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Recent Advances in Centrifuge Modeling of Seismic Shaking

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Recent Advances in Centrifuge Modeling of Seismic Shaking

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(State of the Art Paper)

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SYNOPSIS: This "State-of-the-Art" paper focuses primarily on aspects of dynamic centrifuge modeling related to simulation of earthquake effects. New shaker mechanisms and model containers are described and soil-container-shaker interaction is discussed. Progress in dealing with scale effects such as particle size, rate dependent material properties and conflicts in dissipation and generation time scale factors is also described. Issues related to repeatability and value of model testing are also discussed.

INTRODUCTION

Seismic events in real life may cause extensive damage, but the location of the events are not predictable with certainty. Not knowing exactly where to invest in field instrumentation, engineers are confronted with a lack of data regarding performance of instrumented sites during strong shaking. This fact makes centrifuge modeling of seismic events and especially important source of data.

There are now approximately 60 active geotechnical centrifuge facilities on earth. Sixteen in Japan, fourteen in the USA, seven in the Peoples Republic of China, six in the UK, and three in Canada. Fifteen countries possess at least one geotechnical centrifuge. Approximately fifteen of these centrifuges (in the USA, Japan, UK, and France) have shaking table facilities. Many other geotechnical centrifuges have been used to simulate other dynamic phenomena such as explosion effects, foundation vibration, and pile driving.

Several excellent papers have been published in the last 5 years summarizing the progress in dynamic centrifuge testing (Ko 1994, Scott 1994, and Steedman 1991); there is no need to repeat these reviews here. Many researchers, and perhaps dozens of centrifuge facilities throughout the world, have been involved in soil dynamics research based on centrifuge model tests. Dynamic centrifuge modeling has matured to a stage where it is not possible to provide a comprehensive review in a single paper. In this paper, an attempt has been made to present a somewhat detailed review of progress in a few specific areas in which the author has been involved.

Part of this paper deals with recent advances in centrifuge based shaking table machines. Some detail on flexible model containers useful in simulation of 1-dimensional propagation of shear waves is also presented. The importance of soil-container-shaker interaction is discussed, a factor which has been often ignored in analysis of centrifuge shaking table tests.

Advances in understanding of scale effects and techniques to avoid scale effects are also discussed. All geotechnical engineers deal with soils with a large range in particles sizes; centrifuge modelers are not the only ones that must deal with scale effects. Three aspects of scale effects are discussed. The particle size effect, the strain-rate effect, and the use of viscous pore fluids to avoid the conflict in scaling laws for time in dynamic and consolidation problems.

SHAKERS

Mechanical Shakers

In the last 15 years, many centrifuge based shaking table facilities have been invented, and implemented. Steedman (1991) and Ko (1994) provide a description of many of these facilities, which are only briefly reviewed in this paper. A steady evolution in capabilities and sophistication has occurred. Centrifuge shakers began with Morris (1983) and Ortiz et al. (1983) in the form of cocked spring and mass exciters. Initial experiments were conducted using these apparatus, but it was soon recognized that their characteristic of providing decaying sinusoidal waves of a specific frequency was not especially realistic.

The bumpy road apparatus was then developed at Cambridge University (Schofield 1981 and Kutter 1983). This apparatus has been in use for more than a dozen years, and has probably produced more model "earthquakes" than any other centrifuge-based shaker. The bumpy road system forces a cam roller to follow a curved track mounted on the wall of the centrifuge; the radial vibration of the cam is transmitted via a crank and shaft to shake the model in the circumferential direction. The motion of the model depends on the shape of the track which is changeable (with considerable effort), but only two sinusoidal tracks were implemented. The Bumpy Road system provided a mechanism for adjusting the shaking amplitude and permitted several shaking events to be triggered without stopping the centrifuge. Kimura et al. (1991) describe a rotating cam shaker, which is subject to similar limitations as the bumpy road; in order to change the frequency of the shaking, the cam must be changed. Although it is theoretically possible to make a cam with a combination of frequencies machined onto its profile, the system was only used to produce approximately sinusoidal shaking. The mechanical shakers have certainly provided useful data, but they are all limited in there ability to produce a broad spectrum of shaking frequencies.

Importance of Frequency Content

In a separate paper in this conference, Fiegel et al. (1995) demonstrate the importance of studying models using a variety of realistic, earthquake like motions. They found that spectral ratios (the ratio of the ground surface spectral ordinates to those of the base motion) were dependent on the frequency content, even for input motions with the same peak acceleration. The paper illustrates the usefulness of a shaker with capability to vary frequency content as well as amplitude without stopping the centrifuge.

The importance of frequency content on response characteristics of soil deposits sounds obvious to earthquake engineers, but many modelers continue to use a limited range of input motions in their work. Often models are subject to purely sinusoidal base motions. This is justified by the impression that it will be easier to understand the results if the simple input motions are used. Of course this is true, but dynamic systems have a spectrum of response characteristics; the use of single frequency base motions only provides one data point in the spectrum. Furthermore, the use of sine waves of constant amplitude may give a false impression of the importance of one particular aspect of soil response. For example, even in laboratory element tests, a different character of response may be obtained for cycles of constant amplitude compared to cycles of random amplitude. Kutter and Chen (1994), reported results of a torsional hollow cylinder simple shear test with uniform load cycles as shown in Fig. 1. After a stress reversal, the shear stress and mean normal effective stress rapidly drop near zero and shear strains develop with very little stress. The liquefied soil appears to have some memory of its previous maximum shear strain. As the soil approaches the previous maximum shear strain, the "liquefied" particles engage each other, the tendency to dilate produces negative pore pressures and increases in effective stress causing the shear stress to increase.

This type of repeated liquefaction and dilation in each cycle may not be so prevalent for variable amplitude cyclic loads. Fiegel and Kutter (1994) tested centrifuge models consisting of a layer of Nevada Sand covered by a layer of non-plastic silt. They showed a different character of response for uniform sinusoidal base motions than for more realistic base motions as shown in Fig. 2. The model subject to 10 uniform cycles showed repeated large amplitude acceleration spikes in the Nevada Sand layer. Some spikes also appear in the case of the more realistic

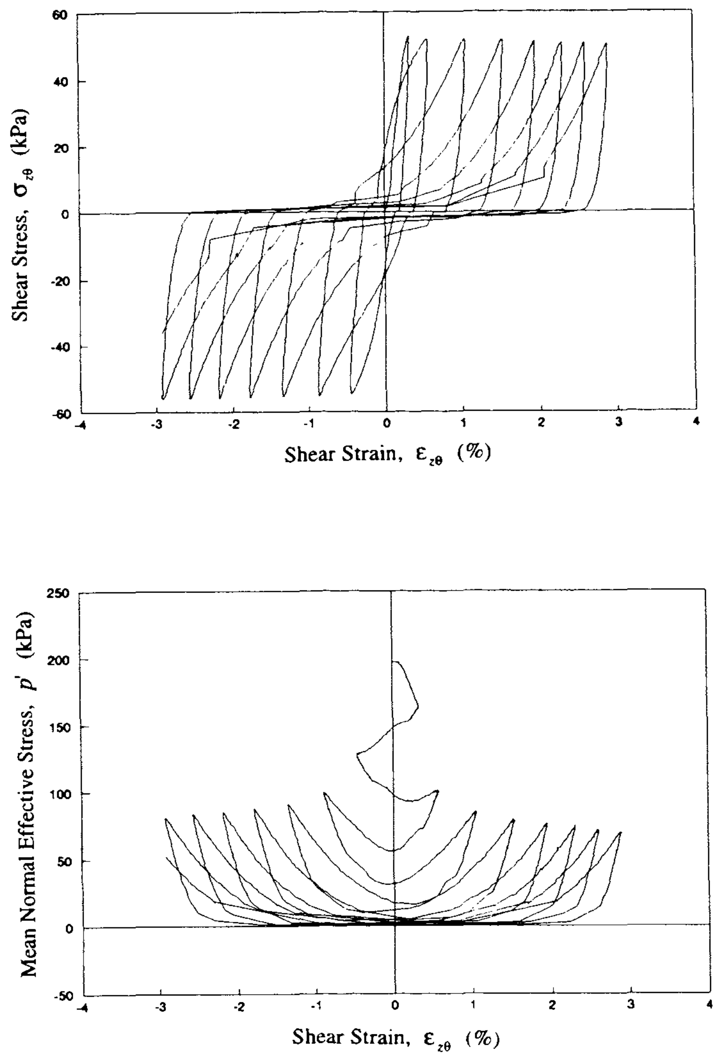

Fig. 1. Torsional simple shear test with uniform load cycles on a fine sand. Kutter and Chen (1994)

base motion, but they are much less pronounced. The peak of the uniform base motion was about half of the peak of the more realistic earthquake, but the peak acceleration in the Nevada Sand was twice as great as in the case of uniform base motion. Similar spiky acceleration traces have been reported and discussed by several other authors. By looking only at one sinusoidal time history, one might be deceived about the importance of the spiky acceleration.

