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SYNOPSIS A study of the effects of dam-foundation interaction on the response of earth dams to obliquely incident P and SV waves is presented. The numerical formulation combines the Finite Element Method and the Boundary Equation Method in a powerful hybrid technique. The FEM has been proven very efficient for finite size elastodynamics problems, but several previously suggested modifications for handling infinite domain dynamic problems seem to be either computationally expensive or have serious limitations on the geometry and excitation. The BEM is employed to solve the halfspace problem using exact Green's functions to compute the halfspace stiffness, which is then incorporated in the FEM solution. This technique proved to be exceptionally powerful as it leads to accurate and very efficient solutions. A preliminary study is undertaken to investigate the response of dams subjected to obliquely incident P and SV waves, propagating across the dam width. The results of the present rigorous study extend the conclusions from earlier studies on the effects of the dam-foundation interaction and the special variability of the ground motion. Moreover, the proposed model provides an efficient tool for dynamic analysis of earth dams and is part of a broader study of 2D and 3D dams using the hybrid formulation in both the frequency and time domains, with final objective the incorporation, in the latter case, of soil nonlinearity.

INTRODUCTION

The seismic response of earth and rockfill dams has been the subject of considerable research in the last twenty years with primary focus on the effects of such factors as: the dam and canyon geometries; the inhomogeneity of the dam material; the nonlinear and inelastic behavior of the material; the relative flexibility of the dam and the canyon or foundation materials; and, to only a limited extent, the spatial variability of the excitation. Detailed accounts of past contributions are available in the state-of-the-art reports by Gazetas (1987) and Gazetas and Dakoulas (1992).

Earlier numerical and theoretical studies of the spatial variability of the ground motion were limited by certain restricting simplifying assumptions. Thus, early finite element studies of plane strain dams subjected to waves traveling across the dam width did not consider the dam canyon interaction. Also, a series of theoretical studies using shear beam type models considered the dam-canyon interaction, but focused only on obliquely incident SH waves propagating along the longitudinal direction of the dam (Dakoulas et al.1992,1994; Dakoulas 1993). These models helped clarify the effects of various parameters, including the canyon geometry and the canyon flexibility. However, the study of waves traveling across the width of the dam could not be realized with the shear beam. To study the response of a "2-D" or "3-D" dam due to travelling waves along any direction, a rigorous formulation consisting of a coupling of the finite element method (FEM) and the boundary element method (BEM) is presented for general soil-structure interaction problems.

Among the various numerical methods employed by many researchers for investigating the problems of soil-structure interaction and wave propagation, are the FEM, combined with a variety of transmitting boundaries, and the BEM. The FEM is capable of handling all geometry configurations, non-homogeneity and non-linearity of structures and the near field soil, but lacks the ability to simulate by itself the infinitely extended far field. For that reason, an artificial boundary has to be introduced, requiring special treatment for preventing fictitious wave reflections. On the other hand, the BEM seems to be an ideal approach for treating infinite domains, as it has the inherent advantage of being able to simulate the radiation conditions at the far field. However, it is not convenient for treating complex geometries, non-homogeneous and non-linear problems.

In order to benefit from the advantages of each of the two methods while avoiding their disadvantages, it is expedient to combine both methods in a hybrid procedure. Zienkiewicz *et al.* (1977) are among the first authors to propose such coupling (Brebbia *et al.* 1984, Beskos 1987).

The initial formulation and applications of the coupling technique were considered in the frequency domain. Some representative studies are by Kobayashi and Mori (1986) and Wang (1992) who employed a combination of the FEM and the BEM to solve some generic 3-D soil-structure interaction problems; Auersch and Schmid (1990) who solved 2-D soil-structure interaction problems; Bielak *et. al.* (1991) who applied this technique to the problem of soil amplification in inhomogeneous alluvial valleys due to incident SH waves; and Mossessian and Dravinski (1987) who investigated wave scattering problems by near surface irregularities.

Transient soil-structure interaction problems were also treated using the same coupling technique in the time domain. Spyrakos and Beskos (1986) investigated flexible strip foundations subjected to external dynamic force as well as seismic waves, while Karabalis and Beskos (1985) examined 3-D flexible foundations in time domain. Von Estorff and Kausel (1989) demonstrated the applicability of the coupling technique in various soil-structure interaction problems such as flexible foundations, open and filled trenches and tunnels.

In the application of the FE-BE coupling technique to soil-structure interaction problems, boundary elements are typically used to discretize the near/far field interface, while the finite elements are used to discretize the whole near field domain.

In this paper, frequency domain formulation is presented and applied to study the seismic response of an idealized earth dam on elastic halfspace subjected to obliquely incident P and SV waves, propagating across the dam width.

