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SIMPLIFIED EVALUATION FOR DYNAMIC LAYERED SOILS-STRUCTURE INTERACTION

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ABSTRACT

An analytical method is presented for the evaluation of dynamic soil-structure response under a simpler condition where a structure is semi-buried in level ground of layered soils. Formulas are proposed to determine the lateral soil-wall displacement and corresponding earth pressures against the sidewalls of the structure during an earthquake. The key factors affecting the dynamic response of the soil-structure system are also discussed.

INTRODUCTION

There are generally two kinds of solution for the evaluation of dynamic soil-structure interaction: complete solutions and numerical solutions. The complete solutions may be regarded as a closed-form, because they can be used to any desired degree of accuracy. So the complete solutions for evaluating dynamic soils-structure interaction have been studied a lot and some methods have been developed since Reissner (1936) obtained the first one. For example, Sun et al. (1993) suggested a simplified complete solution for the response of soils-pile system subjected to a harmonious concentrated load imposing on the pile top. Militano et al. (1999) proposed an analytical method to calculate the dynamic response of an elastic pile under transient torsional and axial loading, in which the dynamic equilibrium equation of the soils is solved in the Laplace domain. These existing solutions however concentrate mainly on the structures excited by dynamic loads.

In practice, the properties of layered soils vary considerably. The inhomogeneities and stratification of the soil deposits surrounding a structure have great effects on the dynamic response of the structure. Sarma (1994) developed the complete solutions for one-dimensional shear response of a visco-elastic soil system of multiple homogeneous layers. The authors (2000) presented a simplified method, for analyzing dynamic homogeneous soil-structure interaction in an earthquake, in which the response of the soils directly excited by an earthquake and its effect on the structure are properly considered.

This paper presents a new, analytical solution for the evaluation of dynamic soils-structure response under a simpler condition where a structure buried in level layered ground. A new method of evaluation is further developed based on the following two formulas: 1) a complete solution for the dynamic response of multi-layer soils to an earthquake (Sarma, 1994); 2) a seismic earth pressure theory for retaining structure under any lateral displacement (Zhang et al., 1998).

ANALYSIS

One-Dimensional Governing Equation

In general, the soils displace far more than the underground structure in an earthquake, because the shear moduli of the soils are far less than that of the structure. In addition, the dimensions of the soils are much larger than those of the structure. As a result, the influence of the structure on the deformation of soils may be ignored in some cases, as a simplified treatment with reasonable accuracy. For a structure semi-buried in level layered soil ground, as shown in Fig. 1, the following simplifications and assumptions were made: 1) the soils and the structure behave as visco-elastic mediums; 2) the soils and the structure can be simplified as a group of shear beams which displace horizontally only; 3) the soils and the structure are homogeneous within the same layer; 4) the structure is fixed on the base rock; 5) seismic excitation is due to shear waves propagating vertically; 6) the dynamic interaction coefficient showing the relationship between the earth pressures and the displacements at the interface of a structure and its surrounding soils in shaking is taken as a constant, that is

$$p = k\Delta \qquad (1)$$

where Δ is the relative displacement between the soil and the structure; p is the deviatoric earth pressure; and k is the dynamic interaction coefficient. Based on these assumptions, the soil-structure system was treated as a one-dimensional problem. The orthogonal coordinate system with the origin at the midpoint of the top of the structure is set up. According to their mechanical and geometrical properties, the structure and the soils are divided into M and $(M\text{-}L)$ layers respectively.

