Array Processing of Rayleigh Waves for Shear Structure

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SYNOPSIS

Site response to earthquakes is strongly dependent on shallow shear wave velocity structure $\beta(z)$, and evidence suggests that soil strength and liquefaction potential depends on it as well. We have determined $\beta(z)$ at several sites by inversion of dispersion data from Rayleigh waves recorded on linear arrays of geophones using artificial sources. Improved methods have been developed for extracting phase and group velocities that lead to significantly more stable and accurate inversion results.

INTRODUCTION

It is often important to know the shallow shear structure at a site. A surface seismic tool for this purpose is the inference of structure from Rayleigh wave dispersion data. Several studies have shown the method to be accurate and reasonably easy to use. Examples are the works by Gabriels, *et al.* (1987), Nazarian (1984), Stokoe and Nazarian (1985), and Barker and Stevens (1983). As with any geophysical method, there are strong points as well as limitations and difficulties. In the following, we discuss aspects of the data acquisition and processing and show how certain pitfalls can be overcome. We also compare the results with those from studies using downhole methods. The paper emphasizes the use of arrays for analyzing Rayleigh waves, which was also discussed by Nolet and Panza (1976).

BACKGROUND

The most persistent source of difficulty and potential source of error in our applications has been the identification of the Rayleigh wave fundamental mode. This is because, in many cases, higher mode amplitudes are as large or larger then the fundamental in the time window and frequency band of interest. The most powerful tool for identifying the fundamental mode and extracting its properties is, in our experience, to deploy linear arrays of sensors, preferably with twelve or more geophones, in the style of a refraction array. The data are then processed as an array.

We compare two approaches to finding dispersion curves from array data: (1) a time oriented approach in which phase and group delays are inferred (in our case, using phase-matched and narrow-band filters) at single stations and the velocities are found from travel-time curves, and (2) an amplitude oriented approach using stacking. We introduce a method for finding group velocities in which the Hilbert transform envelopes of narrow-band filtered seismograms (whose peaks travel at the group velocity) are stacked. The stacking methods have the advantages of reducing the errors due to phase misidentification at individual stations, those due to uncertainties in initial phase and group delays and, for phase velocity, ambiguities of n 2π in estimating the phase. Single station time oriented methods preserve variations in the structure along the survey line, and are less sensitive to signal amplitude fluctuations than the amplitude oriented approach. We have automated both approaches.

We also discuss additional aspects of the processing which we have found to improve the reliability of the inferred structure. We have found that joint inversion of group and phase velocity data significantly reduces the nonuniqueness inherent in the inversion compared with inversion of group or phase velocity alone. Another enhancement is the use of two component (vertical and radial) data, which allows the extraction of group velocities from polarization filtered seismograms.

We compare $\beta(z)$ obtained from the Rayleigh wave inversion with cross-hole and downhole studies at the same sites in the Imperial Valley, California and in the Journada del Muerte Valley, New Mexico. The inversion earth models are smoothed versions of the borehole models.

The methods we describe here can applied in the field using a data acquisition system which has a personal computer with a graphics display, and require relatively little analyst intervention.

METHODS OF ANALYSIS

The basis for the inference of structure from Rayleigh waves is that the waves are dispersed, or that waves of different frequencies travel at different speeds. As a rule, the velocities at higher frequencies are more sensitive to the shallow structure while low frequency data are more sensitive to deeper shear velocities. The frequency dependence is characterized by dispersion curves which show the dependence of either phase or group velocity on frequency. Group velocity is the velocity at which energy propagates and phase velocity is the apparent wave speed or the speed of a Fourier phase at a frequency. The two are interrelated, and in principal, knowledge of either is sufficient to determine the structure. In practice, finding both types of dispersion curves greatly improve the results since the group velocities tend to constrain gradients in the structure better while phase velocities better constrain the absolute values. We note that Rayleigh waves are only weakly dependent on the compressional wave speeds so these methods can be applied to finding shear wave profiles only.

A flow chart showing the steps in the process is shown in Figure 1. The inference of shear structure from Rayleigh waves involves two primary steps: (1) extraction of dispersion information and (2) inversion of dispersion data for structure. We have automated the steps in Figure 1.