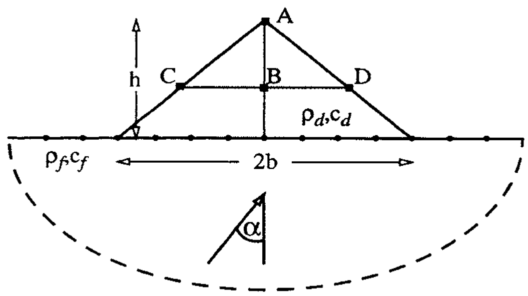

Figure 1 Layout of the idealized dam cross-section.

FORMULATION

Finite element analysis

The FEM is used to discretize the whole domain of the near field into a finite number of elements. The displacements within each element are approximated using shape functions, $u(x,t) = N\hat{u}(t)$ where N is the shape function vector and $\hat{u}(t)$ are the nodal displacements. To minimize the error resulting from the discretization, the Galerkin weighted residual formulation is used, in which the weighting function is taken the same as the shape function. This discretization scheme renders the following matrix differential equation

$$[M]\ddot{u} + [K]u + f = 0 \tag{1}$$

where $[M]$, $[K]$ are the global mass and stiffness matrices and f is the vector of nodal loads. For steady state response, the nodal displacements can be expressed as $u = u(x)e^{i\omega\tau}$ which substituted in (1) leads to

$$[A]u = f \tag{2}$$

where

$$[A] = -\omega^2[M] + (1+2i\beta)[K] \tag{3}$$

and β is the damping coefficient.

Boundary element method

Making use of the dynamical reciprocity theorem with one state being the real state of the problem, and the other being the fundamental singular solution pair expressing the displacements, G_{ij}, and the forces, F_{ij}, the governing equation in frequency domain can be expressed in terms of boundary integral equations as (Eringen and Suhubi 1975)

$$c_{ij}u_j + \int_\Gamma F_{ij}u_j\,d\Gamma = \int_\Gamma G_{ij}t_j\,d\Gamma \tag{4}$$

where u and t denote displacements and tractions; $\Gamma = \Gamma_u + \Gamma_t$, in which Γ_u is the boundary with prescribed displacements, Γ_t is the boundary with prescribed tractions; and c_{ij} is a constant related to the geometric location and smoothness of the boundary at the source points. The fundamental solutions G_{ij} and F_{ij} for 2-D steady state elastodynamics have the form (Cruse and Rizzo 1968)

$$G_{ij} = \frac{1}{2\pi\rho c_2^2}[\Phi\delta_{ij} - \Psi r_{,i}r_{,j}] \tag{5}$$

$$F_{ij} = \frac{1}{2\pi}\left[\left(\frac{d\Phi}{dr} - \frac{\Psi}{r}\right)\left(\delta_{ij}\frac{\partial r}{\partial n} + r_{,j}n_i\right) - 2\frac{\Psi}{r}\left(r_{,i}n_j - 2\frac{\partial r}{\partial n}r_{,i}r_{,j}\right) - 2\frac{d\Psi}{dr}\frac{\partial r}{\partial n}r_{,i}r_{,j} + \left(\frac{c_1^2}{c_2^2} - 2\right)\left(\frac{d\Phi}{dr} - \frac{d\Psi}{dr} - \frac{\Psi}{r}\right)r_{,i}n_j\right]$$

$$\Phi = K_0\left(\frac{i\omega r}{c_2}\right) + \frac{c_2}{i\omega r}\left[K_1\left(\frac{i\omega r}{c_2}\right) - \frac{c_2}{c_1}K_1\left(\frac{i\omega r}{c_1}\right)\right] \tag{6}$$

$$\Psi = K_2\left(\frac{i\omega r}{c_2}\right) - \frac{c_2^2}{c_1^2}K_2\left(\frac{i\omega r}{c_1}\right)$$

where r is the distance between the source and field points,

K_0, K_1, K_1 are modified Bessel functions of the second kind, c_1, c_2 are the P and SV complex valued wave velocities in the elastic body. By discretizing the boundary of the problem into finite number of elements and using the same concept of shape functions as in FE to describe the nodal displacements, (4) can be expressed as a system of linear equations

$$[F]\,u = [G]\,t \tag{7}$$

where $[F]$ is the constant coefficient matrix derived by integrating the left hand side of (4) and adding c_{ij}; $[G]$ is the coefficient matrix derived by integrating the right hand side of (4).

It should be noted that a sufficient portion of the half-space should be modelled, so that any resulting error from the approximation is small enough to be neglected.