Piezoelectric, Explosive, and Electromagnetic Shakers

These types of shakers introduced significant potential for providing variable frequency content model earthquakes, but they also have limitations described below. A piezoelectric shaker was developed by Arulanandan et al (1982) which could provide controllable high frequency shaking, but the performance was limited at the lower end of the important range of frequencies. Simulation of earthquakes using controlled and sequenced detonation of

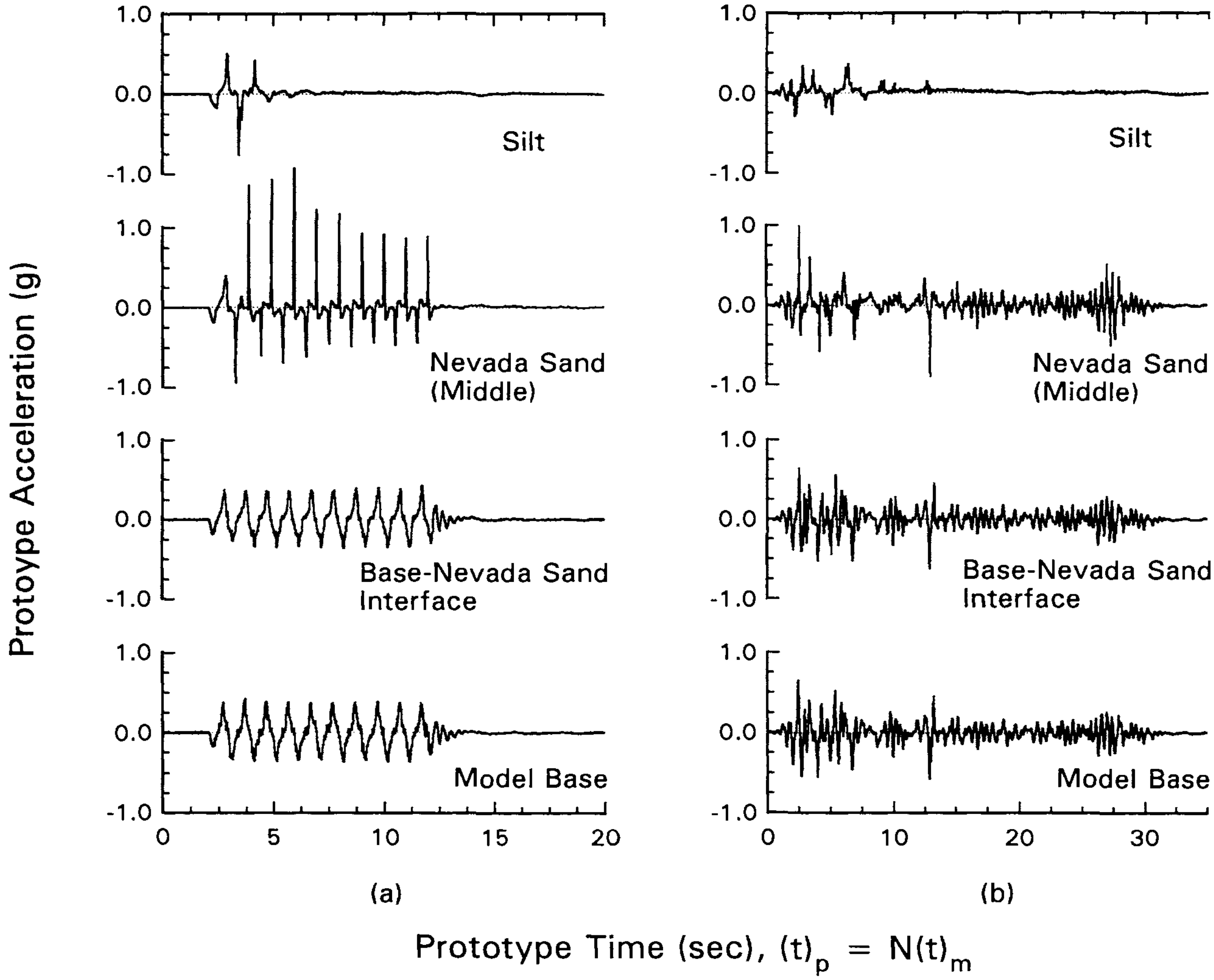

Fig. 2. Difference in character of soil response for similar models under different types of base motion.

explosives was developed by Zelikson et al. (1981), and this system provided some ability to simulate earthquakes with prescribed frequency contents, but simulation of specific time histories of base motion was still not feasible.

Fujii (1991) and Sato (1994) describe an electromagnetic shaking system, which they preferred over servo-hydraulic actuation because it is able to provide controlled excitation over a large frequency range. It appears that frequencies on the order of 350 Hz are achievable. Sato (1994) shows very good replication of actual earthquake recordings. Fujii and Sato have made some important progress in this area, but the fact remains that the electromagnetic shaking mechanism is large and heavy compared to servo-hydraulic actuators with the same capacity. The electromagnetic shaker described by Sato (1994) is able to provide 10 g spikes and 5 g sinusoidal excitations to 300 kg models, which is fine for the study of relatively small earthquakes.

Servo-hydraulic Shakers

Servo-hydraulic actuation has long been recognized as compact, powerful and versatile method of simulating realistic earthquakes on a centrifuge. Relatively small servo-hydraulic actuators have now been implemented on many centrifuges in the US and Japan (Chang 1990, Van Laak et al. 1994, Ketcham et al. 1988, Aboim et al. 1983, and Takemura et al. 1989, Nagura et al. 1994). The success of these servo-hydraulic shaking systems, and the development of larger centrifuge facilities in recent years has led to recent efforts to develop "second generation", larger hydraulic shakers at RPI and UC Davis. These large shakers are just coming on line and few results are available.

The shakers at UCD and RPI have many common design features, primarily because there was collaboration in the design phase of their development. The design of the UC Davis shaker is described in some detail by Kutter et al. (1994), and will be briefly described here. A schematic of the actuator mechanism (patented by Team Corporation, Seattle) is shown in Fig. 3. Each actuator consists of a two stage servo-valve block sandwiched by single acting actuators. Excitation to the voice coil moves a pilot valve which provides hydraulic pressure to actuate the slave

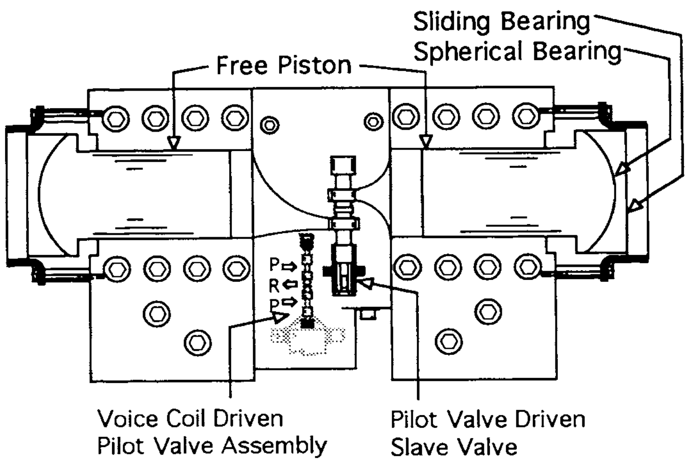

Fig. 3. Schematic of patented split actuator shaker used in "second generation" hydraulic shakers at RPI and UC Davis.

