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SYNOPSIS An analysis for determination of lateral thrust on a rigid retaining wall with reinforced earth backfill under seismic condition is described in this paper. The backfill is reinforced with horizontally laid unattached strips and an equivalent horizontal static force replaces the dynamic force. A Coulomb wedge is assumed to develop at limiting equilibrium. Analysis indicates substantial reduction in pressures on the wall due to reinforcement. An optimum length of reinforcing strips of 60 percent of height of wall is indicated.

INTRODUCTION

Reinforced earth is a composite material formed by the association of granular soil and linear reinforcing elements of metal or heavy synthetic fabrics. Since its invention by Vidal, it has found large scale application in the construction of large and medium retaining walls. The Reinforced earth retaining wall consists of a precast or prefabricated skin to which reinforcing strips are attached and buried in the backfill. The interaction between the strips and the granular backfill soil provides the stabilising forces which holds the reinforced soil mass and the skin in position. For stability, the internal design of Reinforced earth requires strips of length approximately equal to the height of wall. Such large space may not be always available especially in case where the wall is to act as a support structure along narrow highways in hilly regions, as is very common in the sub-Himalayan regions of India. An alternative form of reinforcement for such condition can be the reinforcing of backfill behind conventional walls by unattached strips. The provision of reinforcing strips would reduce the lateral pressures and the overturning moments on the wall and result in thinner section of wall and its base. This type of reinforcing system can also be employed as remedial measures for under-designed walls or walls in distress or in temporary construction. Since the tension in unattached strips would be small, cheap reinforcing materials like treated bamboo strips can be utilized as reinforcement. The work reported here is an extension of the analysis of retaining wall with reinforced earth backfill under static condition (Saran et al, 1979) to determine pressure intensity, resultant pressure and point of action of resultant under seismic conditions.

ANALYSIS

A retaining wall of height H and vertical back retains dry cohesionless backfill with zero surcharge angle and with soil properties of internal friction angle ϕ and dry unit weight γ.

The backfill is reinforced with unattached horizontal strips of length L and width w laid normal to the back of wall at horizontal and vertical spacing of S_H and S_V respectively. A failure wedge of reinforced soil with a plane surface separates from the rest of the backfill as active conditions are reached. Full frictional resistance is assumed to be mobilised in the soil and along the reinforcing strips at failure. The total frictional resistance generated by a strip is computed from its effective length, which is lesser of its two portions - one within and the other without, the failure wedge. If a strip lies wholly within the wedge, it will contribute no frictional resistance.

Considering the equilibrium of an element IJKM in the failure wedge of thickness dy and located at a distance y from the top, the following forces can be identified per unit length of the wall (Saran and Prakash, 1971) Fig.1.

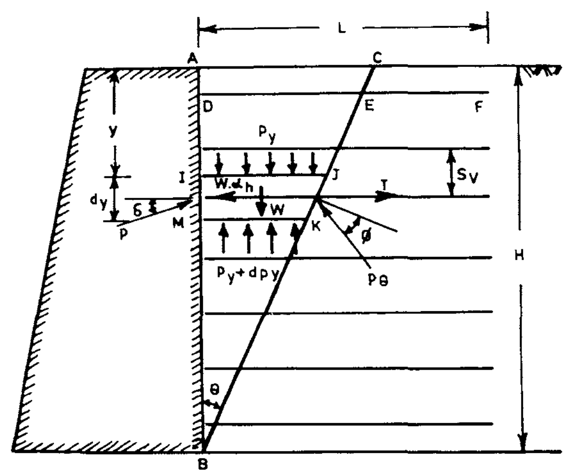

Fig.1 Forces acting on the element IJKM

p_y: pressure intensity acting uniformly on IJ in the vertical direction

p_y+dp_y: intensity of uniform reaction acting on KM in the vertical direction

p_θ : reaction intensity on JK acting at an angle ϕ to the normal to JK

p : pressure intensity on IM acting at an angle δ with the normal to IM

W : weight of the slice IJKM acting downward

α_h : pseudo static coefficient of horizontal pressure due to earthquake

T : Tensile force in the strip assumed transmitted uniformly to soil layer of thickness S_V encompassing the strip.

