS:** Seed's Liquefaction Assessment Chart (SLAC) correlates the corrected SPT blow count  $(N_1)_{60}$  to the cyclic stress ratio required to cause liquefaction for free field level ground conditions. However, the cyclic stress ratio or the liquefaction resistance of a soil under sloping ground (initial static shear) and at depth (high confining pressure) would be significantly different from that derived from the SLAC. To correct the liquefaction resistance for the initial static shear and confining pressure, factors  $K_\alpha$  and  $K_\sigma$  are used respectively. The factors have significant influence on the evaluation of liquefaction resistance against earthquake loading and therefore on the final outcome of the design of the structure. The corrections that are available in the state-of-the-art literature and used by the industry are empirical and inappropriate. The  $K_\alpha$  and  $K_\sigma$  factors can vary profoundly depending on the initial state of the soil in the  $q$ - $p$ - $e$  (stress/void ratio) space and its stress path being in the dilative or contractive domain of the state boundary surface. In this paper, it is demonstrated that it is possible to correlate the correction factors  $K_\alpha$  and  $K_\sigma$  to some fundamental parameters, (state parameter,  $\psi$ ), that govern the strength-deformation of soils using critical/steady state principles.

## INTRODUCTION

Liquefaction of soils due to earthquake has been a subject of active research for the past 25 years. Over this period, a number of different approaches evolved and have been utilized to evaluate the potential for liquefaction of soils in field conditions. Of these several approaches, the steady state (Castro, 1969) and the SPT-based (Seed et al., 1983) approaches have received the most attention by the engineering profession. Although these methods provide some useful framework for understanding and analysis of liquefaction, they have not been put to an acid test or had their validity established in true field design conditions except for a back analysis of a case or two. This is a dilemma in which practising engineers must determine which approach would more realistically predict the potential for liquefaction. It is partly because neither approach in isolation is able to provide both a complete understanding of the strength-deformation behaviour of the soil and an adequate analytical procedure at the same time. However, each method has some elements that would enhance the understanding and application of the other. This paper identifies two such elements in the Seed's analysis: the correction for sloping ground/initial static shear ( $K_\alpha$ ) and the correction for depth/confining pressure ( $K_\sigma$ ) of the soil. These factors have significant influence on the liquefaction resistance and hence the final design of the structure.

A typical SPT-based liquefaction analysis requires the utilization of Seed's Liquefaction Assessment Chart (SLAC) (Fig. 1). This chart was developed by Seed and Idriss (1982) and Seed et al., 1984. The chart correlates the equivalent

cyclic stress ratio required to cause liquefaction to the strength of the soil element depicted by the corrected SPT blow count  $(N_1)_{60}$ . This correlation is based on an assembly of actual data base of case histories of various earthquake affected sites of level ground conditions (Fig. 1). The  $(N_1)_{60}$  value on the abscissa is the field SPT blow count corrected to an effective overburden pressure of 1 tsf and for a hammer delivering 60% of the theoretical free fall energy. The cyclic stress ratio,  $\tau_{av}/\sigma'_o$  required to cause liquefaction is plotted on the ordinate,  $\tau_{av}$  is the equivalent cyclic shear stress induced by an earthquake of magnitude 7.5 and  $\sigma'_o$  is the vertical effective stress.

This correlation was developed for clean sands at an effective confining pressure of 1 tsf and in a level ground condition. For this condition the initial static shear stress is zero on the horizontal plane of the soil element. However, for soil elements at large depths, in slopes, dams, embankments, or near buildings, (Figs. 2a and 2b) the cyclic stress ratio could be different from that required under free field level ground conditions as depicted by the SLAC.

This could be due to the following factors:

- (a) Initial static shear stress on horizontal planes in the soil element (sloping ground, dam, building foundation).
- (b) High effective confining pressure (foundation soil under large dams, tall buildings).

These factors have a significant influence on the actual liquefaction resistance of the soil and

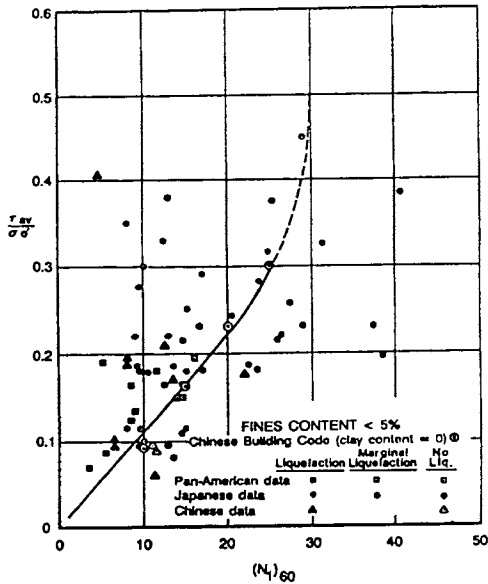


FIG. 1 - RELATIONSHIP BETWEEN CYCLIC STRESS RATIOS CAUSING LIQUEFACTION DURING  $M = 7.5$  EARTHQUAKES AND CORRECTED SPT BLOWCOUNTS IN CLEAN SANDS (AFTER SEED ET AL., 1984) (SEED'S LIQUEFACTION ASSESSMENT CHART - SLAC)

therefore can affect the performance of the structure. The influence of the factors have been demonstrated by field evidence of the 1964 Niigata (Watanabe, 1966) and the 1976 Tangshan (Liu and Qiao, 1984) earthquakes and numerous laboratory studies. In order to obtain the actual liquefaction resistance, the cyclic stress ratio determined from the SLAC should be multiplied by correction factors,  $K_\alpha$  and  $K_\sigma$  for the initial static shear stress and the confining pressure respectively.