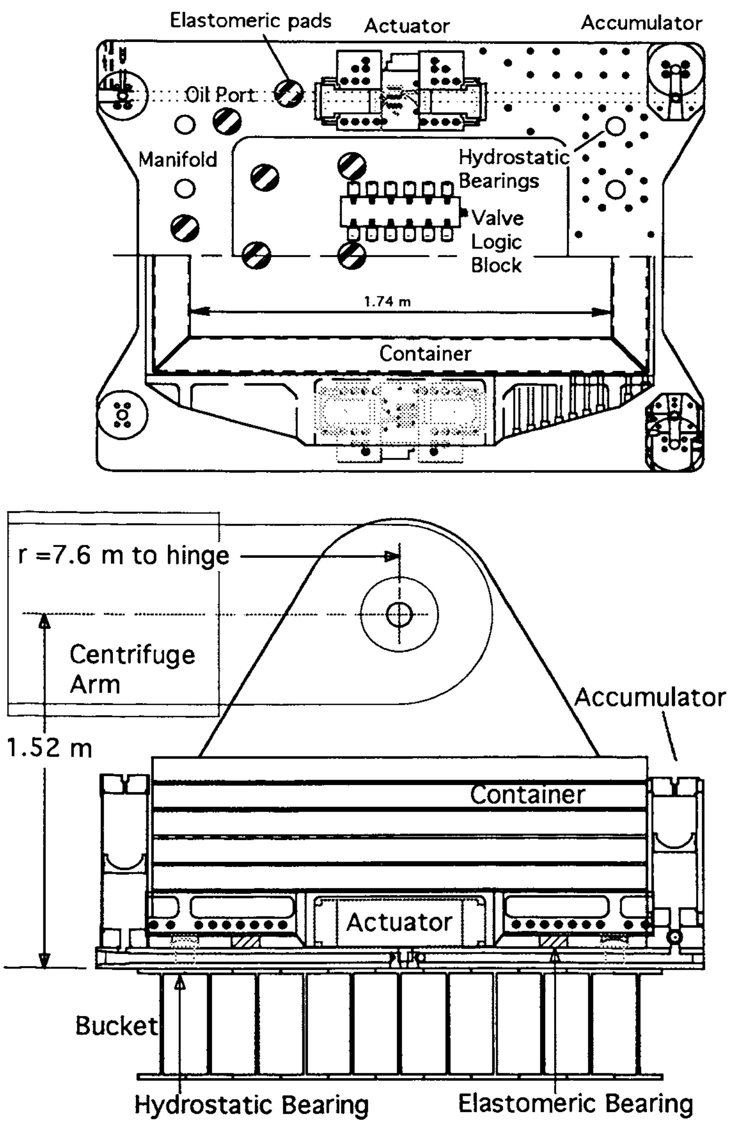

Fig. 4. Plan (top) and elevation (bottom) views of the new UC Davis shaker.

valve. The slave valve supplies pressure to the single acting actuators which in turn move the shaking table. Sliding and spherical hydrostatic bearings are provided to accommodate inevitable distortions of the table and centrifuge platform, without compromising the ability to transmit the shaking forces to the shaking table.

The first generation of hydraulic shakers used one double acting actuator mounted underneath the shaking table, which results in a considerable distance between the center of mass of the load and the line of action of the shaker and introduces uncontrolled annoying rocking motions of the shaking table. As indicated in Fig. 4, the new shaker at UC Davis employs two actuators, mounted at the side of the shaking table, raising the line of action nearer to the center of mass of the model.

Table 1. Specifications of New UC Davis Shaker

Mass of Model and Container (kg)	2700
Max Shaking Acc'n for 2700 kg (g)	15
Max Absolute Velocity (m/s)	1.0
Max Relative Displacement (cm)	2.5
Useful Frequency Range (Hz)	20 to 200
Length of Container (m)	1.75
Width of Container (m)	0.7
Height of Container (m)	0.6
Number of Actuators	2
Actuator Area (m^2)	0.0081
Hydraulic Pressure (MPa)	35
Servo-valve flow rate (l/s)	13
35 MPa Supply Reservoir Vol. (l)	70
1.3 MPa Exhaust Reservoir Vol. (l)	55

The shaking table is supported by a combination of 24 elastomeric bearing pads and 4 hydrostatic bearings. Fig. 4a shows the location of these bearings and the bolting pattern to attach the 90 mm thick manifold/mounting plate to the base of the centrifuge bucket (which is made of I beams). Each actuator is supplied with oil by a 35 MPa accumulator, and drains to a 1.3 MPa exhaust accumulator, which are mounted on the corners of the manifold. The accumulators mainly serve as oil reservoirs, and they are backed up by a separate 70 liter, 35 MPa compressed nitrogen pressure vessel. This pressure vessel acts as a power supply, which is capable of providing approximately 1 Megawatt of power to the servovalves for the 1 second duration of the model earthquake. The pressure vessel can be recharged via on-board pumps over a period of about 10 minutes and then another shaking event can be triggered.

The model container, shown in the bottom half of Fig.4a is described later in the next section of this paper.

MODEL CONTAINERS AND BOUNDARY EFFECTS

Whitman and Lambe (1986), and Steedman (1991) have discussed and summarized different types of model containers for dynamic centrifuge tests. The important boundary conditions to be satisfied by the model container for earthquake simulation are quite different from those for problems, such as foundation vibration, where the dynamic source is within the model. As described by Campbell et al. (1991) and shown by Lenke et al. (1991), an energy absorbing material called "duxseal" placed along the boundaries, and the use of non-cylindrical containers provide very good energy absorbing and scattering properties for foundation vibration type problems.

In some centrifuge shaking table tests, Fiegel and Kutter (1994) indicated that there is a preferential flow path for water along the container walls. In a test with a silt layer covering liquefying sand, water was found to leak along the windows past the silt layer instead of through the silt layer. In studies of layered soils, where pore pressure dissipation rates are considered important, it is important to evaluate the possibility of leakage along the boundaries in addition to evaluation of the mechanical interaction of the container and soil.

Some geotechnical structures, such as embankments, lead to models which do not contact the end walls of the container, avoiding the boundary effect due to interaction of the end walls with the model. Other geotechnical structures, e.g. retaining walls, may contact one end wall and not the other, avoid only half the problem. The next section of this paper discusses mechanical interaction between containers and soil models for the 1-Dimensional shear wave propagation problem, where both ends of the model interact with the container.

Vertically Propagating Shear Waves

To simulate the 1-Dimensional vertically propagating shear waves on of stratum soil, it is desirable to satisfy the several boundary conditions. Significant attention has been paid to this problem, and significant progress has been made. Cheney and Whitman (1983), and Campbell et al. (1991) summarize the desired boundary conditions. Three important aspects in simulation of an infinite soil layer are discussed below with reference to recent advances made in each area.

1) An ideal container should maintain negligible normal strain in the horizontal directions.

In order to accurately maintain Ko conditions during consolidation, the horizontal strain should be negligible. Due to the change in total horizontal stress which may develop during centrifugation or during liquefaction, the sides of the model container will deflect which will affect the horizontal stresses. Van Laak et al. (1994b) describe a laminar box design which limits horizontal normal strains to 0.02%. Intuitively, one may reason that a reasonable limit on the allowable horizontal strain might be that they should be significantly smaller than the vertical strains. One very obvious effect of horizontal strains is their effect on the sample volume; in addition to settlements caused by densification, surface settlements will be partly attributable to lateral expansion of the container.

2) An ideal container will permit shear waves to travel vertically, without allowing significant energy transfer between the soil and the container across the vertical boundaries.