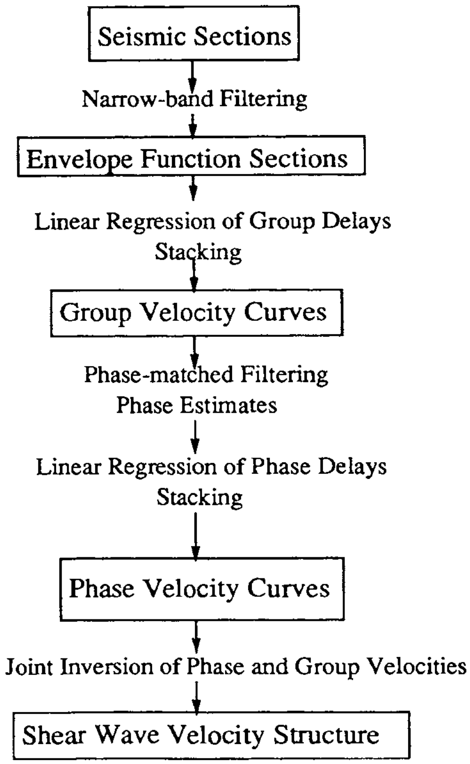

Fig. 1. Flow chart for the determination of shear wave velocity structure from Rayleigh waves.

Although several means for doing step (1), extraction of dispersion, have been proposed, we have implemented methods described by Herrin and Goforth (1977). The methods begin by applying narrow-band filters at a sequence of center frequencies which isolate the wave properties in a narrow frequency band. Next, Hilbert transform envelope functions (described below) are formed from the narrow-band signals for each seismogram in the section. These envelope functions travel with the group velocity for the center frequency used in the narrow-band filter. The time required to travel to an observer is referred to as the group delay, and the group velocity is the source-receiver offset divided by the group delay. Clearly, a seismogram from a single receiver can yield a group dispersion curve since group delays for a family of frequencies at the corresponding offset can be computed. Such a computation is referred to as a single station determination, and in principal, is adequate to find the dispersion curve. In practice, interference from other phases (e.g. higher modes), group delays introduced by instrumentation and lateral variations in earth structure cause errors in single-station estimates. We discuss below how arrays can be used to reduce errors. Once the envelope sections are found for the frequencies of interest, two approaches can be taken. The approaches, one based on travel times and one based on amplitudes, are described below. Similar considerations apply to the extraction of phase velocity dispersion.

ARRAY METHODS

We find group velocity from the envelope functions $e(t; f_n)$ of a seismogram by narrow-band filtering the seismogram at a sequence of center frequencies f_n, and combining them with their Hilbert transforms:

$$e(t; f_n) = [s(t; f_n)^2 + h(t; f_n)^2]^{1/2}, \quad (1)$$

where $s(t; f_n)$ is a filtered seismogram, and $h(t; f_n)$ is its Hilbert transform. The peak of the envelope occurs at the group arrival time.

In Figure 2, we show a seismogram section recorded from an explosive source in the Journada de Muerte Valley, New Mexico. The corresponding envelope function section for a center frequency of 6 Hz is shown in Figure 3. The Rayleigh wave can be easily identified in both the seismogram and envelope sections. By contrast, consider the seismogram and envelope functions in Figures 4 and 5, taken in the El Dorado Valley, Nevada. The Rayleigh wave could not be confidently identified from a single station in this case. These figures illustrate how the envelope functions provide a visual check for a consistently propagating phase in the array, and for initial group velocity estimates which can be used to window seismograms before the automated process.

For each center frequency f_n, we can form a travel time curve from the group delays $t_g(x)$ for the seismogram section, and find the values of group slowness p_g ($1/v_g$, the group velocity) and initial group delay τ_g which provide the best fit to

$$t_g(x) = \tau_g + p_g x \quad (2)$$

This is readily accomplished by linear regression.

An alternative approach is to treat the envelope functions for each center frequency as seismograms and find the group slowness and initial delay which maximize the slant stack, or discrete Radon transform,

$$\tilde{e}(\tau_g, p_g) = \sum_{i=1}^{N_x} e(X_i, \tau_g + p_g X_i) \quad (3)$$

Slant stacking is a common means for velocity estimation in refraction surveys. One simply does a series of trial lags and adds to see which best lines up the envelope peaks, which in turn maximizes the stack.