The influence of the initial static shear stress and the confining pressure can vary profoundly depending on the initial state of the soil element (in  $q$ - $p$ - $e$  space) (Fig. 4) and whether its stress path is in the dilative or contractive domain of the state boundary surface. The correlation charts for the correction factors  $K_\alpha$  and  $K_\sigma$  that are available in the literature have been presented by Seed (1983), Harder (1988) and Rollins and Seed (1990) (Figs. 3a, 3b, 3c and 3d). These correction charts proposed by Seed and his co-workers were developed based on results of triaxial tests, mostly on dilative materials. Their correlations do not depict the differing and sometimes opposite influences of the initial static shear and confining pressures under dilative and contractive conditions of the soil. Also the influences and therefore the factors  $K_\alpha$  and  $K_\sigma$  vary with the degree of dilativeness or contractiveness.

In this paper, the mechanisms of these aspects are demonstrated for dilative and contractive conditions and it is shown that it is possible to correlate these aspects and the correction factors,  $K_\alpha$  and  $K_\sigma$ , to some fundamental parameter, such as the state parameter,  $\psi$ , that govern the strength deformation of soils using critical state/steady state principles.

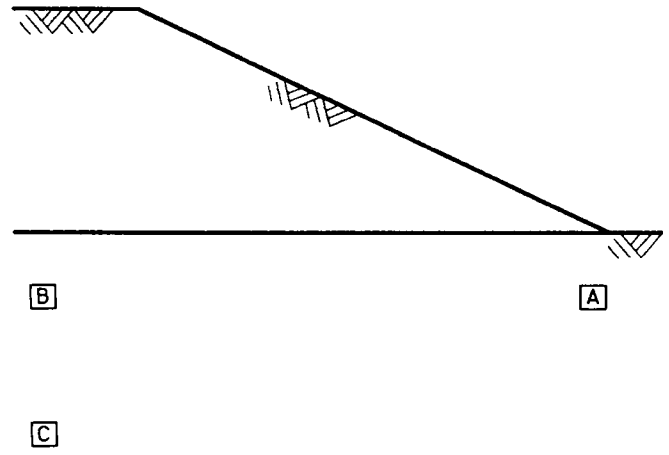


FIG. 2a

- A: SOIL ELEMENTS WITH ALPHA
- B: SOIL ELEMENTS WITHOUT ALPHA
- C: SOIL ELEMENTS WITH LARGE CONFINING PRESSURE

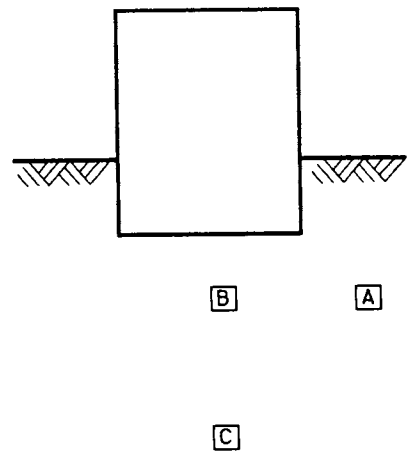


FIG. 2b

- A: SOIL ELEMENTS WITH ALPHA
- B: SOIL ELEMENTS WITHOUT ALPHA
- C: SOIL ELEMENTS WITH LARGE CONFINING PRESSURE

BACKGROUND

$K_\alpha$ : Correction for the Effect of Initial Static Shear

Alpha ( $\alpha$ ) is defined as the ratio of initial static shear stress on the horizontal plane,  $\tau_{o'}$ , divided by the vertical effective stress,  $\sigma'_{o'}$ . For any soil element in the free field level ground conditions, there exists no static shear stresses on the horizontal plane and therefore  $\alpha$  is zero. However, for soil elements in slopes,

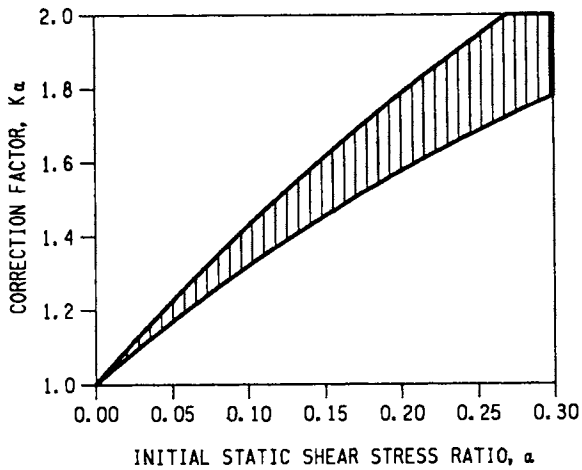


FIG. 3a - RELATIONSHIP BETWEEN  $K_\alpha$  AND  $\alpha$  (AFTER H. B. SEED, 1983)

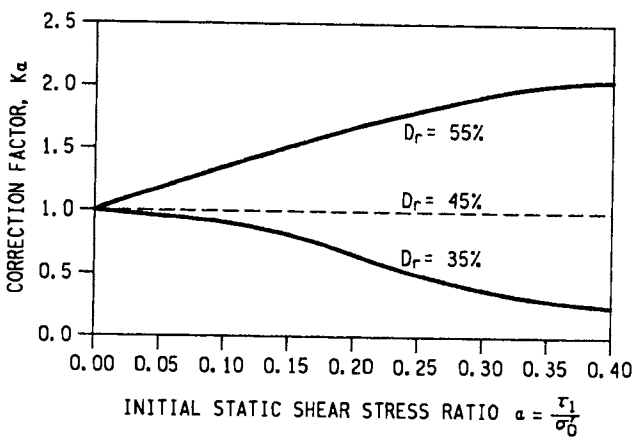


FIG. 3b - RELATIONSHIP BETWEEN  $K_\alpha$  AND  $\alpha$  (AFTER ROLLINS AND SEED, 1990)

embankments, dams and near buildings, there exist initial shear stresses on the horizontal plane, which can significantly affect the liquefaction resistance of the soil.  $K_\alpha$  is defined as the ratio of the liquefaction resistance of the soil at a given initial static shear stress ( $\alpha > 0$ ) to the liquefaction resistance under level ground conditions ( $\alpha = 0$ ).