Fixed end boxes do not permit the lateral deformation to occur at the ends. This constrains the soil unrealistically and allows horizontal p-waves to enter the sample at the ends. It has been argued that fixed end containers have well defined, if not realistic, boundary conditions. If the purpose of the centrifuge test is to validate a finite element procedure, or to observe a failure mechanism then fixed end containers may be adequate.

Laminar boxes (Hushmand et al. 1988, Law et al. 1991, Van Laak et al. 1994b) , stacked ring apparatus (Whitman et al. 1981), and Hinged Plate Containers (Fiegel et al. 1994) have been developed to permit lateral shear deformations to occur freely. The sides of these containers do, however, have mass (usually about 30% of the mass of the contained soil), and will impart lateral inertia loads onto the ends of the specimen. If the purpose of the centrifuge test is to verify an analysis procedure, Van Laak et al (1994b) suggested that the inertia forces can be approximately accounted for in analysis of the test results by lumping the mass of the container walls into the unit weight of the soil in the analysis. The total unit weight of the soil in the calculation is taken as the weight of soil plus the weight of rings divided by the volume of the soil. In an effective stress analysis, the buoyant unit weight used in the analysis must be the same as that for actual soil.

Instead of using roller bearings to permit freedom for lateral deformation, the Equivalent Shear Beam (ESB) developed by Schofield and Zeng (1992) uses rubber sheets of specific stiffness between the rings. The system of aluminum rings and rubber sheets can be designed to have a natural frequency that is similar to that of the soil layer. While this idea has obvious merits, it is not applicable to simulation of highly non-linear soil response; the container designed to match the initial natural frequency of the soil layer cannot match the softening and permanent deformation that might occur in a soil layer.

3) An ideal container will provide complementary shear stresses to the ends of the soil model.

In the Laminar boxes and stacked ring apparatuses described above, the complementary shear stresses are presumed to be transferred by vertical preloaded clamps as indicated schematically in Fig. 5. The upward and downward shear stresses on the ends of the container are either transferred upward through a bearing and clamp or directly downward into the base of the container. Attention should be paid in the design to ensure that the load path for the complementary shear forces is sufficiently stiff. Schofield and Zeng (1992) proposed the use of a "shear sheet" to transfer the complementary shear stresses to the base of the container in their ESB box. A thin sheet of steel, on the order of 0.1 mm thick is clamped to the base of the container, covering the end walls of the container. The sheet is pressed against the rings so that it will not buckle, hence it acts as a compression and tension member that is capable of undergoing large lateral deformations. The idea of a shear sheet has been adopted by Fiegel et al.

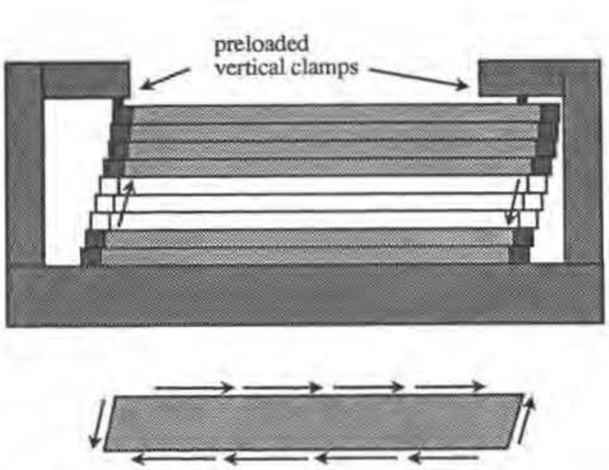

Fig. 5. Complementary shear stresses on sides of soil layer and corresponding forces on ends of laminar box.

(1994), and could possibly be incorporated in laminar box apparatuses to improve the shear stress transfer.

A variation on the ESB concept is now being implemented for the large shaker at UC Davis (Kutter et al. 1994). The large container is made of six rectangular rings made of hollow aluminum tubing, again with layers of rubber glued in between each aluminum ring. This box is to be made softer than the soil layer, hence it has been called a "flexible shear beam" (FSB). Instead of attempting to match the natural frequency of the container to that of the soil layer, the rubber thickness and stiffness are designed to provide a container natural frequency much lower than the initial natural frequency of the soil layer. For liquefaction studies, an ESB container would provide too much constraint to the softened (liquefied) soil. Due to soil non-linearity, it is not possible to create a truly "equivalent" shear beam; it was thought preferable, therefore, to create a "flexible shear beam".

Fiegel et al. (1994) describe a new model container called a Hinged Plate Container (HPC); the mechanism of this container is shown in Fig. 6. It works like a four-level stack of Cambridge type simple shear boxes with each end plate hinged to those above and below. The ability to rotate and provide continuity of displacement on the end boundaries was thought to avoid the local disturbance of the soil caused by intense shearing that must occur near the steps in a laminar box type container. The side plates for each level are independently supported on side rails so they do not rest on each other, minimizing friction and alignment problems. The walls are made up from bolted aluminum tubes and angle sections to minimize weight and maximize bending stiffness. Like the laminar boxes described by Van Laak et al. (1994b) and Law et al. (1991) the HPC provides roller bearing lateral supports to the long sides of the container to minimize deflections. The HPC includes a shear sheet as proposed by Schofield and Zeng (1992).

Fiegel et al. (1994) compare the results from four different types of model containers: a fixed-end box, the HPC, a Caltech style laminar box (Hushmand et al. 1988)

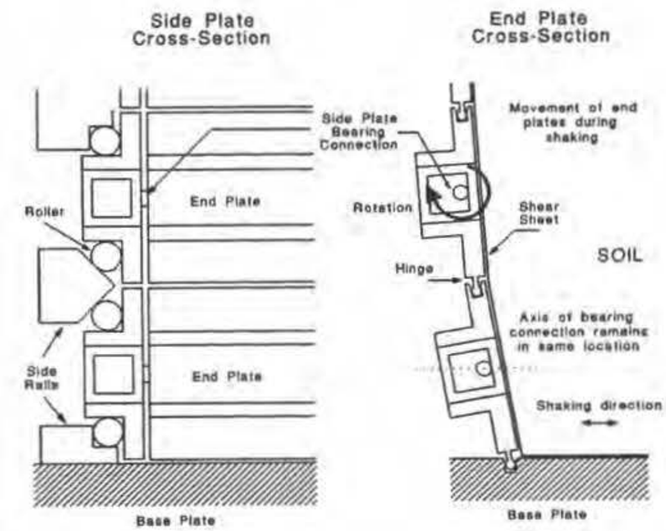

Fig. 6. Hinged plate container concept. Fiegel et al. (1994)

and an ESB container (Schofield and Zeng 1992). They found that the natural frequency of a soil layer in the ESB and Hinged Plate containers was about the same, but in the laminar box the natural period of the soil-container system was about 5% greater. An increase in natural period could be caused by incomplete development of complementary shear stresses. The damping provided by different boxes also seems to be different. The ESB box had less damping than the HPC or Laminar Container. The ESB box was designed to have a similar natural frequency as the soil layer, but no effort was made to make the damping ratio of the box similar to that of the soil; at large strain amplitudes the damping of the rubber in the ESB may be less than that of the soil.

Soil-Container-Shaker Interaction

Fiegel et al. (1994) compared the response a layer of dry sand in a fixed end and in the HPC containers for an impulsive excitation: a step displacement command to the servocontroller. From these results shown in Fig. 7, it is clear that the soil response is dependent on the type of container used. Furthermore, it is apparent that the base motion is dependent on the type of soil container used. The base motion for the Hinged Plate Container (HPC) has a sharp spike and decaying high frequency vibration is seen following the spike. For the fixed end container, the spike is a little wider and it is followed by a decaying frequency, almost identical to the frequency of the surface motion. These results clearly show that the base motion as well as the surface motion are dependent on the type of container used. In the rigid box there is a significant soil-container-shaker interaction. The base motion is not a true "input" since it depends on the soil behavior.