1. Balancing all the forces acting on the slice in the vertical direction gives

$$p_y(H-y)\tan\theta-(p_y+dp_y)(H-y-d_y)\tan\theta+0.5\gamma dy(2H-2y-dy)\tan\theta-pdy\sin\delta-p_\theta dy\sec\theta.\sin(\theta+\phi)=0 \quad \ldots (1)$$

Neglecting small quantities of second order, it reduces to

$$\frac{dp_y}{dy}=\frac{p_y+(H-y)\gamma}{(H-y)}-\frac{p\sin\delta}{(H-y)\tan\theta}-\frac{p_\theta.\sec\theta\sin(\theta+\phi)}{(H-y)\tan\theta} \quad \ldots (2)$$

2. Balancing all the forces acting on the slice in the horizontal direction.

$$\frac{T}{S_V}.dy+pdy.\cos\delta-p_\theta.dy\sec\theta\cos(\theta+\phi)-\frac{1}{2}\gamma.\alpha_h.dy(2H-2y-dy).\tan\theta=0$$

$$\text{or } p_\theta=\frac{t+p\cos\delta-\gamma\alpha_h(H-y-\frac{dy}{2})\tan\theta}{\sec\theta.\cos(\theta+\phi)} \quad \ldots (3)$$

Where $t = T/S_V$

3. Taking moments of all the forces about the mid point of slice between J and K gives

$$p_y(H-y)\tan\theta\,(H-y)\left[\frac{\tan\theta}{2}-\frac{dy\tan\theta}{2}\right]-(p_y+dp_y).\left[(H-y-dy)\tan\theta\right]\left[(H-y-dy)\frac{\tan\theta}{2}+\frac{dy}{2}\tan\theta\right]-pdy\sin\delta.(H-y-dy/2)\tan\theta+\frac{1}{2}\gamma\,dy(H-y-\frac{dy}{2})^2\tan^2\theta=0$$

On simplification and neglecting 2nd order terms of small quantities yields

$$\frac{dp_y}{dy}=\gamma-\frac{2p\sin\delta}{(H-y)\tan\theta} \quad \ldots (4)$$

Combining equations (2) and (3) gives

$$\frac{dp_y}{dy}=\frac{p_y+(H-y)\gamma}{H-y}-\frac{p.\sin\delta}{(H-y)\tan\theta}-\left[t+p\cos\delta-\gamma\,\alpha_h(H-y-\frac{dy}{2})\tan\theta\right].\sec\theta\sin(\theta+\phi)\Big/\left[(H-y)\tan\theta.\sec\theta\cos(\theta+\phi)\right]$$

$$\text{i.e. } \frac{dp_y}{dy}=\frac{p_y+(H-y)\gamma}{(H-y)}-\frac{p\sin\delta}{(H-y)\tan\theta}-\frac{(t+p\cos\delta)\tan(\theta+\phi)-\alpha_h\gamma(H-y)\tan(\theta+\phi)}{(H-y)\tan\theta} \quad \ldots (5)$$

Equating eqns (4) and (5) yields

$$p.\left[\cos\delta.\tan(\theta+\phi)-\sin\delta\right]=p_y\tan\theta-t.\tan(\theta+\phi)+\alpha_h.\gamma(H-y)\tan\theta\tan(\theta+\phi),$$

$$\text{or } p=\frac{p_y\tan\theta}{\cos\delta\tan(\theta+\phi)-\sin\delta}-\frac{t.\tan(\theta+\phi)}{\cos\delta.\tan(\theta+\phi).\sin\delta}+\frac{\alpha_h.\gamma(H-y)\tan\theta\tan(\theta+\phi)}{\cos\delta.\tan(\theta+\phi)-\sin\delta}$$

