Investigation of Stress Waves Reflection Problems From a Free Surface by Dynamical Photoelasticity Method

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INTRODUCTION

Under normal incidence conditions of a plane longitudinal wave on a plane interface between two media having different acoustic impedance $\rho_1 c_2$ and $\rho_2 c_2$ (ρ_1, ρ_2 -media densities, c_1 and c_2 velocities of longitudinal waves in these media), stress amplitudes in an incident σ_0, reflected σ_{rfl} and refracted σ_{rfr} waves are connected by well-known relations

$$\sigma_{rfl} = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_1 c_1 + \rho_2 c_2} \sigma_0 \qquad (1)$$

$$\sigma_{rfr} = \frac{2\rho_2 c_2}{\rho_1 c_1 + \rho_2 c_2} \sigma_0 \qquad (2)$$

Considering a free boundary as an interface between rigid media and air $\rho_2 c_2 = 0$, and $\sigma_{rfl} = -\sigma_0$ and $\sigma_{rfr} = 0$ are obtained. Thus, under normal incident of compression wave, a tensile wave of the same amplitude is reflected and vice versa. Shear waves under such reflection don't occur. The boundary stays free from stresses and its motion velocity is equal to double velocity of particles in an incident wave. Under oblique incidence of plane longitudinal wave on a free surface the boundary conditions can't be met in an assumption that a longitudinal wave only is reflected.

According to the theory of plane elastic waves reflection [1] while incidence on a free boundary semispace of a longitudinal PP and shear PS waves are reflected from a boundary, stresses being determined by the following [2]

$$\sigma_r^{PP} = K_p \sigma_r^{P} \; ; \; \sigma_\theta^{PP} = \frac{\upsilon}{1-\upsilon} \sigma_r^{PP} \qquad (3)$$

$$\tau^{PS} = K_s \sigma_r^{P} = \sigma_r^{P}(1+K_p)\,\mathrm{ctg}2\beta \; ; \; \sigma_r^{PS} = \sigma_\theta^{PS} = 0 \qquad (4)$$

where σ_r^{P} -radial stress in an incident wave P ; σ_r^{PP}, σ_θ^{PP}, σ_r^{PS}, σ_θ^{PS} -radial and tangential stresses in reflected waves PP and PS respectively; τ^{PS}-tangential stress in reflected wave PS; K_p , K_s longitudinal and shear waves reflection coefficients according to stresses, moreover

$$K_p = \frac{\mathrm{tg}\beta\, \mathrm{tg}^2 2\beta - \mathrm{tg}\phi}{\mathrm{tg}\beta\, \mathrm{tg}^2 2\beta + \mathrm{tg}\phi} \qquad (5)$$

$$K_s = (1+K_p)\,\mathrm{ctg}2\beta \qquad (6)$$

$$\phi = \phi_1 \; ; \quad \frac{\sin\phi}{\sin\beta} = \frac{c_v}{c_s} \qquad (7)$$

where - ϕ an angle of incidence on a free boundary of a longitudinal wave P, ϕ_1 -a reflection angle of wave PP, β - a reflection angle of shear wave PS.

The relations given above make possible to determine maximum dynamic stress values in any point of a media beyond an interferention field of an incident and reflected waves, I.e. at some distance from free boundary exceeding impulses lengths.

Normal stresses at a free boundary in case of plane longitudinal wave incidence may be determined by means of equilibrium condition, taking into account the principal planes orientation and correlation of the principal stresses in incident and reflected waves.

Thus, the conclusion is that under slide incidence of a longitudinal wave on a free boundary (ϕ=90) reflection coefficient K_p value is K_p =-1, and contour stresses are equal to zero. Nevertheless, numerous data of field and model tests clarify that under slide incidence (or close to sliding) of P wave (ϕ=80...90) non-zero contour stresses occur at a media free boundary.

An unusualness of plane wave theory for a description of experimental data while plane wave incidence, close to sliding might be explained by inaccordance of a theoretical model in a particular case and conditions of any real experiment. Truly, a plane longitudinal wave slide incidence may be provided by two ways:1) a plane wave source is placed normally to a free boundary of media and crosses it; 2) a plane longitudinal wave, propagating in unlimited media, comes across a cavity, cut,etc. In both cases the first (the nearest to the source) point of longitudinal wave interaction with a free surface is at the same time a source of difragirated waves, the front of which not being plane and their propagation laws being different from that of a plane longitudinal wave. Thus, to obtain interaction of "clean" longitudinal plane wave with a free surface of the media under slide incidence by an experimental way appears not to be possible. It requires a special investigation of laws of a real longitudinal wave interaction with a free surface under incidence similar to sliding.