Coupling of finite and boundary elements

In order to incorporate the wave excitation into the formulation of the coupling procedure, the wave field should by divided in two components: the free field component (superscript *f*) which is the wave field in an equivalent half-space having the same material properties, and the scattered field component (superscript *s*) which is the field of scattered waves due to the existence of the geometrical as well as material irregularities imposed on the half-space. The displacements can then be expressed as

$$u = u^f + u^s \tag{8}$$

The free field component can be calculated analytically at all nodes within the near/far field interface depending on the type and angle of the incident wave. Equation (7) can be rewritten as

$$\begin{bmatrix} F_{ii} & F_{ir} \\ F_{ri} & F_{rr} \end{bmatrix} \begin{bmatrix} u_i^s \\ u_r^s \end{bmatrix} = \begin{bmatrix} G_{ii} & G_{ir} \\ G_{ri} & G_{rr} \end{bmatrix} \begin{bmatrix} t_i - t_i^s \\ -t_r^s \end{bmatrix} \tag{9}$$

where the *i*, *r* subscripts indicate the interaction nodes lying on the boundary and finite elements interface, and the rest of the boundary element nodes, respectively; t_i^s and t_r^s are stresses due to the scattered wave field, and t_i is the stress at the interaction nodes resulting only from the near field region. The unknowns in (9) are the displacement components, and the stress vector t_i.

By using the structural condensation technique, one can eliminate the extra degrees of freedom, keeping only those associated with the interaction nodes. Equation (9) can be expressed as

$$\hat{F} u_i^s = \hat{G} t_i + \hat{C} \tag{10}$$

where

$$\hat{F} = [F_{ii}] - [F_{ir}][F_{rr}]^{-1}[F_{ri}] \tag{11}$$
$$\hat{G} = [G_{ii}] - [F_{ir}][F_{rr}]^{-1}[G_{ri}]$$
$$\hat{C} = -\Big[[G_{ir}]t_r^f + [G_{ii}]t_i^f\Big] + [F_{ir}][F_{rr}]^{-1}[G_{rr}]t_r^f + [F_{ir}][F_{rr}]^{-1}[G_{ri}]t_i^f$$

To ensure compatibility between the far field discretized as boundary elements, and the near field discretized as finite element, a transformation from BE tractions to FE nodal forces is introduced

$$f_i = -Tt_i \tag{12}$$

where T is a transformation matrix constructed using the FE shape functions N and the BE shape functions L as

$$T = \int_{\Gamma_i} N^T L \, d\Gamma_i \tag{13}$$

By making use of (12), equation (10) yields

$$\hat{A} u_i^s = -f_i + \hat{B} \tag{14}$$

in which

$$\hat{A} = [T][\hat{G}]^{-1}[\hat{F}] \tag{15}$$
$$\hat{B} = [T][\hat{G}]^{-1}[\hat{C}]$$

The FE system of linear equations in (2) can also be written as

$$\begin{bmatrix} A_{dd} & A_{di} \\ A_{id} & A_{ii} \end{bmatrix} \begin{bmatrix} u_d \\ u_i \end{bmatrix} = \begin{bmatrix} f_d \\ f_i \end{bmatrix} \tag{16}$$

where the subscript *i* still indicates the interaction on the BE and FE interface, and the subscript *d* indicates the rest of the FE nodes in the near field.

By combining (14) and (16), the complete coupled system takes the form

$$\begin{bmatrix} A_{dd} & A_{di} \\ A_{id} & A_{ii} + \hat{A} \end{bmatrix} \begin{bmatrix} u_d \\ u_i^s \end{bmatrix} = \begin{bmatrix} f_d - A_{di} u_i^f \\ \hat{B} - A_{ii} u_i^f \end{bmatrix} \tag{17}$$

By solving (17), the displacements vector u_d in the FE region and the scattering wave field u_i^s at the interaction boundary are obtained. The interaction tractions t_i, interaction forces f_i and the scattered wave field u_r^s can also be calculated using (9), (10) and (12). The total displacement field is given by (8).

RESPONSE OF EARTH DAM TO P AND SV WAVES

The general formulation presented above has been implemented in a computer program and is used in this paper to study the effects of the spatial variability of the ground motion on the response of earth dams. The dam is idealized as a plane strain linearly-hysteretic elastic body founded on an elastic halfspace. The excitation consists of obliquely incident P and SV waves travelling across the width of the dam cross-section.

The dam has a height of h=100 m and base width of 2b=400 m. The dam material has an elastic Young's modulus $E=8.19 \times 10^8$ N/m^2, mass density ρ_d=1920 kg/m^3, Poisson's ratio $\nu = 1/3$, and shear wave velocity c_d =400 m/s. The damping ratio β is 10%. The dam body is discretized using plane strain four-node isoparametric elements.