To correct liquefaction resistance for sand with a given  $\alpha$  value, Seed (1983) proposed a correlation (Fig. 3a) between  $K_\alpha$  and  $\alpha$  and this was recently refined (Rollins and Seed 1990) (Fig. 3b). The original correlation suggested that the existence of initial static shear in the soil resulted in higher liquefaction resistance as compared to the soil without any initial static shear. However a revised correlation by Rollins and Seed (1990) (Fig. 3b) indicates that for a relative density of  $D_r = 55\%$ ,  $K_\alpha$  is larger than unity and increases with  $\alpha$ . For  $D_r = 45\%$ ,  $K_\alpha = 1$  for all values of  $\alpha$ . For  $D_r = 35\%$ ,  $K_\alpha$  is less than unity and decreases with the increase of  $\alpha$ . This correlation appears to be an advancement over the previous one; in a sense, it recognizes a condition where  $K_\alpha = 1$  and conditions where  $K_\alpha < 1$ . However, their suggested correlation remains empirical and can be misleading.

Effects of initial static shear have been investigated in the laboratory by several research workers and often produced conflicting and confusing results. For dense sands, the cyclic stress ratio required to cause liquefaction was found to increase with the increase of  $\alpha$  (Vaid and Chern 1983; Vaid and Finn 1979; Lee and Seed 1967; Seed et al. 1973; Szerdy 1986). However, dense sands at higher confining pressures ( $\sigma = 16$  ksc) produced the opposite effect (Vaid and Chern, 1985). For loose sands, generally, the cyclic stress ratio required to cause liquefaction tends to decrease with an increase of  $\alpha$  (Vaid and Chern, 1983; Szerdy, 1986; Yoshimi and Oh-Oka, 1975). Also for loose sands there exists evidence that at low confining pressures, the cyclic stress ratio might increase with  $\alpha$  (Lee and Seed, 1967).

The above laboratory test results appeared to be conflicting partly because the researchers based their studies only on relative density and failed to identify the phenomenon or fundamental parameter that govern these effects.

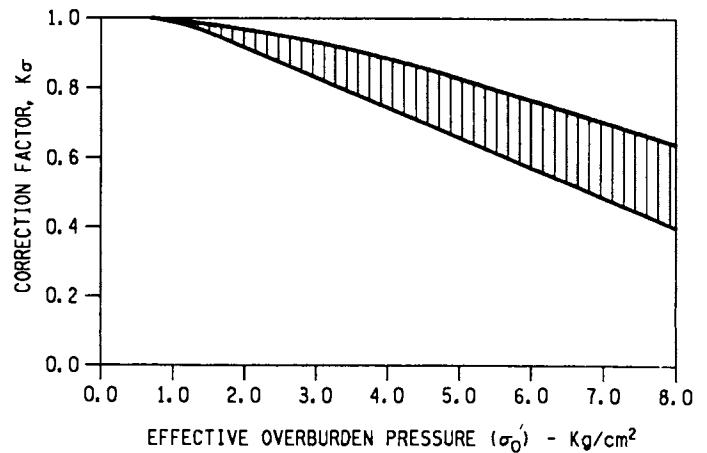


FIG. 3c - RELATIONSHIP BETWEEN  $K_\sigma$  AND  $\sigma'_0$  (AFTER H. B. SEED 1983)

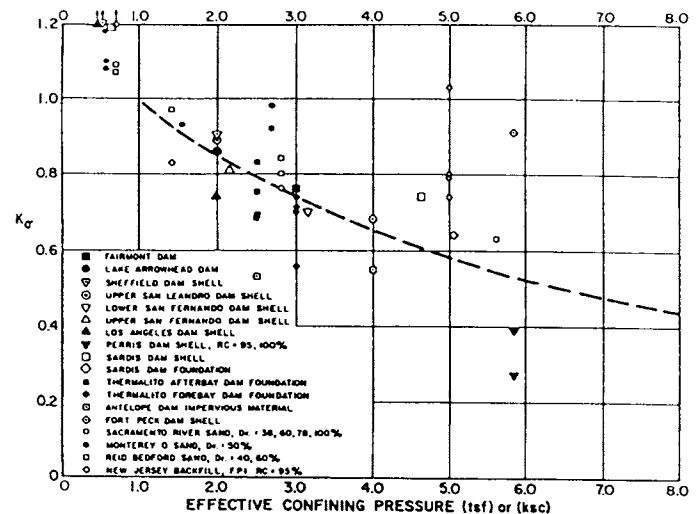


FIG 3d - EFFECTIVE VERTICAL STRESS VERSUS CORRECTION FACTOR,  $K_\sigma$  (AFTER HARDER 1988)

K<sub>σ</sub>: Correction for the Effective Confining Pressure

The chart (SLAC) that was developed by Seed was based on sites where liquefaction had occurred under fairly small overburden pressures, typically less than 1.5 tsf, and the chart is therefore only applicable to those conditions. However, as the overburden pressure on a soil increases, the cyclic stress ratio required to cause liquefaction decreases and therefore a correction must normally be made for this effect. Correction factor, K<sub>σ</sub>, is defined as the ratio of the cyclic stress ratio (liquefaction resistance) at a given confining pressure to that at 1 tsf.