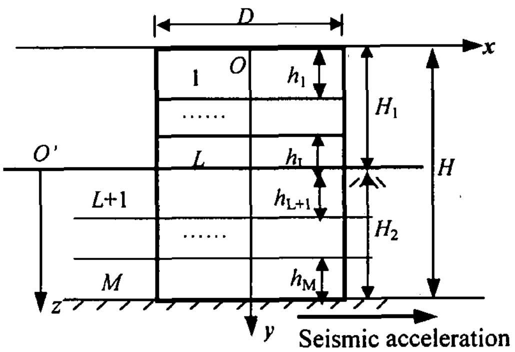

Fig. 1 A structure semi-buried in layered level ground

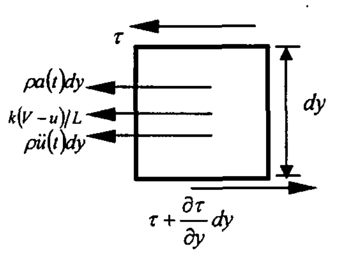

Fig. 2 Dynamic equilibrium analysis for a structure

Assume that the shear stress τ can be expressed by the relation

$$\tau = G\left(\frac{\partial V}{\partial y} + \eta\frac{\partial^2 V}{\partial y \partial t}\right) \qquad (2)$$

where η is the strain-rate damping coefficient. For the structure subjected to a base acceleration $a(t)$, the dynamic equilibrium governing equation considering soils-structure interaction can be derived by analyzing the equilibrium of forces acting on a microelement of the structure, i.e.,

$$\frac{\partial^2 V_i}{\partial t^2} + \frac{k_i}{\rho_i D_i}(V_i - u_i) + a(t) = S_i^2\left(\frac{\partial^2 V_i}{\partial y^2} + \eta_i \frac{\partial^2 \dot{V}_i}{\partial y^2}\right) \qquad (3)$$

with the boundary conditions: 1) zero stress condition at the free surface, 2) no relative motion at the base rock level; and 3) continuity of the shear stress and the displacement at the interfaces of layers. The initial conditions are at rest and can be expressed in the following form.

For t>0:

$$y = 0,\quad \frac{\partial V_i}{\partial y} = 0;\quad y = H,\quad V_i = 0;\quad i = 1,2,\cdots,M \qquad (4)$$

$$y = \sum_{k=1}^{i} h_k,\quad V_i = V_{i+1},\quad \tau_i = \tau_{i+1},\quad i = 1,2,\cdots,M-1; \qquad (5)$$

For $H \geq y \geq 0$:

$$t = 0,\quad V_i = \dot{V}_i = 0,\quad i = 1,2,\cdots,M \qquad (6)$$

In these equations, the subscript i refers to the ith layer; V_i and u_i are, respectively, the horizontal displacements of the structure and the soils relative to the base, which are a function of both time and height of the layer; h_i is the height; D_i is the horizontal width; ρ_i is the mass density; S_i is the shear wave velocity; G_i is the shear modulus given by $G = S^2\rho$; and k_i is the dynamic interaction coefficient.

Analytical Solution

The governing partial differential equations (3)-(6) can be simplified as a one-dimensional shear vibration equation of motion or equation (7), if horizontal layered soils rest on a rigid base, the dynamic interaction coefficient k is set zero, the parameters of the structure are replaced with those of the soils, and the origin of the local coordinate system is moved to the intersection of structure and ground, as shown in Fig. 1.

$$\frac{\partial^2 u_i}{\partial t^2} + a(t) = S_{s,i}^2\left(\frac{\partial^2 u_i}{\partial z^2} + \eta_{s,i}\frac{\partial^2 \dot{u}_i}{\partial z^2}\right) \qquad (7)$$

For $t > 0$:

$$y = 0,\quad \frac{\partial u_i}{\partial z} = 0;\quad y = H_2,\quad u_i = 0;\quad i = L+1,\cdots,M \qquad (8)$$

$$y = \sum_{k=L+1}^{i} h_k,\quad u_i = u_{i+1},\quad \tau_i = \tau_{i+1},\quad i = L+1,\cdots,M-1; \qquad (9)$$

For $H_2 \geq y \geq 0$

$$t = 0,\quad u_i = \dot{u}_i = 0\quad i = L+1,\cdots,M-1 \qquad (10)$$

where the subscript s indicates the soil.