The material of the half-space has a mass density ρ_f=2400 kg/m^3, Poisson's ratio $\nu = 1/3$ and no material damping. The flexibility of the elastic base rock is considered by examining a range of impedance ratio values, defined as

$$IR = \frac{\rho_f c_f}{\rho_d c_d} \tag{18}$$

where c_f is the shear wave velocity of the foundation rock.

The half-space is discretized using two-node boundary elements. The discretized length of the surface of the half-space is taken equal to $10 \times b$.

The results of the parametric study are presented in terms of amplifications, AF, of the motion with reference to the free field surface motion. AF is plotted versus a dimensionless frequency $a_o = \omega h / c_d$.

It is of interest to examine first the effect of the flexibility of the elastic rock halfspace on the dam response. Fig. 2 plots the amplification of the horizontal motion due to vertically incident SV waves, evaluated at the crest of the dam (point A) and at three midheight points (B, C and D) indicated in Fig. 1. The four curves in each of the plots in Fig. 2 correspond to impedance ratios IR = 2, 5, 10 and ∞. The results demonstrate that the flexibility of the base rock has a substantial effect on the dam response near the first resonance in shear. For higher frequencies however the response near the crest and within the dam body seems to be much lower and the effect of the flexible base much smaller, except near the slopes where the second resonance seems to be almost as pronounced as the first. Similar results are obtained also from the plane strain shear beam on elastic foundation for the first resonance, but for higher frequencies the latter predicts higher response. Fig. 3 presents the amplification of the vertical motion at the same points within the dam for the case of vertically incident P waves. The conclusions drawn from this figure are quite similar to those from the SV waves.

Fig. 4 plots the amplification of the horizontal motion at points A, B, C and D of a dam with impedance ratio IR=5, subjected to SV waves incident at angles $\alpha = 0°$, $15°$ and $30°$. The results indicate that, even for vertically incident waves, there are differences in the response at point B and points C and D, associated with reflections on the surface of the two slopes. The maximum amplification at the first resonance decreases only slightly as the angle of incidence increases. At higher frequencies the amplification at the points C and D differs due to the presence of antisymmetric vibrational modes in the transverse direction, induced by the spatially variable base excitation from the SV waves. Fig. 5 plots the amplification of the vertical motion for the same dam subjected to P waves. More significant differences are shown between the response at point B and points C and D for vertically incident P waves than those in Fig. 4.

In addition to the steady state harmonic response, it is of interest to examine the differences in the response in the time domain by using an actual earthquake record. Fig. 6 plots the acceleration time history and the Fourier spectra of the El Centro earthquake record used here as input excitation. By using Fourier analysis, the acceleration histories at the four points are computed and plotted in Fig. 7 for SV waves incident at an angle $\alpha = 30°$ to the vertical. The differences in the acceleration time histories at points C and D reflect the different amplification of the high frequency content of the excitation.

The results from the preliminary and limited study presented here confirm earlier findings about the significance of the flexibility of the foundation rock in reducing the overall response of the dam, by accounting in a rigorous way for the exact amount of energy radiated back into the halfspace. The effects of the spatial variability of the ground motion for P and SV waves travelling across the width of the dam, from the limited results presented here, seem to be important but perhaps less dramatic than those reported previously from FE studies which ignored the dam-foundation interaction and from shear beam studies for SH waves travelling in the longitudinal direction. These studies concluded that the effect of the spatial variability of the ground motion may lead to response values that are substantially higher than those caused by coherent ground motion. A more comprehensive study is in progress using both frequency (as in this work) and time domain formulations of 2D and 3D dams, the conclusions of which will be published elsewhere.