Under incidence on a free boundary of semispace of a plane shear wave SV (when the motion of particles occurs in plane of incidence) according to plane waves reflection theory [1], when angles of incidence $\beta<\beta_{cr}$ (β_{cr}=arcsin c_s /c_p) longitudinal SP and shear SS waves, the stresses of which may be determined by relations similar to (3).-.(7) are reflected from a boundary. The consideration of an element equilibrium at a media free boundary makes possible to determine contour stresses values when angles of incidence don't exceed critical magnitude.

The analysis of dependence for contour stresses testifies that while an angle of incidence approaching to critical magnitude contour stresses value trends to infinity, that doesn't correspond to physical aspect of the task for real media. When $\beta>\beta_{cr}$ plane wave theory doesn't make possible to appreciate stress state in reflected waves. Some theoretical investigations of plane shear waves interaction with a free surface when angles of incidence exceed critical values [3] demonstrate that in a proper case reflected shear wave acquires surface wave properties, i.e. its amplitude damps while removing from free boundary and reflected wave entire energy is concentrated in a surfaceby layer. The determination of a stress state when $\beta>\beta_{cr}$ at a free contour as well as in reflected waves within the media is rather difficult and requires additional experimental investigations.

EXPERIMENTAL DATA AND ANALYSIS.

To determine coefficients of longitudinal wave reflection from a free surface dynamic photoelasticity plane model tests with blowing up microcharge of lead azide have been carried on, reflection coefficients through the whole scale of angles of incidence 0...90 being specified experimentally and dynamic stress peculiarities in a model surfaceby layer where wave field is formed as result of 3 waves interaction (P,PP and PS)being valued. To determine longitudinal K_{pp} and shear K_{ps} waves reflection coefficients experimentally may be done using the analysis of these waves amplitudes changes in some depth Z from a free surface where stress impulses in incident P and reflected PP and PS waves are separated in time.

The relations of dumping for each of the three waves obtained before make possible to pass from measured stresses amplitudes at a free boundary (Z=0), where incident and reflected waves interferention doesn't give an opportunity to specify stresses values in each of tested waves separately.

Reflection coefficients K_{pp} and K_{ps} obtained to experimental data, contour stresses at a free boundary as well as corresponding values calculated using plane wave theory are demonstrated in fig 1 and 2 .

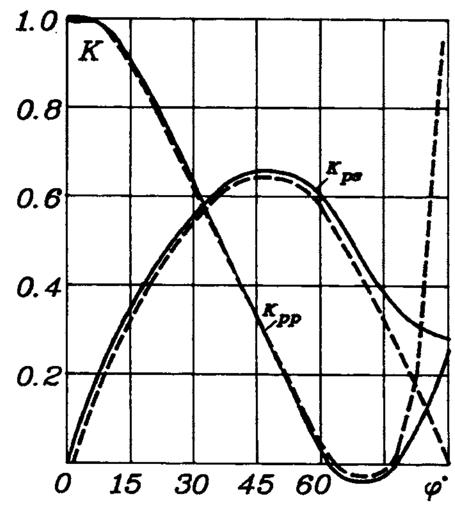

Fig. 1 Reflection coefficient K_{pp} and K_{ps} for for reflected longitudinal PP and shear PS waves under incidence on free boundary of plane longitudinal wave P
——— exp; - - - - - theory [1];

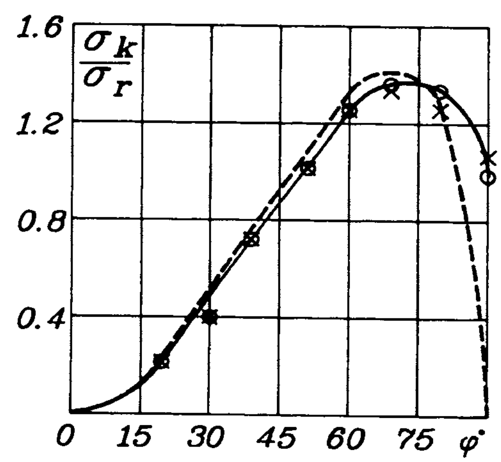

Fig. 2 Contour stresses at a free boundary in a case of incident of plane longitudinal wave P (σ_r-principal stress for incident wave)
——— exp; - - - - - theory [1];

Experimental values of reflection and contour stresses coefficients in a range of angle of incidence $0 < \phi < 70$ are slightly different from theoretical ones obtained by formula (5)and (6)

However,when wave incidence is similar to sliding (ϕ>80) reflected longitudinal wave's amplitude increases up to $|K_{pp}|=1$, while according to experimental data differ from theoretical ones in 4 times (for model material $\nu=0.34$ and $|K_{pp}|=\frac{\nu}{1+\nu}$).

Given experiments figures demonstrate the values of reflection coefficients and relative contour stresses when longitudinal wave being reflected though the whole range of angles of incidence. In a surfaceby layer wave field has a number of peculiarities caused by superposing of incident and reflected waves, in which principal stresses have different directions and their maximums don't coincide in time. When angles of incidence are equal to 50 or less under surface concentration field of principal stresses difference is observed, the position maximum depth charging insufficiently from the angle of incidence and being equal to $0,4\lambda_c$, where λ_c compression stress phase length.