For the specific case of η being the same for all layers, a closed-form solution has been obtained by Sarma (1994)

$$u_i(z,t)=\sum_{j=1}^{\infty}\varphi_{j,i}(z)I_j(t) \qquad (11)$$

where $\varphi_{j,i}(z)$ is the mode shape for the mode j and the ith layer, and I_j is Duhamel's integral of the response of a single degree of freedom system with frequency $\omega_{S,j}$ and damping $\lambda_{S,j}$:

$$I_j=-\frac{1}{\omega_{S,d,j}}\int_0^t a(\tau)e^{-\lambda_{S,j}\omega_{S,j}(t-\tau)}\sin[\omega_{S,d,j}(t-\tau)]d\tau \qquad (12)$$

in which $\omega_{S,j}$ and $\omega_{S,d,j}$ are respectively the undamped and damped natural frequency of the soils for the mode j, i.e.,

$$\lambda_{S,j}=\eta_S\omega_{S,j}/2 \qquad (13)$$

$$\omega_{S,d,j}=\omega_{S,j}\sqrt{1-\lambda_{S,j}^2} \qquad (14)$$

The specific expression of $\varphi_{j,i}(z)$ and the solution for $\omega_{S,j}$ can be found in the paper already presented by Sarma (1994). In particular, for the homogeneous soil,

$$\omega_{S,j}=(2j-1)\pi\sqrt{G/\rho}/2H_2 \qquad (15)$$

where G and H_2 are the shear modulus and height respectively.

If the obtained displacement of the soils is substituted to the governing equations (3)-(6), and then let

$$\bar{k}_i=\frac{k_i}{\rho_i L_i}V_i \qquad (16)$$

where $\bar{k}_i$ is defined as the effective dynamic interaction coefficient, Eq. (3) can be rewritten as follows.

$$\frac{\partial^2 V_i}{\partial t^2}+\bar{k}_iV_i-\bar{k}_iu_i-a(t)=S_i^2\left(\frac{\partial^2 V_i}{\partial y^2}+\eta\frac{\partial^2 \dot{V}_i}{\partial y^2}\right) \qquad (17)$$

In comparison with Eq. (7), there is a new item $\bar{k}_iV_i$ in Eq. (17), which indicates the influence of dynamic soils-structure interaction on the response of the structure. The items $\bar{k}_iu_i$ and $a(t)$ show the effects of the kinematic and inertial interaction. Their corresponding displacements relative to the base of the structure are denoted as $\bar{V}(y,t)$ and $\tilde{V}(y,t)$ respectively. According to the principle of superposition, we can write

$$V(y,t)=\bar{V}(y,t)+\tilde{V}(y,t) \qquad (18)$$

For the specific case of $\bar{k}_i$ being the same for all the layers, a closed-form solution can be obtained.

The solution corresponding to the item $a(t)$ is

$$\tilde{V}_i(y,t)=\sum_{j=1}^{\infty}[\phi_{j,i}(y)\tilde{J}_j(t)] \qquad (19)$$

where the expression of $\phi_{j,i}(y)$ is the same to that of $\varphi_{j,i}(z)$ as long as the parameters of the soils are replaced with those of the structure. $\tilde{J}_j(t)$ can be expressed as Duhamel integral

$$\tilde{J}_j=-\frac{1}{\tilde{\omega}_{d,j}}\int_0^t a(\tau)e^{-\lambda_j\tilde{\omega}_j(t-\tau)}\sin[\tilde{\omega}_{d,j}(t-\tau)]d\tau \qquad (20)$$

$$\tilde{\omega}_j=\sqrt{\omega_j^2+\bar{k}_I{}^2} \qquad (21)$$

$$\lambda_j=\eta\omega_j/2 \qquad (22)$$

$$\tilde{\omega}_{d,j}=\tilde{\omega}_j\sqrt{1-\lambda_j^2} \qquad (23)$$

where ω_j 和 λ_j are respectively the undamped natural frequency and the damping corresponding to inertial interaction for the mode j; $\tilde{\omega}_j$ and $\tilde{\omega}_{d,j}$ are, respectively, modified undamping and damping natural frequency corresponding to inertial interaction of the structure for the mode j; ω_j can also be obtained by the same way as used by Sarma (1994); and $\bar{k}_I$ is the effective dynamic interaction coefficient corresponding to inertial interaction. The determination of $\bar{k}_I$ will be discussed in the next section.