We can use the group velocity to find the phase velocity v_p from the relation

$$v_g = v_p \left(1 - \frac{f}{v_p}\frac{dv_p}{df}\right)^{-1} \quad (4)$$

The phase delays are extracted at each source-receiver offset by integrating the group velocity curve (using Equation 4), yielding a phase travel-time curve $t_p(x)$. As with the group delay curve, we then find the phase slowness p_p and initial phase delay t_p which provide the best fit to

$$t_p(x) = \tau_p + p_p x . \quad (5)$$

Note that an integration of group velocity has an undetermined constant of integration. The travel time approach removes this ambiguity.

An alternative is to create a phase slant stack which is done in the frequency domain. The slant stack maximizes (Nolet and Panza, 1976)

$$\tilde{w}(f;\tau_p,p_p) = \sum_{j=1}^{N_x} e^{i[\phi(f,X_j) + 2\pi f(\tau_p + p_p X_j)]} \quad (6)$$

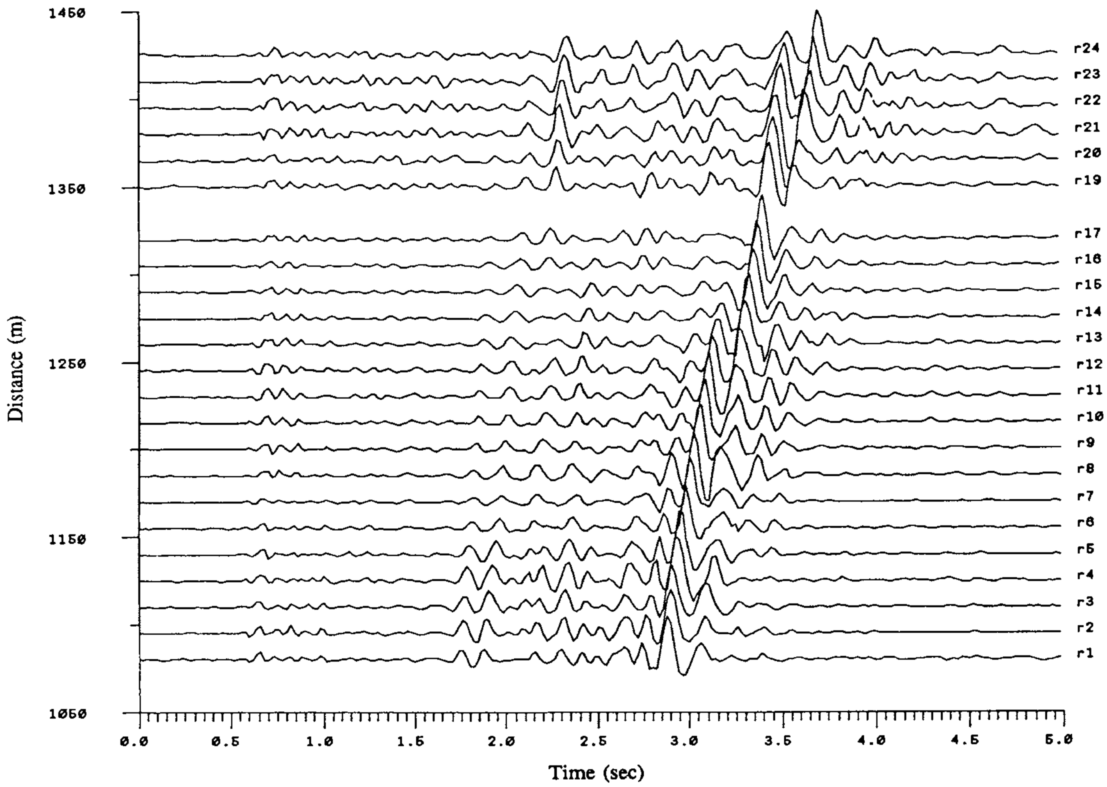

Fig. 2. A seismogram section recorded in the Journada del Muerte Valley, New Mexico.