Of course, one would expect interaction. The masses of soil models, containers, shaking tables and their reaction masses in typical centrifuge shaking systems are all of the

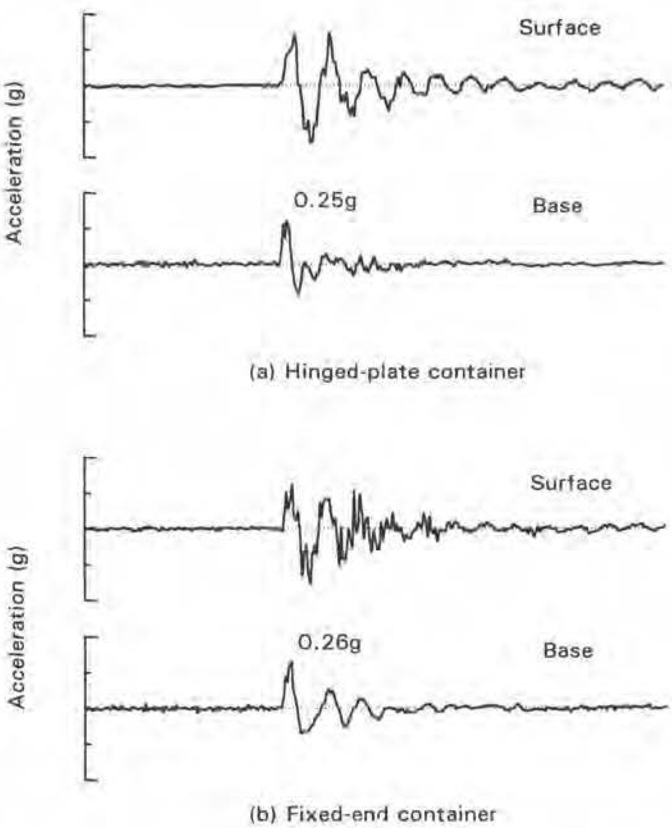

Fig. 7. Response of shaker base and soil surface to a step displacement command. Fiegel et al. (1994)

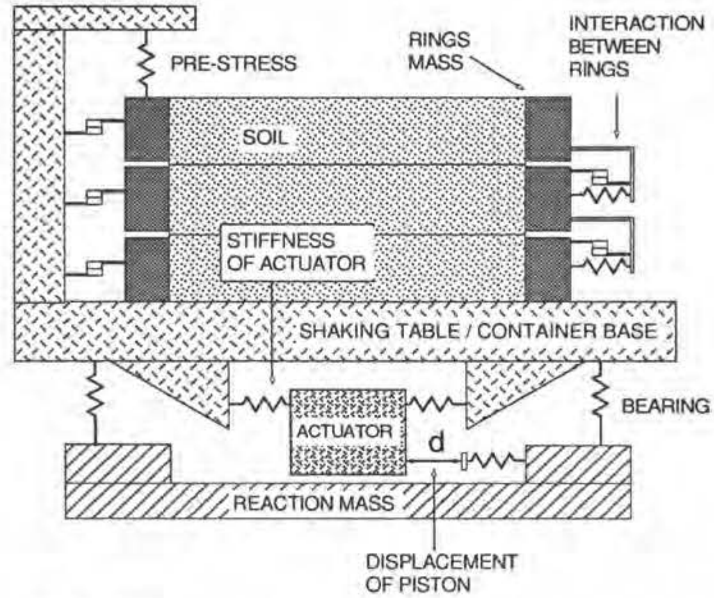

Fig. 8. Soil-container-shaker interaction.

same order of magnitude. Furthermore, centrifuge based shaking tables are being driven below, at, and beyond there own resonant frequencies.

It may be theoretically correct to treat the container base motion as an "input" motion to the soil and container, but we must recognize that there is energy transfer (analogous to radiation damping through compliant boundaries in the field) between the soil model, the shaking table and the reaction mass. The interaction of the model and the shaker system might amplify or attenuate discrepancies between the analytically predicted and experimentally measured response of the physical model. The importance of the interaction of the model with the shaker system should be evaluated, and if necessary, the interaction should be included in the analysis of the model behavior. In the analysis of centrifuge test data, numerical modelers ought to attempt to include the model containers in their analyses.

To be even more rigorous, a dynamic centrifuge test on a shaker could be analyzed as indicated in Fig. 8. The mass of the container and frictional damping between the elements could be included as appropriate for the particular package used. The mass of the shaking table could be included and stiffness of the bearings should be included to incorporate the possibility of rocking of the model. The connection between the reaction mass and the shaking table could be modeled by a spring representing the stiffness of the oil, a spring representing the stiffness of the connection of the actuator to the reaction mass (usually the swinging bucket of the centrifuge) and a specified displacement, d, which represents the flow of oil to alternate sides of the actuator. It may even be important and possible to include the control system and servo valve response characteristics into the analysis. The connection of the reaction mass to the centrifuge arm (not shown) could also be incorporated in the analysis if important natural frequencies of the centrifuge arm are in the range excitation.

We have made significant progress in developing complicated model containers (stacked rings, laminar boxes, ESB, FSB, and HPC) that more accurately simulate 1-dimensional shear wave propagation, but each type of container produces different results; we have not converged on one solution. We ought to carefully evaluate the different types of containers and the soil-container-shaker interaction in order to establish which container is superior, and which aspects of interaction need to be included in our analysis of the test results.

If it is determined that soil-container-shaker interaction is important, it may be preferable, in some cases, to use a container with well defined properties, instead of the most realistic boundaries. The rigid box, ESB and FSB containers have reasonably well defined boundary conditions, which could be simulated in a numerical analyses. It is worth noting that a so called "rigid box" will not be truly rigid, especially at the relatively high frequencies commonly encountered in dynamic centrifuge testing.

SCALING LAWS AND SCALE EFFECTS

As any researcher knows, it is important to be self critical of data. Data should be consistent with existing knowledge, and results must be reliably repeatable. The limitations of any method of investigation must be kept in mind. Particle size effects, strain rate effects, and boundary effects may bias centrifuge test data as they may bias any experimental data. A thorough research project will include some consideration of the possible magnitude of the importance of these errors.

Scale effects are especially deleterious in attempts to directly model a specific prototype event. On the other hand, most centrifuge model tests are conducted to discover mechanisms of behavior, to compare the performance of similar structures, or to verify a numerical method, not to model a specific prototype event.

Problems with scale effects are not unique to physical modelers. Particle size effects also occur in prototypes, "element tests" like triaxial or simple shear tests, and they are apparent to any finite element modeler who attempts to simulate strain softening (the results are often mesh dependent). Particle size effects manifest themselves as a "characteristic length" (perhaps representing the thickness of the shear band) to numerical modelers in simulations involving strain softening. In softening materials, deformations tend to localize on shear bands, the rate of softening depends on the strain in the shear band, which depends on relative displacement between opposite sides of the shear band and on the thickness of the shear band which depends on particle size.

Like centrifuge models, triaxial specimens are much smaller than prototype geotechnical structures. Triaxial specimens of different size will exhibit different strain softening rates, even in monotonic tests; the softening rates may be quite different from those that occur in a prototype. It might be argued that centrifuge models, being larger than most triaxial specimens, are less affected by scale effects than are triaxial tests.

Strain rate effects are also real, they affect results of laboratory tests as well as centrifuge model tests. For example, conventional consolidation tests lasting a week in the laboratory are used to estimate the settlement that will occur 50 years after completion of a prototype construction. In dynamic problems, the frequency and strain rate of cyclic loading are known to affect the stress-strain behavior.

A concise review of scaling laws is necessary as an introduction to the following discussion of Scaling Laws and Scale Effects. Scaling laws have been previously described by Bucky (1931), Pokrovsky (1934), Schofield (1980, 1981), Scott (1988) and others. In the following discussion, scale factors for a quantity will be denoted by the symbol for the quantity with an asterisk, e.g., L^* represents the ratio of L in the model to L in the prototype.