Seed (1983) (Fig. 3c) developed a correlation between K<sub>σ</sub> and the confining pressure  $\sigma_v'$  and made recent refinements (Harder, 1988, Rollins and Seed, 1990) (Fig. 3d). This correlation curve is based on a collection of cyclic triaxial data carried out at various confining pressures and various densities for different types of materials (sands and gravels) and an "average" curve was drawn to indicate the trend. The wide variety of materials were dense and tested mostly under varying dilative conditions, at relatively low confining pressures, generally less than 5 ksc (~5 tsf). For the densities and confining pressures at which the soils were consolidated and tested, the materials could have been very dilative (i.e. initial states located far left of the critical/steady state line). Very little data is available on material tested at very high confining pressures or in contractive states (looser of critical state). As a result, questions on the existing correlations remain unanswered; whether this curve is material specific or can it be generalized for dilative and contractive soils or is K<sub>σ</sub> dependent on a more fundamental parameter.

INTERPRETATIONS

General

Influences of K<sub>σ</sub> and K<sub>σ</sub> on the liquefaction resistance of sands are studied and interpretations made using available published data based on critical state/steady state principles. Although there may exist some differences, for the analysis and interpretations herein, critical state and steady state are considered the same for sands on both q-p and e-p space. Details of critical state and steady state principles are described elsewhere in references (Schofield and Wroth, 1968, Atkinson and Bransby, 1978) and (Castro and Poulos, 1977 and Poulos, 1981) respectively. State parameter concepts are described in reference (Been and Jeffries, 1985). However some essentials of these principles that relate to factors K<sub>σ</sub> and K<sub>σ</sub> are briefly discussed here. It is widely recognized that the soil behaviour is dependent on the initial state of the soil (in q-p-e space) (Fig. 4) and its stress path. Depending on whether the initial state of the soil is in the contractive or dilative domain of the state boundary surface, the factors K<sub>σ</sub> and K<sub>σ</sub> can vary profoundly. These aspects can be demonstrated in 3-dimensional q-p-e space using critical state/steady state principles.

All soil states are contained within a 3-D ellipsoid-shaped geometry or state boundary

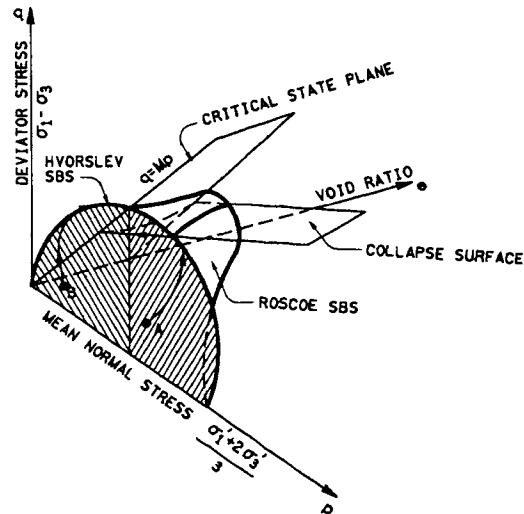


FIG. 4 - TYPICAL STATE POINTS IN CONTRACTIVE (A) AND DILATIVE (B) DOMAINS IN 3-D q-p-e SPACE

surface containing the p-axis (Fig. 4). The critical state plane ( $q=Mp$ ) divides this state boundary surface into two domains (dilative/contractive). The dilative domain is bounded by the Hvorslev surface and the contractive domain by the Roscoe surface (Atkinson and Bransby, 1978). Upon prolonged shearing, all soils regardless of their initial state, from the dilative or contractive domain strive to reach the ultimate critical state plane ( $q=Mp$ ). However, the soil would have profound differences in strength deformation behaviour during the interim shear depending on whether the initial state of soil is from the contractive or dilative domain. The differences in the strength behaviour, in effect, have different influences on the factors K<sub>σ</sub> and K<sub>σ</sub>.

Firstly, these factors are affected by the origin of the state of the soil whether it is in the dilative or contractive domain. This is determined by whether the state of the soil is located to the right or the left of the critical/steady state line (CSL/SSL) on the e-log p space (Fig. 5a). Secondly, these factors are dependent on the degree of dilativeness or contractiveness. This is determined by the relative location of the initial state of the soil with respect to the steady state line. This can be defined by the state parameter,  $\psi$ , (Been and Jeffries, 1985). A constant degree of contractiveness can be defined by a constant positive state parameter. Similarly, a constant degree of dilativeness can be defined by a constant negative state parameter. Alternatively, a constant contractiveness or dilativeness of the soil can be defined by (over-consolidation) lines which are parallel to the normal consolidation line (NCL) or the critical/steady state line. These lines represent constant over-consolidation ratios (OCR). Although NCL per se, for sands are controversial and difficult to determine in laboratory conditions because of high stresses involved, there appears to exist an apparent or pseudo NCL for sands (Fig. 5a). This pseudo NCL could form a reference line or trace of all apparent pre-consolidation pressures on e-p space

(Fig. 5a). For clays, the NCL occurs in the general engineering working stress range and it is measurable; and the significance of OCR controlling the strength-behaviour is well recognized. For sands, although OCR appear to play a similar significant role, very little experimental work has been done, partly because large structures involving high stresses are rare and experimental difficulties associated with the testing.

There exists limited data on the determination of NCL using high pressure triaxial cells on Toyura sand (Miura et al., 1984, Pillai, 1985) and Chatacochee sand (Vesic and Clough, 1968). For these sands the critical state line that separates the dilative states from the contractive states occur when the OCR of the soil is about 2 to 3 (say 2.5) in the  $e$ - $\log p$  space. If the initial state of the soil is located to the right of the CSL, then the OCR of the soil is less than 2.5 and the soil would be contractive. For the initial state located to the left of the CSL, the OCR would be larger than 2.5 and the material would be dilative.