On differentiation and adopting proper sign for dynamic increment gives

$$\frac{dp}{dy}=\frac{dp_y}{dy}.\frac{\tan\theta\cos(\theta+\phi)}{\sin(\theta+\phi-\delta)}-\frac{dt}{dy}.\frac{\sin(\theta+\phi)}{\sin(\theta+\phi-\delta)}+\frac{\alpha_h.\gamma\tan\theta.\sin(\theta+\phi)}{\sin(\theta+\phi-\delta)} \quad \ldots (6)$$

Combining equation (4) with (6) gives

$$\frac{dp}{dy}=\frac{\gamma\tan\theta.\cos(\theta+\phi)}{\sin(\theta+\phi-\delta)}-\frac{2p\sin\delta\cos(\theta+\phi)}{(H-y)\sin(\theta+\phi-\delta)}-\frac{dt}{dy}.\frac{\sin(\theta+\phi)}{\sin(\theta+\phi-\delta)}+\frac{\alpha_h.\gamma\tan\theta\sin(\theta+\phi)}{\sin(\theta+\phi-\delta)}$$

$$\text{or } \frac{dp}{dy}=-C_1\frac{p}{H-y}+C_2-C_3\frac{dt}{dy} \quad \ldots (7)$$

Where $C_1 = 2\sin\delta\cos(\theta+\phi)/\sin(\theta+\phi-\delta)$

$C_2 = \tan\theta\;\cos(\theta+\phi)+\alpha_h.\sin(\theta+\phi)\;/\sin(\theta+\phi-\delta)$

$C_3 = \sin(\theta+\phi)/\sin(\theta+\phi-\delta)$

At limiting equilibrium, the tension T in the strip can be assumed as

$$T = 2wf.\sigma_v\frac{l'}{S_H} \quad \ldots (8)$$

where l' = effective length of strip

σ_v = vertical stress on strip (equal p overburden stress)

f = coefficient of friction between soil and reinforcement

$$\text{or } T = 2wf\gamma(y+\frac{dy}{2}).\frac{l'}{S_H}$$

where γ = unit weight of soil.

The values of l' will vary from strip to strip and will depend upon angle θ which the failure plane makes with the vertical. Three case may arise (Fig.2)

Case 1: $H\tan\theta \leq L/2$

Effective length, $l'=(H-y)\tan\theta$ for all strips.

Case 2: $L/2 \leq H\tan\theta \leq L$

Effective length, $l'=L-(H-y)\tan\theta$ for $y \leq Z_1$

and $l'=(H-y)\tan\theta$ for $y > Z_1$

Case 3: $H\tan\theta > L$

Effective length $l'=0$ for $y \leq Z_2$,

$$\frac{1}{2+C_1}(M_1-M_6) - \frac{1}{1+C_1}(M_1-M_6) + \frac{M_1}{6} - \frac{M_6}{12}\Bigg] \Bigg/ P_2 \qquad \ldots(16)$$

where $M_1 = K_1 \cdot H$.

Case 3. For $y \le Z_2$

$$p_3 = -\frac{C_2 H\gamma}{1-C_1}\left[(1-y/H)-(1-y/H)^{C_1}\right] \qquad \ldots(17)$$