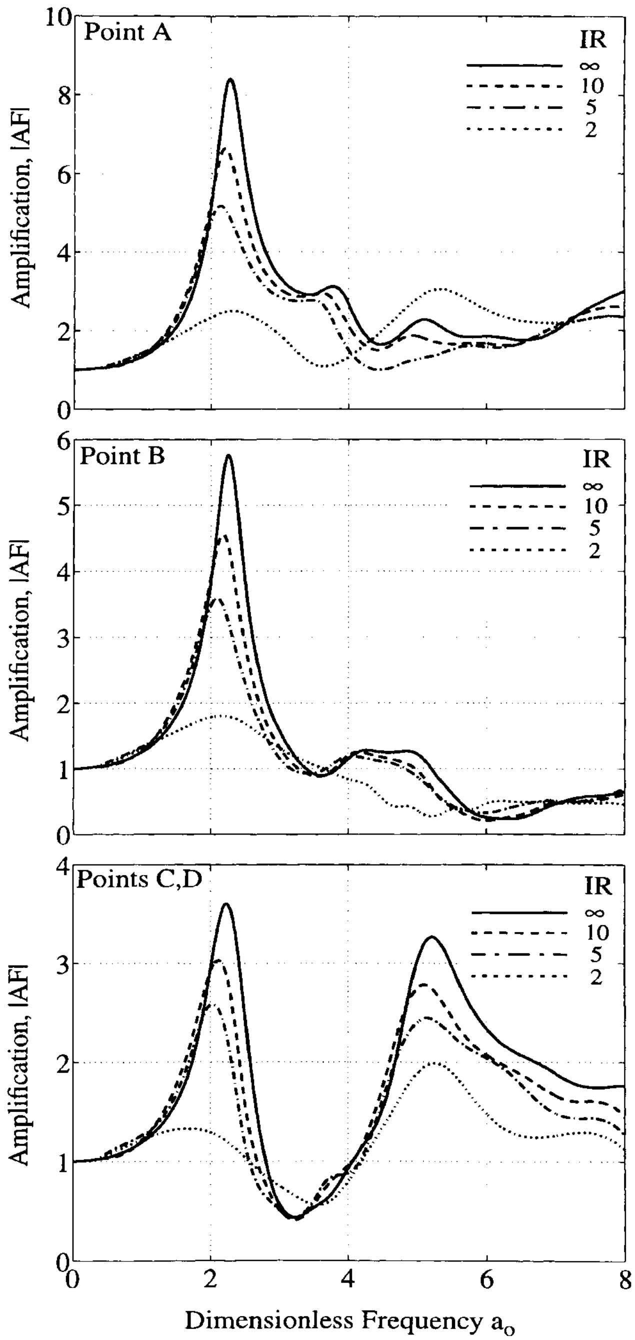

Figure 2 Horizontal amplification at different points within the dam body due to vertically incident SV waves for impedance ratios IR = 2,5,10, and ∞.

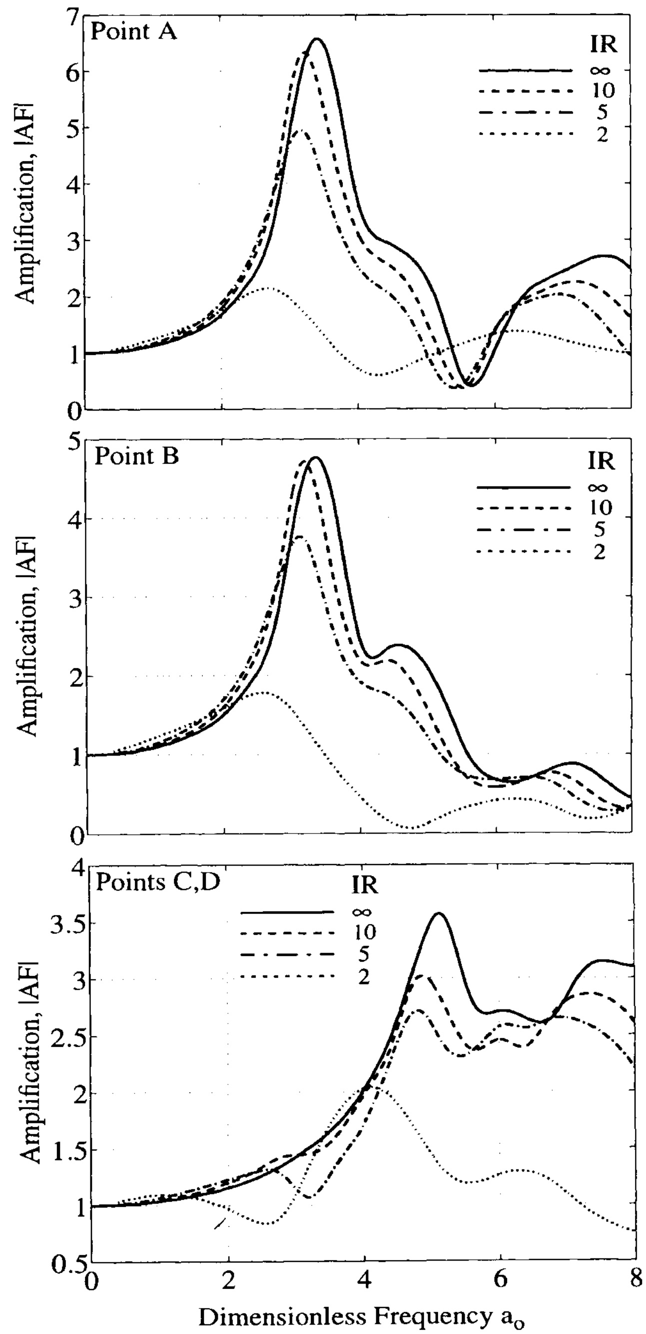

Figure 3 Vertical amplification at different points within the dam body due to vertically incident P waves for impedance ratios IR=2,5,10, and ∞.