With or without a previous stress history, loose sands at low confining pressures have an inherent tendency to dilate or behave as over-consolidated. This is often observed while walking on beach sands. This is because the confining pressures are low enough that the initial state of soils is left of the CSL and the actual OCR is in fact large so that the soil is able to produce a dilative behaviour for that state. On the contrary, a dense sand under a large confining pressure becomes weak or contractive because the OCR has decreased or the initial state has moved right of the CSL. Therefore, for sands, relative density (or void ratio) alone cannot depict strength-deformation behaviour. It is important to recognize that strength-deformation behaviour depends on the initial state of the soil as defined by the OCR or a fundamental parameter such as the state parameter  $\psi$  (Been and Jeffries, 1985), which is more readily measurable.

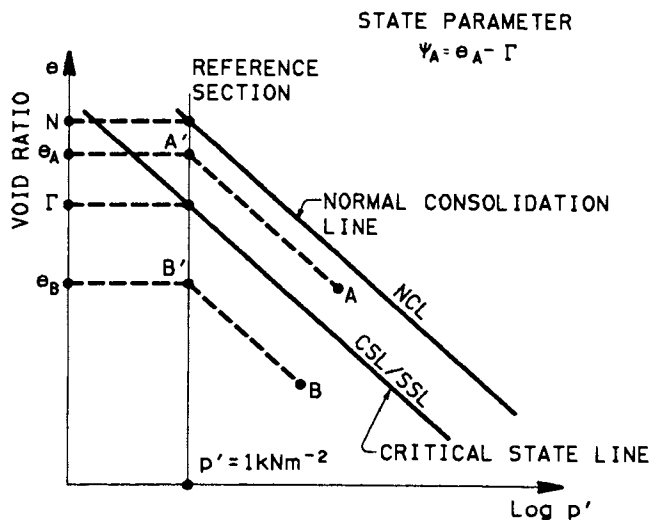


FIG. 5a METHOD OF CORRECTING STATE POINTS ONTO CONSTANT  $p'$  REFERENCE SECTION

### Interpretations of $K_\alpha$ - $\alpha$ Correlations

For Sacramento River sand, Fig. 6b shows a plot of the steady state line (SSL) on  $e$ - $\log p$  space and the initial states of three sets of tests. Fig. 6a shows  $K_\alpha$ - $\alpha$  curves for the three sets of tests. For a dilative state B, ( $\psi_i = -.07$ ),  $K_\alpha$  is represented by the curve, B; for the critical state, C ( $\psi_i = 0$ ),  $K_\alpha = 1$  is represented by the flat line C; for a contractive state A, ( $\psi_i = +.04$ ),  $K_\alpha$  is represented by the curve A. This is based on a replot of essentially the same data used by Rollins and Seed to develop the relationship of  $K_\alpha$  with relative density,  $D_r$  (Fig. 3b).

Figures 6a and 6b illustrate that for all soil states denser than critical, ( $\psi_i < 0$ ) the soil would be dilative and  $K_\alpha$  would increase with  $\alpha$  from unity. For all states looser than critical, ( $\psi_i > 0$ ), the soil would be contractive;  $K_\alpha$  would be less than 1.0 and decrease with  $\alpha$ . For all states, ( $\psi_i = 0$ ), (soil being neither contractive or dilative),  $K_\alpha$  would remain the same at unity for all values of  $\alpha$ . That is, for soil at critical state, the cyclic resistance is not affected by the initial static shear stress.

The above illustrate that  $K_\alpha$  is a unique function of the initial state parameter and it is possible to have a unique  $K_\alpha$ - $\alpha$  curve for a constant  $\psi_i$ . That is, if the dense Sacramento River sand (Fig. 6b) with  $D_r = 55\%$  is subjected to a high confining pressure of about 500 psi, the soil would become as contractive as the soil state represented by "A" ( $\psi_i = +.04$ ) and yields the same  $K_\alpha$ - $\alpha$  curve (A). Similarly, if the loose sand with  $D_r = 35\%$  is tested at a low confining pressure of about 7 psi, it would be as dilative as dense sand B ( $\psi_i = -.07$ ) and yield the same  $K_\alpha$ - $\alpha$  curve B, corresponding to the  $D_r = 55\%$  sand at 50 psi.

This is somewhat illustrated by another plot of experimental data on very dense tailings sand with a relative density  $D_r = 70\%$  (Figs. 6c and 6d). At an initial confining pressure of 2 ksc (~30 psi), (the state parameter  $\psi_i = -.10$ ),  $K_\alpha$  increased with  $\alpha$ . This result is as expected for a soil with a relative density of  $D_r = 70\%$ . However, when the same tailings sands with the same relative density ( $D_r = 70\%$ ) was subjected to a large confining pressure of 16 ksc (~240 psi) (Fig. 6c), the material became contractive (state