For $Z_2 \le y \le Z_3$

$$p'_3 = \frac{K_1}{H(1-C_1)}\cdot\left(1-\frac{y}{H}\right) - \frac{2C_4H^2}{2-C_1}\left(1-\frac{y}{H}\right)^2 + D_2\cdot H^{C_1}\left(1-\frac{y}{H}\right)^{C_1} \qquad \ldots(18)$$

and for $y > Z_3$

$$p''_3 = \left(-\frac{C_2 H\gamma}{1-C_1} - \frac{C_4H^2}{1-C_1}\right)(1-y/H) + \frac{2C_4H^2}{2-C_1}(1-y/H)^2 + D_3\cdot H^{C_1}(1-y/H)^{C_1} \qquad \ldots(19)$$

where
$$D_2 = -\frac{C_2\gamma H^{1-C_1}}{1-C_1}\left[\left(1-\frac{Z_2}{H}\right)^{1-C_1} - 1\right] \qquad \ldots(20)$$

and

$$D_3 = \frac{K_1}{H^{1+C_1}}\left(1-\frac{Z_3}{H}\right)^{1-C_1} - \frac{2C_4H^{2-C_1}}{2-C_1}(1-Z_3/H)^{2-C_1} + D_2 + \frac{C_2\gamma H^{1-C_1}}{1-C_1}\cdot\left(1-\frac{Z_3}{H}\right)^{1-C_1} + \frac{C_4H^{2-C_1}}{1-C_1}(1-Z_3/H)^{1-C_1} - \frac{2C_4H^{2-C_1}}{2-C_1}\left(1-\frac{Z_3}{H}\right)^{2-C_1} \qquad \ldots(21)$$

The resultant P_3 is derived as

$$P_3 = \frac{(2Q)^{1+C_1}}{1+C_1}\left(K_4 + D_2H^{1+C_1}\right) + \frac{Q^{1+C_1}}{1+C_1}\left(D_3-D_2\right)\cdot H^{1+C_1} + \frac{Q^2}{2}(3K_1-3K_4-K_5) - \frac{Q^3}{3}\cdot 6K_6 + K_4\left(\frac{1}{2} - \frac{1}{1+C_1}\right) \qquad \ldots(22)$$

and

$$h_3 = H - \Bigg[\frac{(2Q)^{2+C_1}}{2+C_1}\cdot(M_4 - D_2H^{2+C_1}) + \frac{(2Q)^{1+C_1}}{1+C_1}(M_4 + D_2H^{2+C_1}) + \frac{Q^{2+C_1}}{2+C_1}\cdot(D_2-D_3)H^{2+C_1} - \frac{Q^{1+C_1}}{1+C_1}(D_2-D_3)H^{2+C_1} + \frac{Q^2}{2}\cdot(-3M_4+3M_1-M_5) + \frac{Q^3}{3}\left\{7(M_4-M_1)-6M_6+M_5\right\} + \frac{Q^4}{4}\cdot 14M_6 + M_4\left(\frac{1}{6} - \frac{1}{1+C_1} + \frac{1}{2+C_1}\right)\Bigg]\Bigg/ P_3 \qquad \ldots(23)$$

PARAMETRIC STUDY

The pressure intensity, resultant pressure and the height of application of the resultant thrust on a retaining wall 4m high and supporting backfill reinforced with steel strips 6.0 cm wide and spaced 50 cm horizontally have been determined by maximizing P by varying angle θ, for the following soil conditions.

$\phi = 30^\circ$, $f = 0.5$, $\delta = \frac{2}{3}\phi$, $\gamma = 1.8$ t/m^3

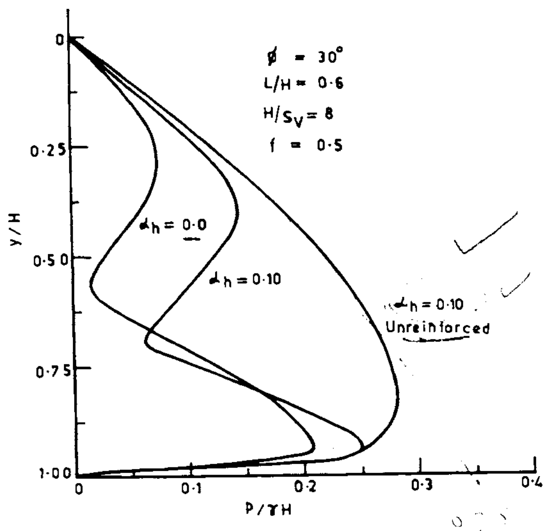

Fig.3 Pressure Intensity on the Retaining Wall for Different α_h

The equivalent static force and length of reinforcements were varied as indicated in Table I.