The basic objective of using a centrifuge is to establish in a reduced scale model identical strength, stiffness and stress as that which exists in a much larger prototype. In other words, we require the scale factor for stress, $\sigma^* = 1$. The scale factor for length, L^*, is determined by the size of the prototype and the size of the available centrifuge containers. Though some researchers (e.g. Stewart et al. 1994), have scaled the density of the material used in the model, it is considered desirable to use identical materials in model and prototype to make it simpler to obtain identical mechanical properties such as friction angle and elastic moduli in model and prototype. If identical materials are used, the density of soil in the model will be identical to the density of soil in the prototype: $\rho^* = 1$. Given that σ^*, L^* and ρ^* are established as discussed above,

$$\sigma^* = 1 \tag{1}$$

$$L^* = \frac{1}{N} \tag{2}$$

$$\rho^* = 1, \tag{3}$$

the scale factor for gravity can be calculated as shown below:

$$\sigma_m = \rho_m\, g_m\, h_m \tag{4}$$

$$\sigma_p = \rho_p\, g_p\, h_p \tag{5}$$

$$\sigma^* = \frac{\sigma_m}{\sigma_p} = \rho^*\, g^*\, h^* \tag{6}$$

In the above, h represents a depth which will scale as any other length; $h^* = L^*$. Rearranging gives

$$g^* = \frac{\sigma^*}{(\rho^*\, L^*)} \tag{7}$$

As explained earlier, σ^* and ρ^* are usually unity, hence,

$$g^* = \frac{1}{L^*} \tag{8}$$

In other words, gravitational acceleration must be increased by the same factor ($g^* = N$) that lengths have been reduced ($L^* = \frac{1}{N}$).

If dynamic stresses are to scale as self weight stresses do, dynamic acceleration should scale as gravitational acceleration does, $a^* = g^* = N$. Considering the basic equation of kinematics:

$$L = 0.5\, a\, t^2 \tag{9}$$

provides the scale factor for time as follows:

$$L^* = a^*\, t^{*2} \tag{10}$$

$$t^* = \left(\frac{L^*}{a^*}\right)^{0.5} = \frac{1}{N} \tag{11}$$

If the rate of pore pressure dissipation is to be simulated in a reduced scale model, the consolidation time factor, T must be the same in model and prototype, providing $T^* = 1$;

$$T^* = \frac{(c_v^*\, t^*)}{L^{*2}} = 1 \tag{12}$$

Equation 12 may be rearranged as:

$$t^* = \frac{(L^{*2})}{c_v^*} \qquad (13)$$

If the same soil and pore fluid are used in model and prototype, then $c_v^* = 1$, and hence for consolidation problems:

$$t^* = L^{*2} = \frac{1}{N^2} \qquad (14)$$

In words, the duration of consolidation in the model is N^2 times less in the model than in the prototype.

Three obvious scaling problems may be apparent at this stage. Before discussing these problems in some detail, it is important to emphasize that the problems are critical if direct modeling of a particular prototype is required. If, however, the model test is viewed as an experiment instead of a direct model, conflicting scaling laws do not render useless the model test.

The scaling problems to be discussed are particle size effects, strain rate effects, and the conflict of time scale factors for dynamics and consolidation.

Time Scale Factor Conflict

If identical soil and pore water are used in model and prototype, the time scale factor for consolidation is different from that for dynamics, as can be seen by comparing equation 11 with 14. This is only a serious problem if the time scales for diffusion and dynamics are of the same order. Examples of cases where the time scale factor conflict is negligible are saturated clay which will not consolidate significantly during a dynamic event, and dry sand or gravel, for which dynamic pore pressures are negligible. It is known that the time required for re-consolidation of liquefied sand is similar to the duration of shaking in typical centrifuge models involving earthquake excitation. A significant amount of dissipation often occurs during shaking in the model, while in the prototype, dissipation during shaking may be unimportant. Arulanandan and Sybico (1992) have discussed how dissipation during shaking can significantly affect the magnitude of settlements induced by ground shaking.

It has often been argued that liquefaction phenomena may be studied without reference to a specific 1 g prototype. Hence, the scaling of permeability is an unnecessary complication. In cases where scaling of permeability is considered important, modelers have made some advances in the techniques for accomplishing this.

As is apparent from eq. 13, it is possible to slow down the consolidation so that the time scale factors for consolidation and for dynamics are both given by $t^* = \frac{1}{N}$. Two different methods have been used to slow down consolidation. The first is to model the pore water of the prototype using a viscous fluid in the model. If $\mu^* = N$, the coefficient of consolidation of the model soil is smaller than that of the prototype, $c_v^* = \frac{1}{N}$.

Glycerin and Silicon Oil have been found to be useful model pore fluids, but these two replacement fluids have certain disadvantages. The density of glycerin-water mixtures are significantly lower than the density of pure water; the buoyant density of soil will be increased, violating the assumption of $\rho^* = 1$. The density of silicon oil is a better match to that of water, and it can be obtained with a wide range of viscosity, but clean up and disposal of the oily sand has been found to be difficult. It is usually impractical to reuse the oily sand, and silicon oil is considered to be a hazardous waste.

An alternative viscous pore fluid can be obtained using Methyl Cellulose mixed with water. This chemical is often used as a food additive. This type of solution was proposed by Kimura (1993) and Ko (1994) presents some results of model tests using hydroxypropyl methylcellulose (metolose). Different grades of methyl cellulose can be obtained, which when mixed in a 1 to 4 percent solution with water, may increase the viscosity of water by factors of 10 to 1000. Because small concentrations of methyl cellulose are sufficient, the pore fluid retains many of the chemical characteristics of water (Ko, 1994); this may be especially important for tests involving silt or clay.

Preliminary tests have been conducted at Davis using a methyl cellulose obtained from Sigma Corporation. Initial experiments indicate that methyl cellulose may tend to clog pores in fine sands (e.g. Nevada Sand with a mean grain size of 0.15 mm. It has been found that the type of methyl cellulose tested at Davis will clog an 0.075 mm sieve, but will flow without clogging through a 0.15 mm sieve. Allard and Schenkeveld (1994) describe the use of a solution of unspecified composition that probably is a type of methyl cellulose. They conducted permeability tests on a fine sand and did not report any problem with clogging.

The second method for reducing the rate of consolidation of sandy model soils is to reduce the grain size. According to Hazen's equation, permeability is proportional to D_{10}^2. To achieve a factor of 50 reduction in permeability, D_{10} in the model should be the square root of 50, times finer than the soil of the prototype. From the perspective of consolidation and seepage, it may be argued that a coarse sand in the prototype can be modeled by a fine sand in the model. If particle size is scaled, it is difficult to ensure that the mechanical properties such as friction angle, Young's modulus, and the stress ratio to cause liquefaction will be the same as that of the prototype soil. If this could be ensured, then scaling of grain size might be preferable to scaling the pore fluid; this would reduce some particle size effects described in the next section.

Particle Size Effects

As will be seen in the discussion below, particle size effects do occur in scale model tests. It is a mistake, however, to believe often heard simplistic view: "A sand in the centrifuge represents a gravel in the prototype." A sand is a sand whether it is spinning or not. Interparticle contact forces (and hence particle deformations) depend on stress and the number of interparticle contacts per unit area, which depend on absolute particle size, not on the scaled particle size, and not on gravity! If the same soil is used in model and prototype, and the stresses are the same in model and prototype, then interparticle contact forces will be the same in model and prototype. From the perspective of the stress-strain behavior, the tendency for rolling, sliding, compression and crushing of particles is modeled best if the same particles are used in the centrifuge and the prototype.

Stone and Wood (1992), Kutter et al. (1994a), Bolton and Lau (1988), Tatsuoka et al. (1991), Yamaguchi et al. (1986), and Hettler and Gudehus (1981) among many other researchers have studied particle size effects in static centrifuge tests. In a strain hardening, stable soil, it is only necessary to require the model to be sufficiently large to ensure that a statistically significant number of particles are involved in the problem. For example, Fuglsang and Ovesen (1988) suggested that a model footing diameter ought to be 30 times the particle size for modeling to be accurate.