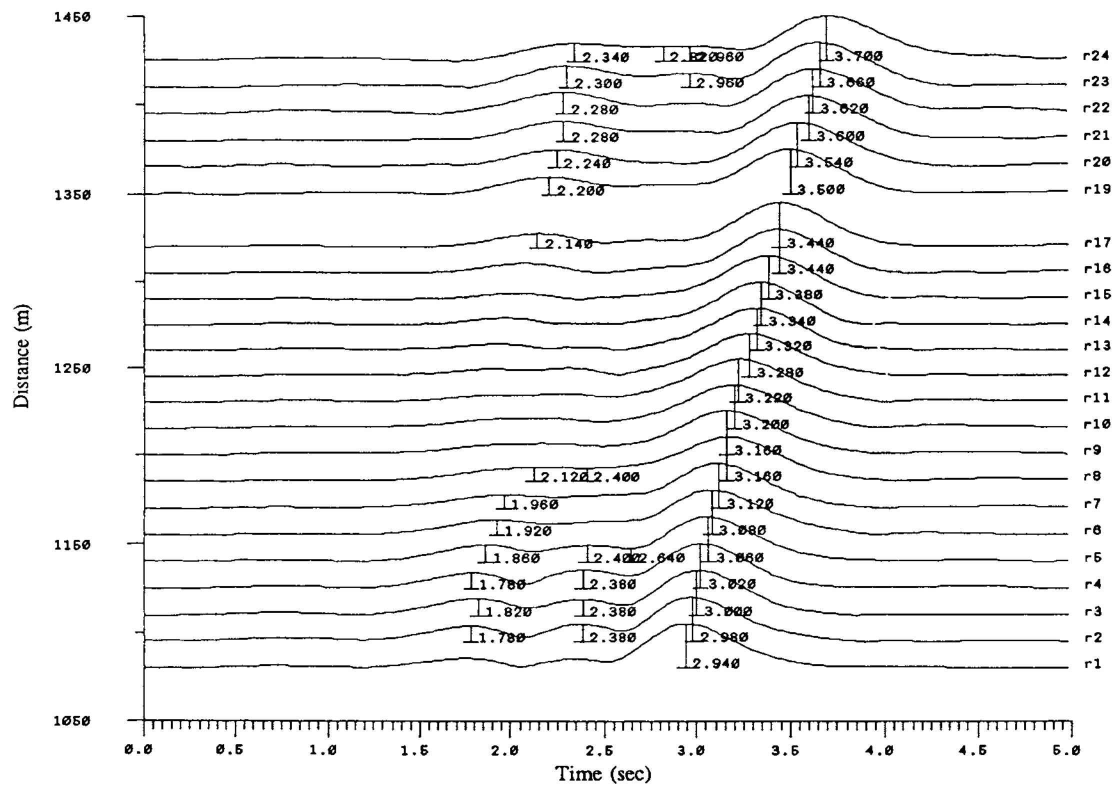

Fig. 3. A section of envelope functions for the seismograms on Figure 2 is shown. The center frequency of the narrow-band filter is 6 Hz. The arrival times of the peaks are indicated.

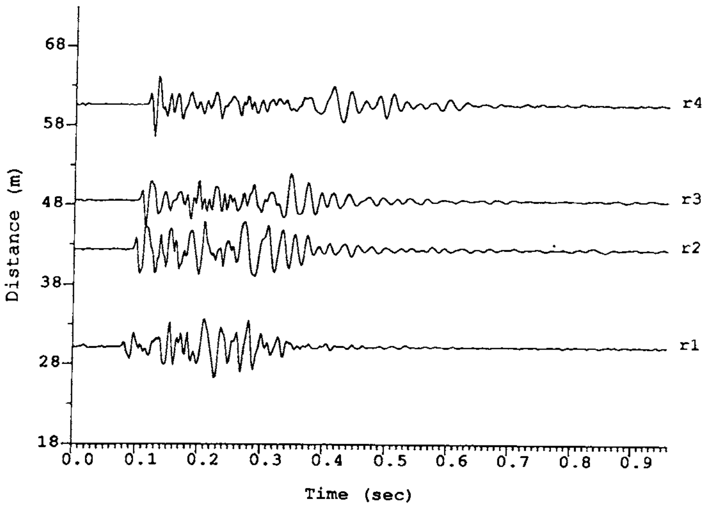

Fig. 4. A seismogram section recorded in the El Dorado Valley, Nevada.