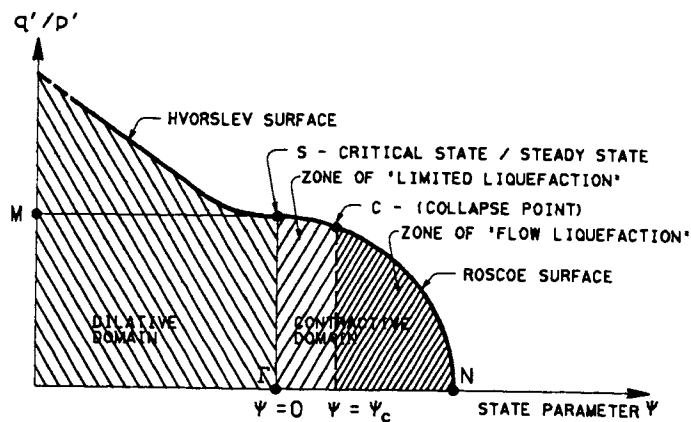


FIG. 5b REFERENCE SECTION IN  $q'/p'$ : $\psi$  SPACE

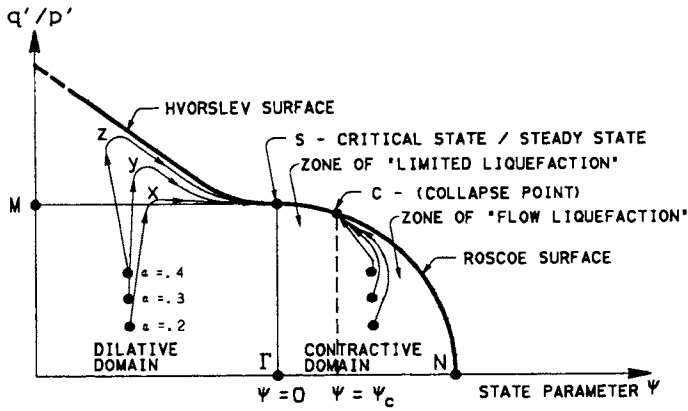


FIG. 5c SCHEMATIC ILLUSTRATION OF EFFECTIVE STRESS PATHS AND THEIR PEAKS FOR VARIOUS ALPHAS IN DILATIVE AND CONTRACTIVE DOMAINS

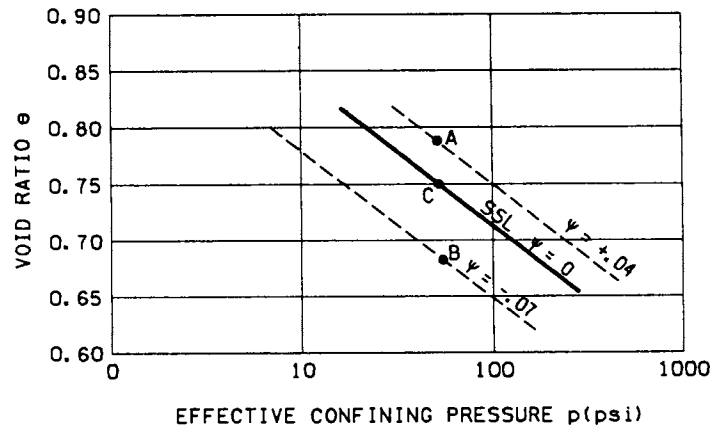


FIG 6b - INITIAL STATES OF SOIL ON e-LOG p SPACE FOR THREE CURVES ON FIG 6a (DATA FROM SZERDY, 1986)

parameter  $\psi = +.16$ )  $K_\alpha$  decreased significantly with increase of  $\alpha$ , contrary to the expectation as postulated on Fig. 3b (Rollins and Seed, 1990). This indicates clearly that  $K_\alpha$  is a function of the initial state of the soil as defined by the state parameter rather than the relative density as postulated in the current literature by Seed and his co-workers.

The influence of static shear stress on dilative and contractive soils and its mechanisms are illustrated schematically on Figs. 5a, 5b and 5c. Fig. 5b shows a normalized plot of the state boundary surface (SBS) for the dilative ( $\psi < 0$ ) and contractive ( $\psi > 0$ ) domains. This is a condensed version of the 3-D (q-p-e) stress-void ratio space (Fig. 4). Any soil state (Fig. 5a) in the e-log p space can be condensed to a single parameter,  $\psi$ , (Been and Jeffries, 1985, Atkinson and Bransby, 1978) and is represented on the abscissa (Fig. 5b). The corresponding shear stress in q-p space is reduced to a single ( $q'/p'$ ) ratio and is represented on the ordinate (Fig. 5b). In the  $q'/p':\psi$  space,  $q'/p'$  would represent the stress ratio during monotonic or cyclic loading, at any stress level [initial ( $\alpha$ ), peak or residual] while  $\psi$  would represent the changing states along the effective stress path.

On the normalized state boundary surface, (Fig. 5b), control features such as the intersection of the SBS with the critical state plane ( $q=Mp$ ), collapse surface (Sladen et al., 1985) can be represented by single points such as "S" for the critical state and "C" for the collapse surface. In the case of the latter, point C on the Roscoe surface (Fig. 5b) represents the limit of all states at which a "flow liquefaction" or a process of large strain softening is triggered. That is, although soils could be contractive for all states with values of  $\psi_i > 0$ , only those soils with initial states  $\psi_i > \psi_c$  (very loose or critical) would have the potential for "large strain softening" or "flow liquefaction". For contractive soil states located between  $0 < \psi_i < \psi_c$ , potential for only "limited strain softening" or "limited liquefaction" may exist. For initial states with  $\psi_i < 0$ , the soil would be dilative and a failure due to earthquake loading could be possible only due to cyclic mobility. The state parameter at collapse or  $\psi_c$  is an easily determinable single parameter and could be used in delineating contractive soils that may undergo "flow liquefaction" from that of "limited liquefaction". The parameter  $\psi_c$  appears to represent the "L line" in the e-log  $\sigma'_3$  space defined by Castro, (1969).

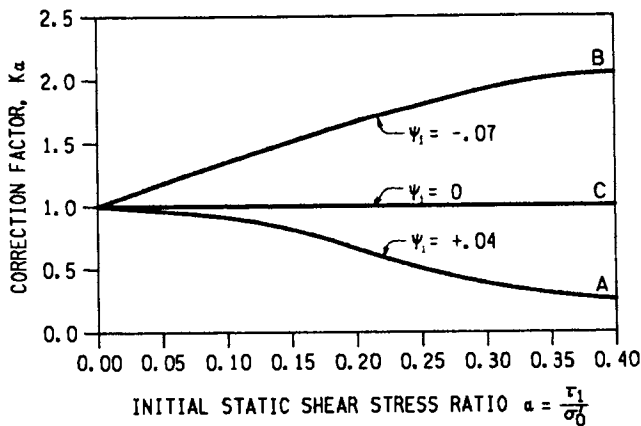


FIG. 6a - CORRELATIONS OF  $K_\alpha$ , ALPHA  $\alpha$  AND STATE PARAMETER,  $\psi$  (SACRAMENTO SAND) (DATA FROM SZERDY, 1986)