Table I Range of Parameters Studied

Parameter	Range	Interval
L/H	0.0, 0.2-0.8	0.1
α_h	0.0 - 0.15	0.05

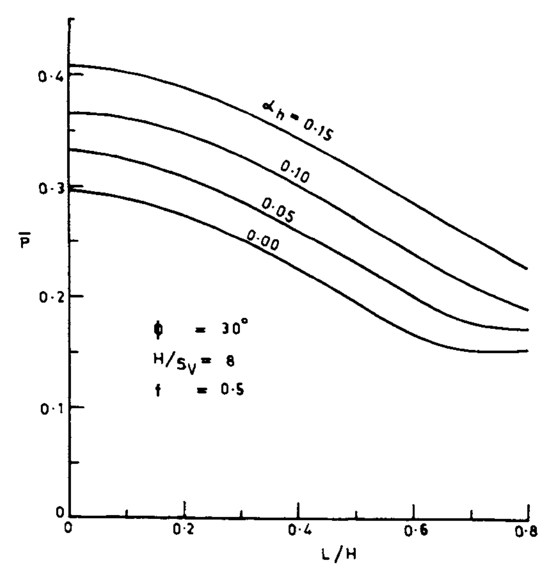

Fig.4 Non-dimensional Pressure $\overline{P}$ Acting on the Wall

$$l' = L-(H-y)\tan\theta \text{ for } Z_2 \leq y \leq Z_3,$$

and $$l' = (H-y)\tan\theta \text{ for } y > Z_3 .$$

The expression for pressure intensity, p, resultant P and its point of action is determined as under:

Case 1.

$$T = 2wf\gamma\left(y + \frac{dy}{2}\right)(H-y)\tan\theta/S_H$$

and $t = 2wf\gamma\left(y+\frac{dy}{2}\right)(H-y)\tan\theta/(S_H.S_V)$. After neglecting small quantities of second order

$$\frac{dt}{dy} = k(H-y)$$

where $k = 2wf\gamma\tan\theta/(S_H.S_V)$

Equation (7) then becomes

$$\frac{dp}{dy} = -C_1\frac{p}{H-y} + C_2\gamma - C_4(H-2y)$$

where $C_4 = C_3.k$.

The solution of this differential equation for the boundary condition $p=0$ at $y=0$ is

$$p_1 = -\left[\left(1-\frac{y}{H}\right)-\left(1-\frac{y}{H}\right)^{C_1}\right]\left[\frac{C_2 H\gamma}{1-C_1}+\frac{C_4H^2}{1-C_1}\right]+\frac{2C_4H^2}{2-C_1}.$$
$$\left[(1-y/H)^2-(1-y/H)^{C_1}\right] \quad \ldots(9)$$

Resultant pressure $P = \int_0^H p_1\,dy$

$$\text{i.e. } P_1 = \frac{1}{1+C_1}\left[\frac{C_2H^2\gamma}{1-C_1}+\frac{C_4H^3}{1-C_1}-\frac{2C_4H^3}{2-C_1}\right]+$$
$$\frac{2C_4H^3}{3(2-C_1)}-\frac{C_2H^2\gamma}{2(1-C_1)}-\frac{C_4H^3}{2(1-C_1)} \quad \ldots(10)$$