To obtain the solution corresponding to the item $\bar{k}u_i$, the variable substitution was made, and the solution is got as

$$\begin{cases}\bar{V}_i(y,t)=\sum_{j=1}^{\infty}[\bar{\phi}_{j,i}(y-H_1)\bar{J}_j(t)] & H_1\le y\le H\\ \bar{V}_i(y,t)=\sum_{j=1}^{\infty}[\bar{\phi}_{j,i}(0)\bar{J}_j(t)] & 0\le y\le H_1\end{cases} \qquad (24)$$

where $\bar{\phi}_{j,i}(y)$ is determined for the buried section of structure ($H_1\le y\le H$), that is, as shown Fig. 1, the origin of the local coordinate system is moved to the intersection of structure and ground, then $\bar{\phi}_{j,i}(y)$ can be calculated by the same way as used by Sarma (1994). $\bar{J}_j(t)$ can be expressed by Duhamel integral

$$\bar{J}_j=-\frac{\bar{k}}{\bar{\omega}_{d,j}}\sum_{k=0}^{\infty}\beta_{j,k}\int_0^t I_k(\tau)e^{-\hat{\lambda}_j\bar{\omega}_j(t-\tau)}\sin[\bar{\omega}_{d,j}(t-\tau)]d\tau \qquad (25)$$

The coefficient $\beta_{j,k}$ can be determined by

$$\varphi_{j,i}(y)=\sum_{k=0}^{\infty}\beta_{j,k}\bar{\phi}_{k,i}(y) \qquad (26)$$

In particular, define

$$I_0(t)=1\,, \qquad \phi_{0,i}(y)=1 \qquad (27)$$

$$\bar{\omega}_j=\sqrt{\hat{\omega}_j^2+\bar{k}_K{}^2} \qquad (28)$$

$$\hat{\lambda}_j=\eta\hat{\omega}_j/2 \qquad (29)$$

$$\bar{\omega}_{d,j}=\bar{\omega}_j\sqrt{1-\hat{\lambda}_j^2} \qquad (30)$$

where $\hat{\omega}_j$ and $\hat{\lambda}_j$ are respectively the undamped natural frequency and the damping corresponding to kinematic interaction of the structure for the mode j; $\bar{\omega}_j$ and $\bar{\omega}_{d,j}$ are respectively the modified undamping and damping natural frequency corresponding to kinematic interaction for the mode j; $\hat{\omega}_j$ can also be obtained from the same way as used by Sarma(1994); and $\bar{k}_K$ is the effective dynamic interaction coefficient corresponding to kinematic interaction. The determination of $\bar{k}_K$ will be discussed in the next section.

Application

Practical seismic waves are complex and random. To get Duhamel integral's analytical solution, a base input seismic acceleration history can be expressed with finite Fourier series as follows.

$$a(t) = \frac{A_0}{2} + \sum_{k=1}^{(N-1)/2}\left[A_k\cos(D_k t) + B_k\sin(D_k t)\right] \tag{31}$$

$$A_k = \frac{2}{N}\sum_{m=0}^{N-1} a_m \cos\frac{2\pi km}{N} \qquad B_k = \frac{2}{N}\sum_{m=0}^{N-1} a_m \sin\frac{2\pi km}{N}$$

$$D_k = \frac{2\pi k}{N\Delta t} \qquad k = 0,1,2,\cdots,\frac{N-1}{2} \tag{32}$$

where a_m is the acceleration value of a sampling point at the time t_m and N is the number of all sampling points. Duhamel integral can be explicitly integrated when a sine or cosine wave is input. Thus the complete solution can be obtained for the condition where a practical earthquake acceleration history is used as input seismic wave. $I_j(t)$ may be dealt with in a similar approach.

In addition, when the soils are homogeneous,

$$\varphi_{j,i}(y) = \bar{\phi}_{k,i}(y) \tag{33}$$

$\bar{J}_j$ can be simplified as

$$\bar{J}_j = -\frac{\bar{k}}{\bar{\omega}_{d,j}}\int_0^t I_k(\tau)e^{-\lambda_j\bar{\omega}_j(t-\tau)}\sin\left[\bar{\omega}_{d,j}(t-\tau)\right]d\tau \tag{34}$$

This equation is also approximately applicable for the case of layered soils. Obviously, this disposal will produce errors and the errors can become significant when the properties of the soils and the structure vary at a different tendency.