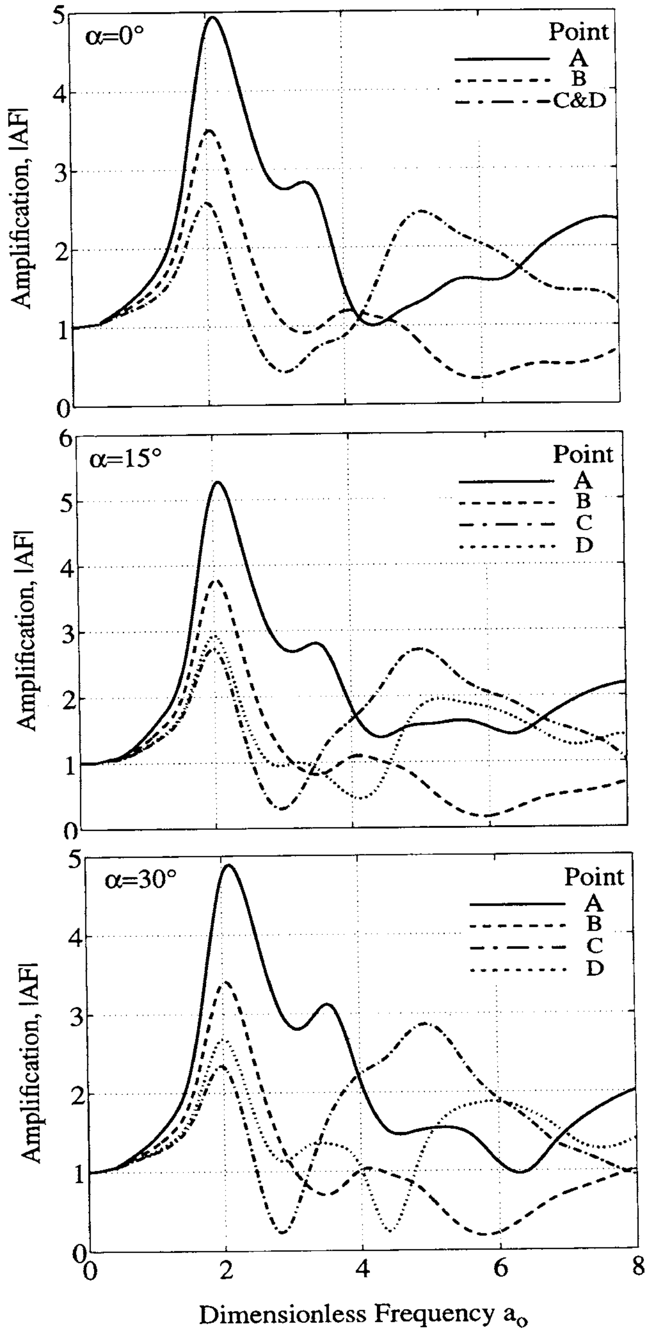

Figure 4 Horizontal Amplification at different points of the dam body due to incident SV waves with impedance ratio IR=5.