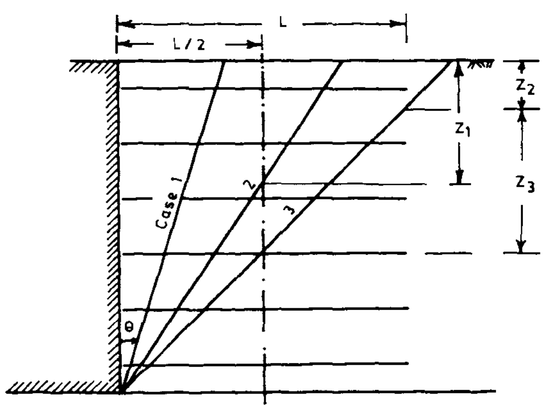

Fig.2 Variation of Effective Length of Strips

and height of action of P above the base,

$$h_1 = H - \frac{\int_0^H p_1 y\,dy}{\int_0^H p_1\,dy}$$

$$\text{or } h_1 = H-\left[\frac{1}{2+C_1}(M_4-M_5+M_6)-\frac{1}{1+C_1}(M_4-M_5+M_6)+\right.$$
$$\left. M_4/3-M_5/6+M_6/12\right]\Big/P_1 . \quad \ldots(11)$$

where $M_4 = \frac{-C_2\gamma H^3}{1-C_1} = K_4.H$; $M_5 = \frac{C_4H^3}{1-C_1} = K_5.H$

$$M_6 = \frac{2C_4H^3}{2-C_1} = K_6.H$$

The pressure intensity for case 2 and 3 are similarly determined and are given below

$$p_2 = \frac{H}{1-C_1}(-C_2\gamma - C_4L/\tan\theta + C_4.H).$$
$$\left[\left(1-\frac{y}{H}\right)-\left(1-\frac{y}{H}\right)^{C_1}\right]+\frac{2C_4H^2}{2-C_1}.$$
$$\left[\left(1-\frac{y}{H}\right)^2-\left(1-\frac{y}{H}\right)^{C_1}\right] . \quad \ldots(12)$$

for the condition $y \leq Z_1$, and

$$p_2' = \frac{H}{1-C_1}(1-y/H)(-C_2\gamma - C_4H)+\frac{2C_4H^2}{2-C_1}(1-y/H)^2+$$
$$D_1.H^{C_1}(1-y/H)^{C_1} \quad \ldots(13)$$

where

$$D_1 = \frac{H^{1-C_1}}{1-C_1}(-C_2\gamma + C_4\,L/\tan\theta + C_4H)\left[(1-Z_1/H)^{1-C_1}-1\right]-$$
$$\frac{2C_4H^{2-C_1}}{2-C_1}\left[(1-Z_1/H)^{2-C_1}-1\right]-\frac{1}{H^{C_1}}\left[\left\{-\frac{C_2H\gamma}{1-C_1}-\frac{C_4H^2}{1-C_1}\right\}\right.$$
$$\left.(1-Z_1/H)^{1-C_1}+\frac{2C_4H^2}{2-C_1}(1-Z_1/H)^{2-C_1}\right] \quad \ldots(14)$$

for the case $y > Z_1$.

$$P_2 = \int_0^{Z_1} p_2\,dy + \int_{Z_1}^{H} p_2'\,dy$$
$$= \frac{Q^{1+C_1}}{1+C_1}\left(K_1-K_6+D_1.H^{1+C_1}\right)+\frac{Q^2}{2}(K_4-K_1-K_5)+\frac{Q^3}{3}.2K_6+$$
$$\frac{1}{1+C_1}(K_6-K_1)+\frac{K_1}{2}-\frac{K_6}{3} \quad \ldots(15)$$

where $Q = \frac{L}{2H}\cot\theta$,

$$K_1 = (-C_2\gamma + C_4\,L/\tan\theta + C_4H)\frac{H^2}{1-C_1}$$

and other terms are as already defined.

$$h_2 = H-\left[\frac{Q^{2+C_1}}{2+C_1}\left(M_6-D_1.H^{2+C_1}-M_1\right)+\frac{Q^{1+C_1}}{1+C_1}.\right.$$
$$(M_1-M_6+D_1H^{2+C_1})-$$
$$\frac{Q^2}{2}(M_1-M_4+M_5)+\frac{Q^3}{3}(M_1+2M_6-M_4+M_5)-\frac{Q^4}{4}.2M_6+$$

RESULTS AND DISCUSSIONS

Typical results of the analysis are shown in Figs.3-5.