The present solution can be applied to the case where the structure is not built on the base rock. In this case, the structure is assumed to be fixed on the soil layer at the bottom level of the structure. The reference system with the origin at the bottom of the structure (where the displacement of the soil is u_0) is set. The displacement of the soil, u_i, is replaced with the displacement relative to the origin ($u_i + u_0$), and the input acceleration a is replaced with $a + \ddot{u}_0$ in the analysis using the above approach.

PARAMETER

Damping

The present solution is derived under the assumption that the strain-rate damping coefficient of the soils or the structure is the same for all layers. This means that the damping increases with high modes of vibration. Therefore an over-damped situation for very high modes, which appears to be unrealistic, may be produced. However, in common cases, the dynamic response of the soils or the structure will decrease rapidly as the order of the mode increases. The over-damping for high modes has little effect on the analytical results. Moreover, the value of the damping used in the solution is an artificial one determined from an equivalent system.

Effective dynamic interaction coefficient

Based on extending a seismic earth pressure theory for retaining structure under any lateral displacement developed by Zhang (1998), it was found that the dynamic interaction force between the structure and its surrounding soil is approximately proportional to their relative displacement in seismic shearing. So a new method for determination of the dynamic interaction coefficient was derived here. The dynamic interaction coefficient at the height y can be expressed as

$$k = \gamma z k_{ES} + \gamma(H_2 - z)k_{ED} \tag{35}$$

$$\begin{cases} K_{ES} = \cos i\left(K_{passive} - K_{active}\right) \\ K_{ED} = (1-\cos i)\left(K_{passive} - K_{active}\right) \end{cases} \tag{36}$$

$$K_E = \frac{2\cos^2(\phi'-i)}{\cos^2(\phi'-i)(1+R) + \cos i\cos(\delta_{mob}+i)(1-R)\left[1+\sqrt{\frac{\sin(\phi'+\delta_{mob})\sin(\phi'-i)}{\cos(\delta_{mob}+i)}}\right]^2}$$

$$(-1.0 \le R \le 1.0)$$

$$K_E = 1 + \frac{1}{2}(R-1)\left(\frac{\cos^2(\phi'-i)}{\cos i\cos(\delta_{mob}+i)\left[1-\sqrt{\frac{\sin(\phi'+\delta_{mob})\sin(\phi'-i)}{\cos(\delta_{mob}+i)}}\right]^2} - 1\right)$$

$$(1.0 \le R \le 3.0) \tag{37}$$

$$\tan i = k_h \tag{38}$$

$$R = \begin{cases} -\left(\frac{|\Delta|}{\Delta_a}\right)^a & (-\Delta_a \le \Delta \le 0); \\ -1 & (\Delta < -\Delta_a) \end{cases} \quad R = \begin{cases} 3\left(\frac{|\Delta|}{\Delta_p}\right)^p & (0 \le \Delta \le \Delta_p) \\ 3 & (\Delta > \Delta_p) \end{cases} \tag{39}$$

$$\delta_{mob} = \left|\frac{1-R}{2}\right|^{0.5}\delta \tag{40}$$

where K_{active} and $K_{passive}$ are respectively the earth pressure coefficients at the active and passive side, respectively, mainly depending on the relative wall-soil displacement and can be calculated by Eq. (37) ; Δ_a and Δ_p are respectively the absolute value of the minimum wall-soil displacement required to develop the active or passive state; φ' is the angle of internal friction of the soil; and δ is the wall friction angle mobilized at the active or passive state.

Take a series of k_h, and then calculate the values of k using the following steps: 1) Determine φ', φ_h', Δ_p / Δ_a, and calculate the values of K_{ES} and K_{ED} for different Δ–values by Eq. (36); 2) Determine the two linear relationships between Δ and K_{ES} and between Δ and K_{ED}, and then by curve fitting, obtain the values of k_{ES} and k_{ED} that are respectively the slopes of the two relation curves; 3) Calculate the dynamic interaction coefficient by Eq. (35). As a result, the relationship between the dynamic interaction coefficient k and horizontal seismic acceleration coefficient k_h can be obtained.