Fig. 3 Streak photography of photoelastic fringe pattern, incident P wave, ϕ=7

Under similar experimental conditions in brittle material (glass) in depth $0,4\lambda_c$ cracks occur which propagated parallel to free surface in direction from explosion epicenter. When incidence is similar to normal, concentration zone occurs as a result of longitudinal waves P and PP interaction. The angles of incidence being to ϕ=30...50. under surface maximum value is effected by reflected shear wave PS, that is confined by the fact that in this angles scale shear wave reflection coefficient K_{ps} has the most values.

Fig. 4 Streak photography of photoelastic fringe pattren incident P wave, ϕ=30

Experimental study of shear wave reflection from a free boundary are of great interest because plane wave theory in the particular case may be applied only for angles of incidence $\beta<\beta_{cr}$, i.e. for models material (ν=0.34) calculation may be done only for angles $\beta<34$.

Experimental data on models while plane shear wave incident on a free boundary through the whole scale of incidence ($0<\beta<90$) have shown that reflected plane shear wave may be observed not only when $\beta<\beta_{cr}$ but also when $\beta>\beta_{cr}$.

Reflected plane longitudinal wave almost in all cases has stresses amplitude (in fringes order) much less than reflected shear wave and is observed practically when angles of incident is not large (β=15...30).

Experimental and theoretical dependences of shear wave reflection coefficient and that of corresponding contour stresses of angle of incident are given in fig. 5,6. If $\beta<30$, experimental values of above magnitudes practically coincide with theoretical ones. Angles of incidence being close to critical (β=30...34) plane waves theory gives too high values of reflection coefficient K_{ss} and infinite values of contour stresses.

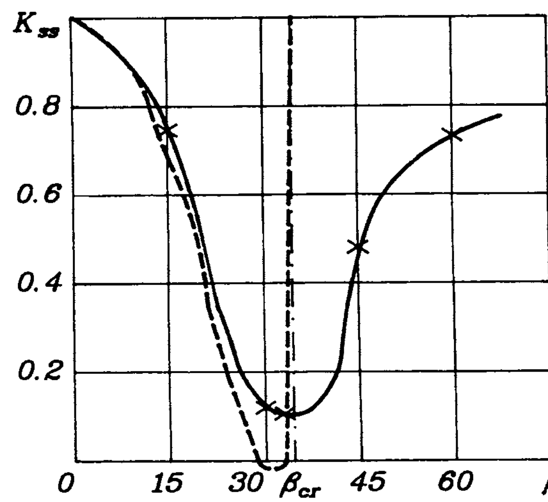

Fig. 5 Reflection coefficients K_{ss} for reflected shear SS wave under incidence on free boundary of plane shear SV wave ———— exp; - - - - - theory [1]

The experiment shows shear wave angle of incidence being equal to critical, reflection coefficient takes maximum value.

Beyond critical angle reflection coefficient increases. Contour stresses approach maximum when $\beta=\beta_{kr}$, their minimum values correspond to angles of incidence β=45...50.

Photoelastic fringe patterns analysis in a surfaceby layer has shown that beyond critical angle of incidence Rayleigh wave takes sufficient part in forming stress state in the proper field.

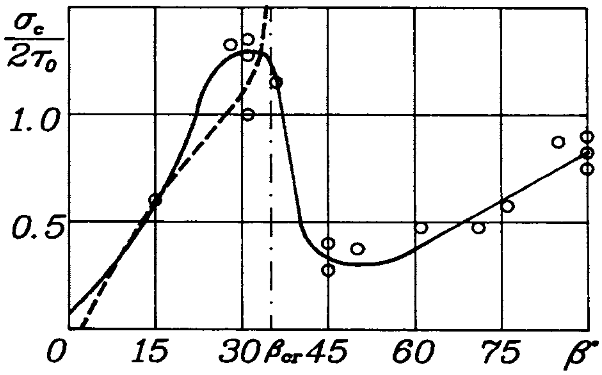

Fig. 6 Contour stresses at a free boundary in a case of incidence of plane shear SV (τ_0-max. tangential stress for incident wave).
——— exp; - - - - - theory [1]

CONCLUSIONS.

The obtained dependencies for reflection and contour stresses coefficients while P and S waves incidence may be applied to specify seismic wave stress of a half-space in proximity tc daily surface when longitudinal and shear wave effect directions are different. The stresses dynamic field in a surfaceby zone being formed as a result of incident and reflected waves superposition, values and directions of principal stresses in each of the proper waves and time displacement between the moments of th waves appearance in a particular point should b taken into account. Calculations show that ignorance of seismic waves reflected from daily surface result in design stresses in a surfaceb zone being considerably lower.

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