Fig. 5c illustrates schematically the effects of various levels of initial static shear,  $\tau/\sigma' = q'/2p'$ , for soils in both dilative and contractive states. For the dilative states, this is illustrated by the rising peaks (x, y, z) of three undrained effective stress paths for increasing levels (three) of static shear, or  $\alpha$ , indicating that  $K_\alpha$  would increase with  $\alpha$ . This is for a given initial negative state parameter (e.g.  $\psi_i = -.07$ ) as depicted by curve B on Fig. 6a. When the initial state occurs more to the left, the soil would have more dilative potential and the peaks of the stress paths would be higher than (x, y, z) that correspond to the same  $\alpha$  for the initial state  $\psi_i = -.07$ . Consequently there would be a  $K_\alpha - \alpha$  curve higher than the curve corresponding to  $\psi_i = .07$ . In addition to the effect of  $\alpha$ , Fig. 7 shows a plot of increasing cyclic stress ratio (resistance) with increasing dilativeness or decreasing  $\psi_i$  for Ottawa sand, when  $\alpha$  is kept constant. The phenomena of increasing cyclic or monotonic stress ratios with an increase in  $\alpha$  and/or decrease in initial state

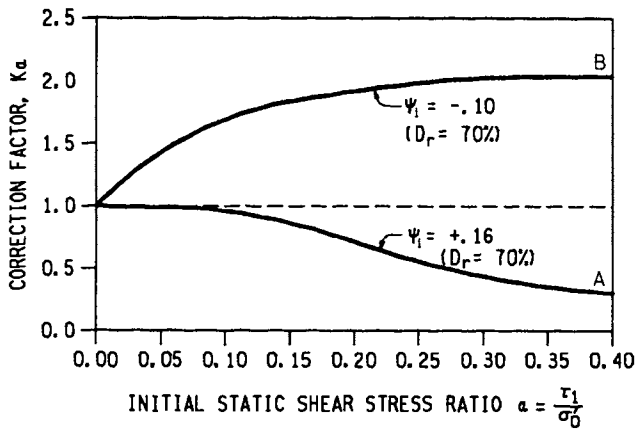


FIG. 6c - CORRELATIONS OF  $K_\alpha$ , ALPHA  $\alpha$  AND STATE PARAMETER,  $\psi_i$  (TAILINGS SAND) (DATA FROM VAID AND CHERN, 1985)

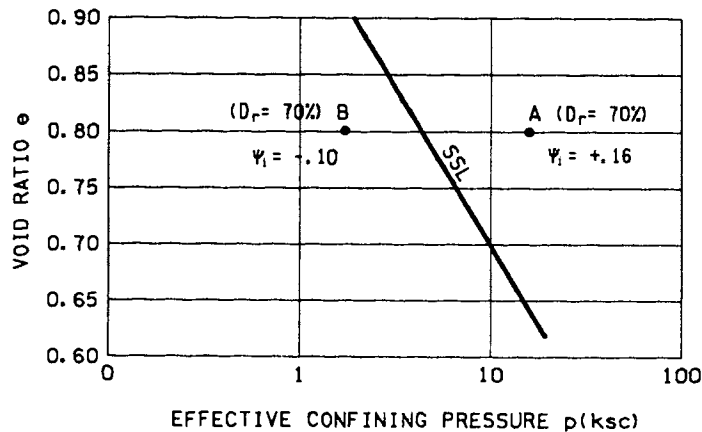


FIG 6d - INITIAL STATES OF SOIL ON  $e$ -LOG  $p$  SPACE FOR TWO CURVES ON FIG 6c (DATA FROM VAID AND CHERN, 1985)

parameter ( $\psi_i$ ) are due to the rising nature of the ultimate stress ratio ( $q'/p'$ ) envelope or the normalized Hvorslev state boundary surface.

On the other hand, near the lower limit of the dilative states (Fig. 5c), that is when  $\psi_i=0$  and the initial state is near critical, peaks of all three stress paths as indicated by x, y, z will be the same and coincide at the point "S" (critical state). That is, the cyclic or monotonic resistance is not affected by the levels of initial static shear ( $\alpha$ ). This is demonstrated by the experimental data on Sacramento River sand (curve C) on Fig. 6a.

For all contractive states ( $\psi_i > \psi_c$ ), the peak of all undrained stress paths, monotonic or cyclic, is limited by a single point (C on Roscoe surface) (Fig. 5c). This point represents not only the peak stress ratio but also the shear strains that could be accumulated through cyclic or monotonic loading before the onset of liquefaction or the large strain softening process. Unlike for the dilative states where the peak is limited by a rising curve (Hvorslev surface), the peak in the contractive domain is

limited by a single point. Consequently any increasing static shear ratio or  $\alpha$  would proportionately reduce the available cyclic or monotonic stress ratio before the onset of large strain softening. If the initial static shear stress ratio is large enough to reach the point it would trigger a static liquefaction involving flow deformation. This was evidenced as in the case of hydraulic sand fill islands in the Beaufort Sea (Nerlek) (Mitchell, 1984).

For contractive states, most of the available experimental data appears to fall in a zone slightly looser than critical ( $0 < \psi < \psi_c$ ) where only a limited liquefaction is possible. As discussed previously, the initial static stress ratio ( $\alpha$ ) has little effect on  $K_\alpha$  particularly when  $\alpha$  is small or when the soil is marginally contractive or the initial state is located only slightly to the right of CSL. This is a possible reason for the flat portion of the  $K_\alpha$ - $\alpha$  curves when  $\alpha$  is small (Fig. 6a and 6c) for contractive materials. However, with increasing  $\alpha$ ,  $K_\alpha$  decreases accordingly (Fig. 6a and 6c). Also with increasing initial state parameter  $\psi_i$  particularly for  $\psi_i > \psi_c$ ,  $K_\alpha$  would decrease proportionately with  $\alpha$  and the corresponding  $K_\alpha$ - $\alpha$  curves would likely be lower than that shown on Figs. 6a and 6c without the flat portion. There is little available published data for this very contractive zone  $\psi_i > \psi_c$ .