According to Eq. (16) and Eq. (35), the effective dynamic interaction coefficient $\bar{k}$ is a function of height z. In general, $\bar{k}$ is not constant for each homogeneous layer; moreover, $\bar{k}$ is also a function of time t, because it is relevant to specific input acceleration history. To determine $\bar{k}$ in the solution, the value of $\bar{k}$ at the midpoint of all the soil layers is taken as an approximate value of $\bar{k}_K$. $\bar{k}_I$ can be determined as follows.

$$\bar{k}_I = \bar{k}_K \cdot H_2 / (H_1 + H_2) \tag{41}$$

The above simplified evaluation sometimes has a harmful effect on the accuracy of the structure's response, especially when kinematic interaction becomes significant. The displacements of the soils are commonly far more than those of the structure and therefore the influence of the simplification on the accuracy of earth pressures is very small.

DISCUSSION

Dynamic soils-structure interaction

To understand how much dynamic soil-structure interaction has effects on the dynamic response of the structure, a comparison was carried out. In the comparison, two conditions considering and not considering dynamic interaction were selected. For the reason that the seismic wave can be expressed as a sum of sine and cosine waves, a sine wave with a period of 0.2 s and an amplitude of 2 m/s^2 was selected here as a simple example of calculation. The duration of vibration is 4 s. The structure and the soils are both 4 meters high and the structure is 10 meters wide. $\delta = 20°$, $\Delta_a = 0.02m$, $\Delta_a = 0.2m$. Other parameters for the calculation are shown as Table 1.

The response acceleration time histories for the above two conditions are shown in Fig. 3. It is found that not only the peak value, but also the period of the response acceleration obviously tend to reduce if the dynamic interaction is considered in the calculation. The distribution of the displacement, acceleration and shear stress with height is shown in Fig. 4. It can be seen that the dynamic interaction obviously reduced the maximum displacement, acceleration and shear stress of the structure. This implies that the dynamic interaction should be properly considered in the analysis of the dynamic response of a soils-structure system.

Table 1 Parameters of structure and soils

	ρ/g·cm^{-3}	G/MPa	λ	φ' / °
Soils	1.5	5.0	0.2	40
Structure	2.5	1000	0.1	/

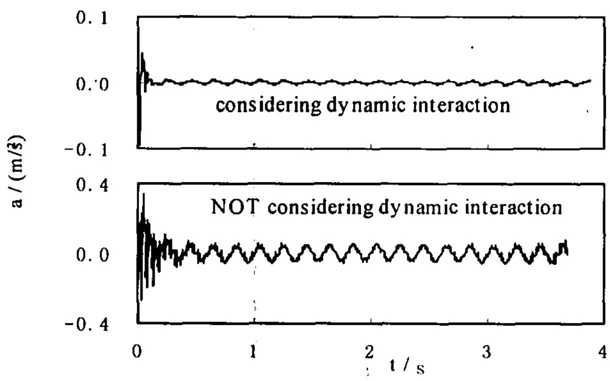

Fig. 3 response Acceleration on top of structure

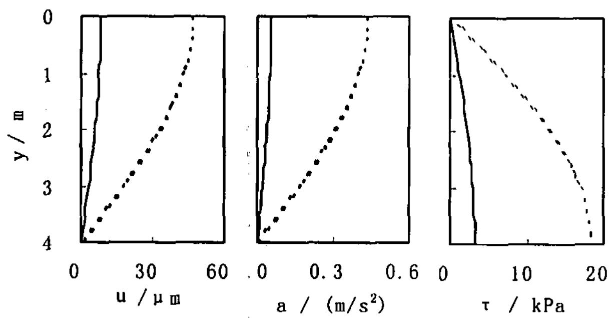

Fig. 4 Peak value of dynamic response of the structure considering dynamic interaction (continuous); Not (dashed:)