In strain softening materials, it is known that localization of shear strains occurs in both model and prototype. As explained by Bolton and Lau (1988) or Stone and Wood (1992), the thickness of a shear band is strongly influenced by the size of the particles. Roscoe's (1970) observation that a shear band in sand has a thickness of about ten grain diameters has been re-observed by many researchers. The rate of formation and softening of a shear band depends on the absolute relative displacement across the shear band, which does not scale if particle size is not scaled. Deformation of structures involving localization of strains in softening materials are expected to exhibit behavior which is sensitive to the characteristic size of the shear zone. In some geotechnical applications, the problem may be solved by scaling the particle size (e.g. Kutter 1994a), but it is not recommended to scale particle size without carefully ensuring that the "continuum" properties (for example, moduli) of the scaled and original soils are identical or unimportant.

Much empirical work has been done to study the effect of particle size on liquefaction susceptibility. It is known that liquefaction involves softening of soil, and important localizations such as cracks and boils are often observed as a result of liquefaction. It has been established that cracking and boils can be created in the centrifuge (e.g., Fiegel and Kutter 1994), but it has not yet been shown how these phenomena may be scaled to prototype dimensions. Model testing will improve our understanding of the mechanisms of liquefaction, including cracking and boiling, but direct extrapolation to prototype scale is at present, not validated.

It is difficult to characterize strain softening materials and localization phenomena in physical models, other laboratory tests, and even numerical models. The influence of particle size on liquefaction processes is not clearly understood at the prototype or the model scale. A shaking table on a centrifuge provides a tool which may re-create the basic mechanisms of liquefaction, a tool which has been found to be useful to study the fundamental mechanisms of pore pressure generation, redistribution, deformations, and particle size effects.

Rate Effects

A common opinion of geotechnical physical modelers is that the time dependent behavior cannot be modeled and hence, time dependent behavior will result in a lack of similitude. It will be argued below, as explained by Sathialingam and Kutter (1994), that rate dependent behavior of a model can be in similarity with rate dependent behavior in a prototype if the void ratio is slightly altered.

Stress, according to the above derived scaling laws, is not scaled in centrifuge model tests. As seen in eqs. 11 or 14, the duration (time) of an event in a centrifuge model is smaller than the duration of the corresponding event in the prototype, therefore, the rate of change of stress is greater in the centrifuge model as compared to the prototype. It is also clear that if geometry is to be identical in model and prototype, the strains must be identical, and to preserve similarity of strains while time is being scaled, results in scaling the rate inversely as time is scaled:

$$\dot{\sigma}^* = \dot{\varepsilon}^* = N \quad \text{for dynamic problems, and} \tag{15}$$

$$\dot{\sigma}^* = \dot{\varepsilon}^* = N^2 \quad \text{for consolidation problems} \tag{16}$$

Whitman (1957), Mesri and Castro (1987), Mitchell (1964), Adachi and Oka (1982) and many other researchers have demonstrated that the stress-strain behavior of clayey soils depends on the strain rate.

As strain rate increases by a factor of 10, the strength increases by about 5 to 10% (Craig 1982). For a typical centrifuge model, at a scale factor of N = 50, the dynamic strength of the model may therefore be about 5 to 15% greater than the strength of the prototype. The change in strength and stiffness caused by the scaling of time is in conflict with the basic premise that $\sigma^* = 1$ (eq. 1). As was the case for particle size effects, the strain-rate effects are especially important when attempting to directly model a specific prototype event. Rate effects are less critical when attempting to observe basic mechanisms of behavior, to compare relative merits of similar structures (parametric studies), or to verify a numerical procedure. In the case of verification of numerical procedures, the numerical model can be verified by comparison to model data, with recognition that the model is not a perfect simulation of a prototype; the constitutive law used in the numerical procedure should account for the differences in strain rate in model and prototype.

Recognizing the rate dependent mechanical properties of clay, Sathialingam and Kutter (1994) have proposed that the effects of the change in strain rate may be counteracted by a carefully determined change in the void ratio of the model. Scott (1988) and Iai (1989) made analogous proposals for 1 g model tests on sand. They suggested that the tendency for dilatancy and friction angle to increase at low stress levels could be counteracted by increasing the void ratio.

In the case of rate dependent behavior, the void ratio of the physical model may be increased in such a way that either the time dependent consolidation (secondary compression and creep) can be simulated or, in dynamic problems, the shear strength of the model soil may be reduced. Specifically, Sathialingam and Kutter (1994) proposed the following scaling law for void ratio:

$$e_{model} = e_{prototype} + C_\alpha \log \left(\frac{1}{t^*}\right) \quad (17)$$

Which, for consolidation problems becomes:

$$e_{model} = e_{prototype} + C_\alpha \log (N^2) \quad (18)$$

and, for dynamic problems

$$e_{model} = e_{prototype} + C_\alpha \log (N) \quad (19)$$

where C_α is the conventional coefficient of secondary compression, and N is the factor by which g is increased in the model. Derivations of scaling laws for void ratio were based on concepts of Bjerrum (1967), critical state soil mechanics (Schofield and Wroth, 1968), and visco plasticity.

Bjerrum suggested that the location of the normal consolidation line (NCL), as shown in Fig. 9, is time dependent. For a factor of 10 increase in time, the normal consolidation line shifts downward by an amount, $\Delta e = C_\alpha$.

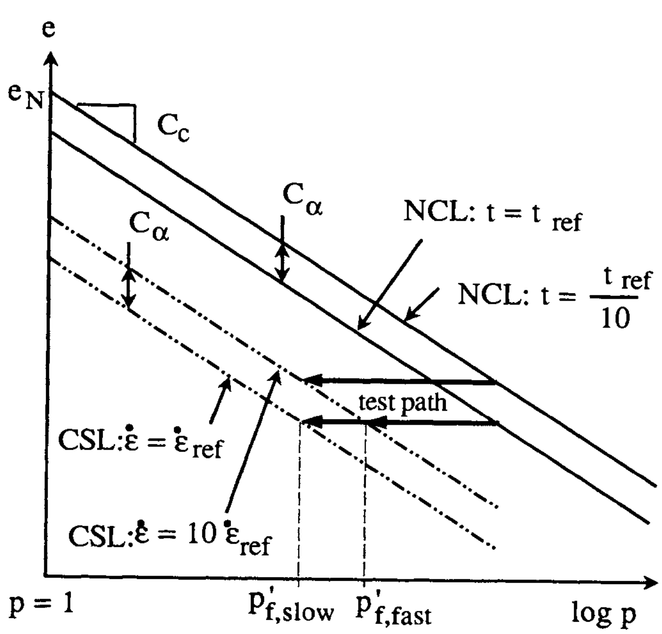

Fig. 9. Dependence of locations of NCL and CSL on time and strain rate.

The dependence on time may also be interpreted as a dependence on strain rate; for every factor of 10 decrease in strain rate, the normal consolidation line shifts downward by the same amount $\Delta e = C_\alpha$. In the model by Kutter and Sathialingam (1993) the location of the critical state line (CSL) is dependent on the rate of strain in the same way; the critical state line shifts downward by $\Delta e = C_\alpha$ as the rate of strain decreases tenfold as seen in Fig. 9. From Fig. 9 it can be seen that a factor of 10 change in strain rate leads to an increase in effective stress by a factor:

$$\frac{(p'_{\dot\varepsilon=10})}{(p'_{\dot\varepsilon=1})} = 10^{C_\alpha/C_c} \quad (20)$$

Kutter and Sathialingam assume that the critical state friction angle is independent of strain rate, hence the increase in strength is proportional to the increase in effective stress, given by eq. 20. Mesri and Castro (1987) suggests that for a majority of inorganic soft clays,

$$0.03 < \frac{C_\alpha}{C_c} < 0.05 \quad (21)$$

This ratio varies over a relatively small range. Inserting the extreme values of $\frac{C_\alpha}{C_c}$ from eq. 21 into eq. 20, provides the result that the increase in effective stress due to a tenfold increase of strain rate is in the range

$$1.07 < \frac{(p'_{\dot\varepsilon=10})}{(p'_{\dot\varepsilon=1})} < 1.12 \quad (22)$$

Which is in general agreement with the empirical data on the effect of strain rate on undrained shear strength (Craig 1982). The constitutive model proposed by Kutter and Sathialingam (1993) was tested by comparison with a variety of other types of test data from various soils. It is interesting that a property, C_α, measured in one dimensional consolidation test was found to be useful for quantification of the effect of strain rate on the strength in undrained shear.