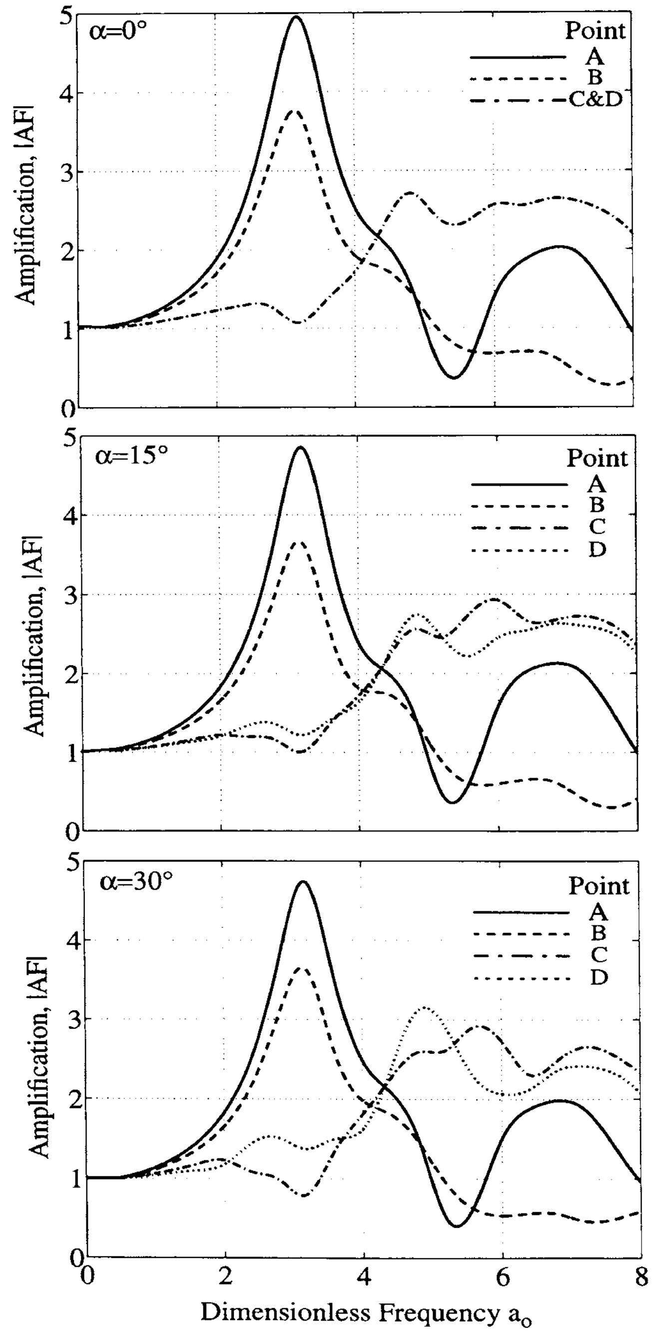

Figure 5 Vertical Amplification at different points of the dam body due to incident P waves with impedance ratio IR=5.

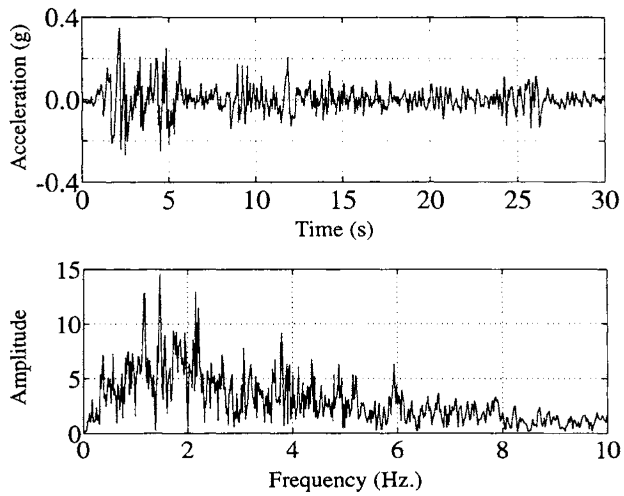

Figure 6 El Centro earthquake record and Fourier spectra.

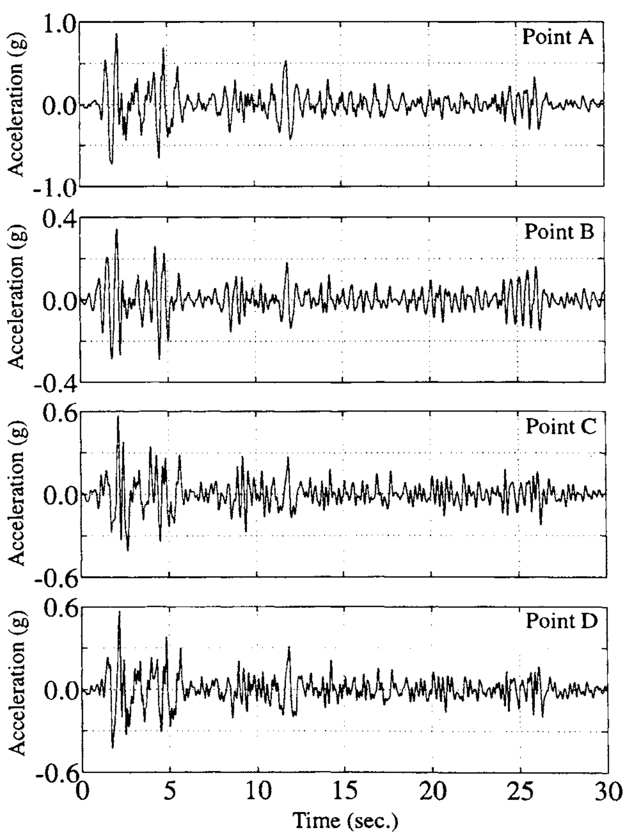

Figure 7 Acceleration response at different locations due to EL Centro earthquake as 30° incident SV waves.