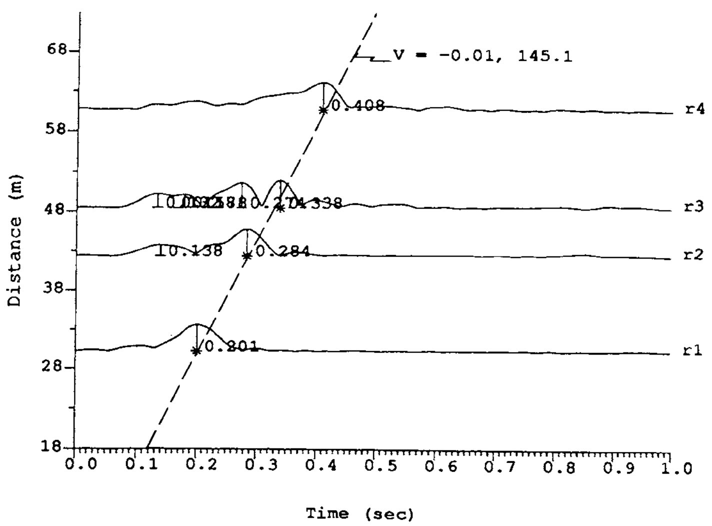

Fig. 5. A section of envelope functions for the seismograms in Figure 4. The center frequency of the narrow-band filter is 15 Hz. The arrival times of the peaks are indicated. The dashed line indicates the least-squares fit to the times, with the initial group delay (sec) and velocity (m/sec) shown at top.

where $\phi(f, Xj)$ is the phase measured from the seismogram at distance X_j.

The philosophy behind these steps is that errors in single station measurements are reduced by making array calculations. In addition, the Hilbert envelope functions are useful graphic tools for inspecting a record section for a consistent phase across the section.

TWO COMPONENT DATA

We have found that if both vertical and radial components c motion are recorded, further improvements in extractin dispersion data can be made by forming polarization filtere time series. This is done by adding the vertical componer seismograms to the Hilbert transform of the radial motion (see for example, Harris, 1981). This has the effect of enhancing th elliptical motion of the Rayleigh wave relative to other phases. I the few data sets we have had available for this processin (including the data shown in Figure 4), the improvements hav been substantial.

INVERSION

We have so far emphasized the extraction of dispersion data since the inversion for structure cannot be any better than the dispersion. In the following, we address briefly the inversior process. First, the importance of using both group and phase velocity should be stated. Dispersion data are by necessity band limited. At high frequencies, intrinsic attenuation anc scattering reduce the fundamental mode while at lov frequencies, geophone response and structural excitatior diminish amplitudes. Thus, aspects of dispersion curves can be misinterpretated. For example, a minimum in the group velocity may indicate a low velocity zone or a gradient in the shear profile (causing an Airy phase). Joint inversion of both phase and group velocity resolves the ambiguity.

Inversion of dispersion data (and all geophysical data) is nonunique. One must make compromises between physically realizable models and models that fit every nuance of the data. Among the methods for doing the inversion, we have found that a process which simultaneously minimizes data misfit and model roughness in a Backus-Gilbert approach is best. This is superior because it requires no *a priori* assumptions about the earth structure and finds the smoothest model consistent with the data (eliminating artificial oscillations in the resulting profile) (Bache, Rodi, and Harkrider, 1978).

COMPARISON WITH DOWNHOLE MEASUREMENTS

In Figure 6, we compare shear velocity structures obtained by the Rayleigh wave methods described above with those found by vertical seismic profiling (VSP). The sites, in the Imperial Valley of California, are the locations of Strong Motion Accelerographs operated by the U. S. Geological Survey. In each case, the Rayleigh wave profiles are smoothed versions of the VSP profiles. Similar comparisons of been made, with similar outcomes, at other sites in the western United States.