#### Interpretations of $K_\alpha$ - $\sigma'$ Correlations

This is an important correction particularly for a deep deposit of foundation soil supporting a large structure (e.g. high embankment dam, tall building). The correlation curves that depict the effect of confining pressure on the liquefaction resistance and that are presently available are shown on Figs. 3c and 3d. These correlations were developed by Seed and his co-workers, based on data obtained from a wide variety of materials (sands and gravels) tested mostly under varying dilative conditions. In effect, the wide scatter of data came from tests on soils with varying initial state parameter but always less than zero ( $\psi_i < 0$ ) or denser than critical state. The "average" curve indicates that  $K_\alpha$  tends to decrease with increasing confining or effective overburden pressure.

Fig. 5c illustrates schematically that for initial soil states denser than critical ( $\psi_i < 0$ ), the state boundary surface or the peak stress ratios ( $q'/p'$ ) corresponding to a preset strain level (e.g.  $e_a = \pm 2.5\%$ ) tend to decrease with increasing initial state parameter or confining pressure. Again, for a particular initial state parameter,  $\psi_i$ , the peak stress ratio or cyclic resistance ratio is constant for a given initial static shear and strain level. This is further shown by a plot of data on Ottawa sand (Vaid and Chern, 1985) which indicates that cyclic stress (resistance) ratio is the same for all soil states with a constant initial state parameter,  $\psi_i$  (Fig. 7). This is a plot of cyclic stress (resistance) ratios on  $e$ -log  $p$  space corresponding to various initial states denser than critical. It indicates that constant cyclic resistance ratios follow contours along lines parallel to the steady state line. Since they parallel CSL/SSL, these contours are lines of



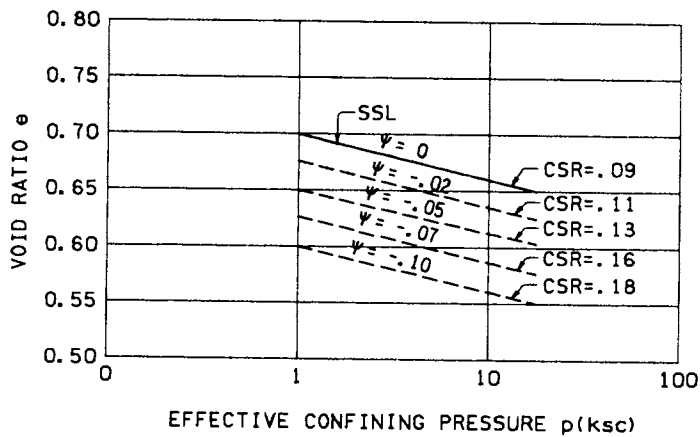


FIG. 7 - LIQUEFACTION RESISTANCE (CSR) VERSUS STATE PARAMETER ( $\psi_i$ ) FOR OTTAWA SAND (DATA FROM VAID AND CHERN, 1985)

constant state parameters. Fig. 7 further indicates that with increasing initial state parameter or decreasing dilativeness, the cyclic stress ratio decreases as illustrated on Fig. 5c. It appears that  $K\sigma$  is primarily a phenomenon of dilative soils and a function of the initial state of soil as defined by the effective confining pressure and void ratio.

For contractive states ( $\psi_i > 0$ ), there is little data available to indicate that there exists a similar decreasing trend for  $K\sigma$  as for the dilative states. The existing correlations of  $K\sigma$  with confining pressures are exclusively based on dilative or very dilative materials and may not be appropriate for materials in contractive state.

#### SUMMARY AND CONCLUSIONS

Based on the foregoing interpretations, concepts and discussions, the following are concluded:

1. Initial static shear stress and confining pressure can significantly affect the cyclic stress ratio required to cause liquefaction. Therefore evaluation of liquefaction resistance of soil under structures against earthquake using SPT-based analysis or SLAC requires appropriate determination of the correction factors  $K\alpha$  and  $K\sigma$ .
2. For the determination of  $K\alpha$  and  $K\sigma$ , the correlation charts that are available in the state-of-the-art literature (developed by Seed and his co-workers) are empirical and inappropriate as these correlations were developed based on a soil state defined only by "relative density" or tests on soil with initial state denser (drier) than critical.
3. For a particular soil,  $K\alpha$  appears to be a unique function of the initial state parameter,  $\psi_i$ , which is a function of relative density (void ratio) as well as effective confining pressure. Also there exists a unique  $K\alpha$ - $\alpha$  curve for each constant initial state parameter ( $\psi_i$ ), both in dilative and contractive domains.

4. The normalized state boundary surface on stress ratio ( $q'/p'$ ): state parameter ( $\psi$ ) space provides the unique point "C" with a unique single parameter,  $\psi_c$ , in the contractive domain, which separates the state that has the potential for "flow liquefaction" ( $\psi_i > \psi_c$ ) from those of "limited liquefaction" types ( $0 < \psi_i < \psi_c$ ).
5. For states denser than critical, contours of constant initial state parameter would yield a constant cyclic stress (resistance) ratio. That is, liquefaction resistance is the same along each line parallel to and located on the left of CSL/SSL, when all other factors such as  $\alpha$  are kept constant. This indicates that for dilative states  $K\sigma$  is a function of not only the confining pressure but also the void ratio. The existing correlations of  $K\sigma$  with confining pressures are exclusively based on dilative or very dilative material and may not be appropriate for contractive materials which have the greater potential for liquefaction failure than those in the dilative state (cyclic mobility).

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