Characteristics of soil layers

Five cases were calculated in order to examine the influence of the characteristics of layered soils on the dynamic interaction. In these cases, the height h_i and shear modulus G_i are chosen in such a way that the average value $\overline{G}$ of the shear moduli is the same for all the cases, while the total height is four meters. The mass densities and all other parameters are assumed the same as shown in Table 1. The average shear modulus $\overline{G}$ is given by

$$\overline{G} = \sum_{k=1}^{n} G_k h_k \Big/ \sum_{k=1}^{n} h_k \tag{42}$$

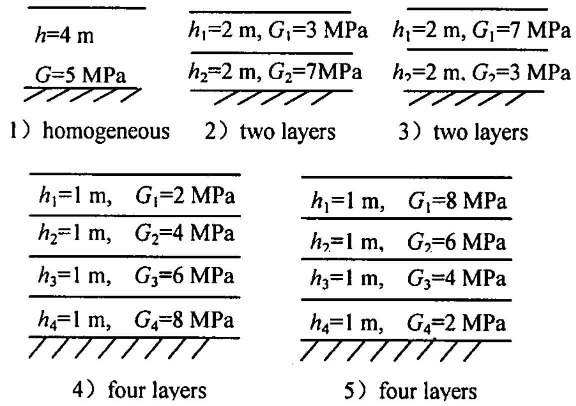

Fig. 5 the soils for calculation

The distribution of the displacement and acceleration as well as the deviatoric dynamic earth pressure with height is shown in Fig. 6. It is found that the displacements of the structure for all cases differ a little, but the deviatoric dynamic earth pressure and response acceleration varied greatly for all cases. For the case 5 where the shear modulus is larger in the upper layer, the response acceleration is less and the deviatoric

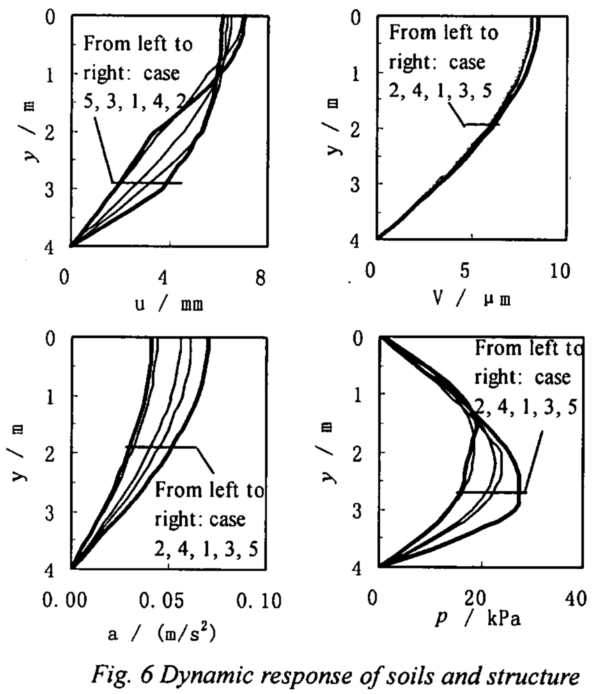

Fig. 6 Dynamic response of soils and structure

dynamic earth pressure is larger. This is because the shear modulus of the structure is far larger than that of the soils, and thus, the deformation of the soils has little effect on the response of the structure, while the deviatoric earth pressure is dependent on the relative displacement between the structure and its surrounding soils. It is shown that the difference in the properties of the layered soils should be considered carefully.

CONCLUSIONS

This paper presents a new analytical method for the evaluation of dynamic soils-structure response under a simpler condition where a structure is semi-buried in level ground of layered soils. By this method, the relative soil-structure displacement and the seismic earth pressure against the sidewalls of the structure at any time and any height can be calculated for practical seismic input waves and layered soils.

Dynamic soils-structure interaction is properly considered in the analysis of the response of the semi-buried structure in layered soils to an earthquake. The properties of soil layers can significantly change the peak value and distribution of the response acceleration of the structure and the seismic earth pressure against the structure.

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