1. Pressure Distribution: The pressure distribution for unreinforced and reinforced soil with length of strips equal to 0.6H and vertical spacing of 0.5m (H/S_V=8) is shown in Fig.3. The use of reinforced earth backfill significantly reduces pressure on the wall both for seismic or non-seismic condition. Major reductions are observed in the middle portion of the wall due to the fact that the effective length of strips contributing to stability is maximum in the middle portion than towards the top or bottom of the wall. The pressures are positive all along the wall height. However in heavily reinforced backfill, negative pressures may develop in the upper portion of the wall. In such a case the analysis can be modified and the height of backfill experiencing negative pressures can be assumed as a uniform surcharge for the rest of the wall height for purposes of analysis.

2. Resultant Pressure: The resultant lateral pressure on the wall represented in a non-dimensional form ($\bar{P} = P/0.5\ H^2$) is shown in Fig.4. For the values of α_h considered, a sharp decrease in normalised pressure is evident at L/H 0.6, which may be taken as the optimum length of reinforcement for the value of parameters considered in this investigation.

3. Point of Action of Resultant: Since most of reduction in the pressure intensity occurs, generally in the mid-portion of the wall, the normalised height above base of point of action, $\bar{H}$ (= h/H), is not much influenced by provision of reinforcement (Fig.5). There is a small increase in $\bar{H}$ for lower values of L/H which is followed by sharper decrease around L/H = 0.6.

4. Tensile Force in Strips: Maximum stresses would develop in the strips at mid-height and will be of the order of 0.3 to 0.4 tonne for the 4m high wall necessitating strip thickness of 0.3 to 0.4 mm for an allowable stress of 14 kg/mm^2.

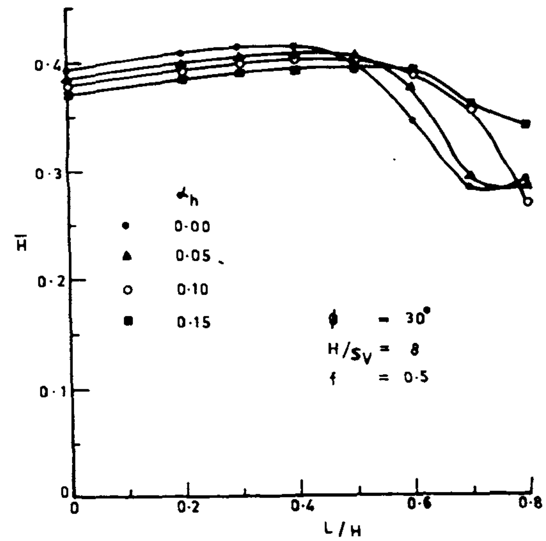

Fig.5 Variation of Height of Point of Action of Resultant Pressure

CONCLUSIONS

The pressure on a rigid wall under seismic conditions can be appreciably reduced by reinforcing the backfill by unattached horizontal strips of moderate strength placed normal to the back of wall. The performance of the reinforcement can be further improved by increasing the soil-reinforcement friction by use of ribbed strips. The method of reinforcement is very simple and economical and materials of low strength can be employed.

REFERENCES

Saran,S. and Prakash,A.(1971),'Seismic Pressure Distribution in Earth Retaining Walls',Fourth European Symposium on Earthquake Engineering, Sofia, Bulgaria.

Saran,S.,Talwar,D.V.and Prakash,S.(1979),'Theoretical Earth Pressure Distribution on Retaining Wall with Reinforced Earth Backfill', Proc.International Conf.on Soil Reinforcement-Reinforced Earth and Other Techniques,Paris, Vol.1, pp.139-144.