CONCLUSIONS

A hybrid numerical formulation has been used, combining the Finite Element and the Boundary Equation methods to study the effects of dam-foundation interaction on the response of earth dams subjected to obliquely incident P and SV waves. The hybrid technique proved to be exceptionally powerful because the infinite region is treated accurately by using the exact Green's functions and the incorporation of the FEM and the BEM in a stiffness substructuring scheme renders problems that can be solved accurately and efficiently. The results showed the significance of the flexibility of the foundation rock in reducing the overall response of the dam, by accounting rigorously for the energy radiated back into the halfspace. From the limited results presented here, the effects of the spatial variability of the ground motion for P and SV waves travelling across the width of the dam seem to be also important, but less dramatic than those reported in previous studies, which concluded that such effects may lead to response values substantially higher than those caused by coherent ground motion. Results from a more comprehensive parametric investigation, still in progress, will be published elsewhere.

REFERENCES

1 Auersch, L. and Schmid, G., (1990) 'A simple boundary element formulation and its application to wavefield excited soil-structure interaction', *Earthquake Eng. Struct. Dyn.* 19, 931-947.

2 Beskos, D.E., (1987) 'Boundary element methods in dynamic analysis', *Appl. Mech. Rev.* 40, 1-23.

3 Bielak, J.; MacCamy, R.C.; McGhee, D.S. and Barry, A., (1991), 'Unified symmetric BEM-FEM for site effects on ground motion-SH waves', *J. Eng. Mech.*, ASCE 117, 2265-2285.

4 Brebbia, C.A.; Telles, J.C.F. and Wrobel, L.C., (1984), *Boundary Element Techniques*, Springer, Berlin, Heidelberg, New York.

5 Cruse, T.A. and Rizzo, F.J., (1968), 'A direct formulation and numerical solution of the general transient elastodynamic problem. 1', *Int. J. Math. Anal. Applic.* 22, 244-259.

6 Dakoulas, P. and Hashmi, H. (1992), "Wave Passage Effects on the Response of Earth Dams", *Journal of Soils and Foundations,* Vol. 32, No. 2, 97-110.

7 Dakoulas, P. (1993), "Response of Earth Dams in Semi-Cylindrical Valleys Subjected to Plane SH Waves", *Journal of Engineering Mechanics*, ASCE, Vol. 119, No. 1, pp. 74-90.

8 Dakoulas, P. (1993), "Earth Dam - Canyon Interaction Effects For Obliquely Incident SH Waves", *Journal of Geotechnical Engineering Div.*, ASCE, Vol. 119, No. 11,

1696-1716.
9 Dakoulas, P., and Hsu (1994), "Response of Earth Dams in Semi-Elliptical Flexible Canyons to Oblique SH Waves", *Journal of Engineering Mechanics*, ASCE, (in press).
10 Eringen, A.C. and Suhubi, E.S., (1975), *Elastodynamics*, Vol II, Academic Press, New York.
11 Gazetas, G. (1987) "Seismic Response of Earth Dams: Some Recent Developments" *Journal of Soil Dynamics and Earthquake Engineering,* 6(1), 1- 47.
12 Gazetas, G. and Dakoulas, P. (1992), "Seismic Analysis and Design of Rockfill Dams: State-of-the Art" , *Journal of Soil Dynamics and Earthquake Engineering*, 11(1), 27-61.
13 Karabalis, D.L. and Beskos, D.E., (1985), 'Dynamic response of 3-D flexible foundations by time domain BEM and FEM', *Soil Dyn. Earthquake Eng.* 4, 91-101.
14 Kobayashi, S. and Mori, K., (1986), 'Three dimentional dynamic analysis of soil-structure interaction by boundary integral equation-finite element combined method.' *Innovative Numerical Methods in Engineering*, Springer, Berlin, Heidelberg, New York, 613-618.
15 Mossessian, T.K. and Dravinski, M., (1987), 'Application of hybrid method for scattering of P, SV, and Rayleigh waves by near-surface irregularities', *Bull. Seism. Soc. America,* 77, 1784-1803.
16 Spyrakos, C.C. and Beskos, D.E., (1986), 'Dynamic response of flexible strip-foundations by boundary and finite elements', *Soil Dyn. Earthquake Eng.* 5, 84-96.
17 von Estorff, O. and Kausel, E., (1989), 'Coupling of boundary and finite elements for soil-structure interaction problems', *Earthquake Eng. Struct. Dyn.* 18, 1065-1075.
18 Wang, S., (1992), 'Coupled boundary and finite elements for dynamic structure (3D)-foundation-soil interaction', *Computers & Structures* 44, 807-812.
19 Zienkiewicz, O.C. (1977), *The Finite Element Method,* 3rd edn, McGraw-Hill, New York.
20 Zienkiewicz, O.C.; Kelly, D.M. and Bettes, P. (1977), 'The coupling of the finite element method and the boundary solution procedures', *Int. J. Num. Meth. Eng.* 11, 355-376.