Sathialingam and Kutter (1994) proposed the scaling law given in eq. 19 by following the implications of their "verified" constitutive model. The constitutive model is relatively simple, with only 7 parameters, and these parameters have already been proposed in conventional soil mechanics literature. The important conclusions of the work by Sathialingam and Kutter (1994), for centrifuge modelers are:

1) The strength increase due to strain rate increase can be counterbalanced by increasing the void ratio of the soil to be slightly greater than that of the prototype. It turns out to be fairly convenient to use reconstituted soils with appropriately increased void ratio by simply controlling the duration of secondary compression during preparation of the centrifuge models.

2) Secondary compression can be modeled in the centrifuge, just like it has been modeled in 1 dimensional consolidation tests in the past.

VELACS

A major collaborative research project, involving centrifuge model tests by 7 universities (Caltech, UC Davis, Cambridge University, RPI, University of Colorado, MIT, and Princeton) has recently been completed. The results of this study were compiled into a two volume symposium proceedings by Arulanandan and Scott (1993, 1994). The intention of VELACS is described by the words comprising the acronym: VErification of Liquefaction Analyses by Centrifuge Studies.

There were two phases to the VELACS project. The first phase was to demonstrate the reliability of centrifuge model tests by blind comparisons of centrifuge test results from different facilities. The first phase models consisted of a submerged, level layer of Nevada Sand covered with a layer of saturated silica flour. The models were prepared following a standard procedure specification and tested in identical boxes by different researchers using different shakers and centrifuges. The comparisons of pore pressure, acceleration, and settlement results of the first phase are described by Arulanandan et al. (1994). The conclusion of this comparison was that there was considerable scatter in the results. The scatter was attributed to differences in the input motions produced by the different centrifuge shaker systems as well as different soil preparation techniques. The method specified for placement of the silt resulted in different silt void ratios at different centrifuges. The differences in input motions can attributed to idiosyncrasies of various shaker systems: the soil-container-shaker interaction discussed earlier in this paper. Despite the explainable sources of scatter, it was felt that the test results were similar enough to justify the use of centrifuge data for evaluation of numerical procedures, which led to the second phase of VELACS.

The second phase of the VELACS project involved:

1. Development of specifications for a set of nine different centrifuge model tests involving liquefaction.
2. Conducting standard laboratory classification and triaxial tests on the soils to be used in the model tests.
3. Submission of "Class A" predictions of experiments based on results of 1 and 2 above.
4. Performance and repetition of the model tests at a primary and, in most cases, two other universities.
5. Comparison of predictions and experiments in a symposium.

It is difficult to give quantitative grades, and it is certainly inappropriate to declare a winner amongst the numerical methods; VELACS was a learning process, not a contest. Scott (1994a), upon reviewing the comparisons between the "class A" numerical predictions and the experimental results, stated "there is some indication that fully coupled codes do better than partially coupled codes, which in turn, perform better than uncoupled programs".

The resolve to improve the repeatability of centrifuge data (compared to that achieved in the first phase) by tighter specifications was counterbalanced by a trend that most of the second phase models were more complicated than the first phase control test; scatter in experimental results obtained at different universities was again apparent. As in the first phase, differences in input motion and sample preparation techniques could explain most of the scatter. The repeated tests were not duplicates of one and other. They were, however, generally consistent with each other.

On their own, the scatter in the centrifuge data may seem alarming. Any experimentalist who has engaged in comparison of blind experiments, however, is aware of the difficulty of obtaining better comparisons. Miura et al. (1994), compared results of cyclic triaxial tests on saturated Toyoura Sand (D_r = 70%) performed by thirty seven different laboratories. All data points for tests which did not meet specifications (more than half of the data) were then removed. The scatter of the remaining data suggest that

1. The stress ratio to cause 5% double amplitude strain in 5 cycles is somewhere between 0.17 and 0.26.
2. The number of cycles to cause 5% double amplitude axial strain is somewhere between 2 and 100 cycles.

There is variability of triaxial test results obtained at different laboratories, just like there is variability of centrifuge test data observed in the VELACS project.

Furthermore, the scatter in the VELACS experimental data is, on average, smaller than the scatter in the VELACS numerical predictions. This is illustrated by Table 2, adapted from Wilson et al. (1994), which summarizes the mean and standard deviation of a few key parameters from two different models sketched in Fig. 10. Comparisons are provided for vertical displacements measured at the crest and near mid-slope of a sand embankment with a silty core, and settlements of the top of a block which rests on level stratified ground.

Table 2. Statistics of some VELACS data and predictions.

	EXPERIMENTS			PREDICTIONS		
	num of samples	mean (cm)	Std. Dev.	num of samples	mean (cm)	Std. Dev.
Model 7 Crest Settlement	3	22	13	8	15	22
Model 7 Slope Settlement	3	15	3	8	2	6.7
Model 12 Structure Settlement	3	15	7	3	34	16

In the case of the embankment, the standard deviations of the predicted settlements are greater than the mean of the predicted settlements. It is recognized that the number of data samples used for calculation of the statistics are not as large as desired on a line by line basis. But the same pattern repeats itself for each of the 3 quantities compared

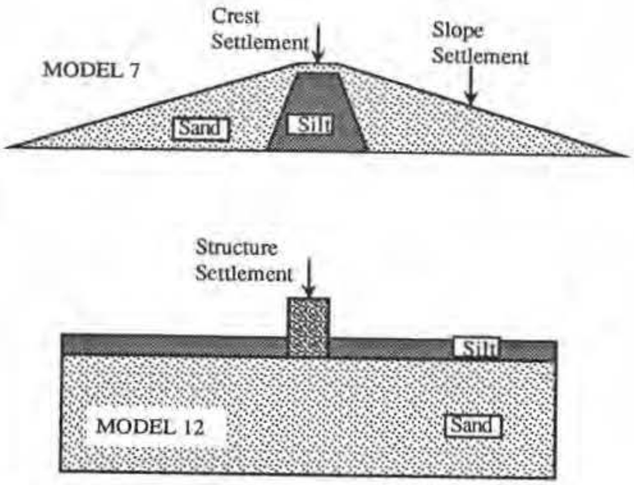

Fig. 10. Sketches of VELACS models 7 and 12.

in Table 2. The variability of centrifuge data is much less than the variability of different methods of prediction; theoreticians still have something to learn from centrifuge experiments.

SUMMARY

In the beginning, centrifuges were wonderful tools for discovery. Modelers were enthralled by the details and dramatic failures that could be reproduced in these machines; new failure mechanisms were discovered and new methods of analysis were devised. This type of exciting exploration will and ought to continue, but modeling has matured to a stage where we also need to produce complete sets of repeatable, reliable data in order to prove or disprove fundamental tenets of soil mechanics and to make an impact in the improvement of engineering practice.

Scott (1994) stated: "One might say that the old happy pioneering days of centrifuge testing vanished as soon as VELACS commenced. Before, we would think of an experiment and do it because of its intrinsic interest, without much concern about its replication or predictability. After its initiation, issues of quality control became paramount."

Progress is being made in understanding particle size effects and strain-rate effects. Techniques for scaling pore fluid viscosity are progressing. Centrifuges, shakers and model containers are becoming more versatile. With the progress in these fundamental areas, and with the scientific approach fostered by VELACS, to study the reliability of data by blind comparisons, we are advancing the science of geotechnical engineering.

Despite the difficulties, the centrifuge is especially useful in earthquake engineering. Real seismic events are costly but infrequent; full scale data is useful but inconclusive. Full scale seismic events are not repeatable. The centrifuge provides an environment which accurately reproduces most of the important physical processes that occur during seismic events. Centrifuge data is repeatable and useful for extending our knowledge of soil behavior under seismic loading.

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