CONCLUSIONS

Although Rayleigh waves have been used to infer shallow shear structure in numerous studies for geotechnical applications, their use is not widespread. The results shown here indicate that it is a viable alternative to downhole methods. However, our experience, as reported here, shows that some care must be taken in the the application of the method. The method has been automated, thereby reducing much of the work by the analyst. The method has the potential for being made routine, and as more data sets are analyzed, the details for this will emerge.

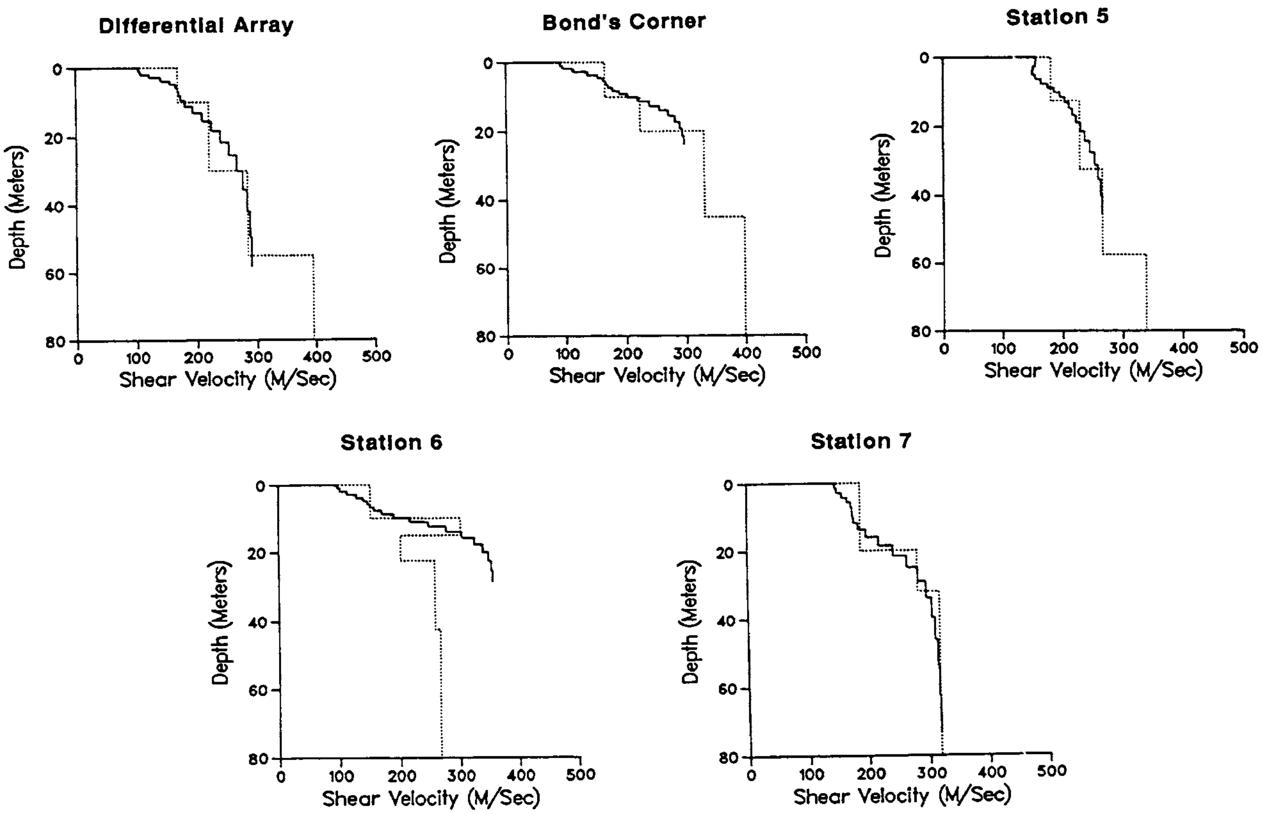

Fig. 6. Comparison of shear wave velocity structures at five USGS El Centro SMAC sites inferred from Rayleigh wave inversion (solid lines) and VSP by Porcella (1984) (dashed lines).

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