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ASSESSING PROBABILISTIC METHODS FOR LIQUEFACTION POTENTIAL EVALUATION – AN UPDATE

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ABSTRACT

This paper presents an assessment of existing probabilistic methods for liquefaction potential evaluation. Emphasis is placed c comparison of probabilities of liquefaction calculated with four different methods. Two of these methods are based on SPT, and tl other two are based on CPT. In both SPT- and CPT-based evaluations, logistic regression and Bayesian techniques are applied to ma factor of safety to probability of liquefaction. The present study shows that the Bayesian approach yields more conservative resul than does the logistic regression approach, although results from the two approaches are quite comparable. Discussion of tl procedure for risk-based liquefaction potential evaluation is also presented.

INTRODUCTION

Site-specific probabilistic assessment of earthquake-induced liquefaction potential of soils using in situ test data is an important task facing geotechnical engineers. Results of such probabilistic assessments may lead to better engineering decisions.

Liao et al. (1988) provided a foundation for probabilistic assessments of liquefaction potential based on logistic regression analyses of the SPT-based field performance records. Youd and Noble (1997) extended this approach by considering earthquake magnitude in the logistic regression analysis and using an extended database. Toprak et al. (1999) conducted logistic regression analyses of the field performance database compiled by the U.S. Geological Survey. The study by Toprak et al. (1999) also extended the approach to deal with CPT-based database.

Unlike the aforementioned approach, Juang et al. (1999; 2000a) used reliability analysis and Bayes' theorem to derive the probability of liquefaction. With their approach, a Bayesian mapping function that relates reliability index and/or factor of safety to the probability of liquefaction was first developed based on field performance data. The Bayesian mapping function can then be used to predict the probability of liquefaction for a given set of site specific information.

Juang and Jiang (2000) extended earlier studies on Bayesian mapping function, and found that mapping function could be developed using the distributions of the calculated factors of safety, instead of reliability indexes. They have developed Bayesian mapping functions for the SPT-based method l Seed et al. (1985), and for the CPT-based method l Robertson and Wride (1998). Juang and Jiang (2000) al compared probability curves developed from their Bayesi mapping functions with those obtained by Toprak et al. (199 from logistic regression.

Juang and Jiang (2000) adopted the magnitude scaling fact (MSF) defined by Idriss (1999). The mapping functions th developed were compared to logistic regression equatio developed by Toprak et al. (1999) using different sets of d than those used for the development of the mapping functio In the present paper, the work by Juang and Jiang (2000) refined. Here, the MSF recommended by the 1996 Natio Center for Earthquake Engineering Research (NCEE workshop (Youd and Idriss, 1997) is followed. In additic the logistic regressions are performed using the same data s as used in the development of mapping functions.

For the convenience of description, the CPT-based method Robertson and Wride (1998) is referred to hereinafter as CP RW method. The updated Seed and Idriss (1971) SPT-bas method, as presented in Youd and Idriss (1997), is referred hereinafter as SPT-SI method. The CPT-RW and SPT methods are the only two deterministic methods considered the present study for developing mapping functions, althou other deterministic methods may be used. The Bayes mapping functions developed based on the CPT-RW and SP SI methods are referred to herein as the CPT- and SPT-ba Bayesian mapping functions, respectively.

DETERMINISTIC METHODS

In the liquefaction evaluation, the seismic load is generally expressed in terms of a cyclic stress ratio (CSR), which may be calculated as (modified from Seed and Idriss, 1971):

$$CSR_{7.5} = 0.65\left(\frac{\sigma_v}{\sigma'_v}\right)\left(\frac{a_{max}}{g}\right)(r_d)/MSF \qquad (1)$$

where σ_v is the total vertical stress at the depth in question, σ'_v is the effective vertical stress at the same depth, a_{max} is the peak horizontal ground surface acceleration, g is the acceleration due to gravity, MSF is the magnitude scaling factor, and r_d is the stress reduction factor. The term MSF is used to adjust the calculated CSR to the reference earthquake magnitude of 7.5. Note that the convention for adjusting the effect of earthquake magnitude is to modify the cyclic resistance ratio (CRR) with MSF. However, it is more logical to include MSF in the calculation of CSR, since both are seismic load parameters, whereas CRR represents soil resistance (Juang et al., 2000b).

The term r_d provides an approximate correction for flexibility of the soil profile. In this study, the values of r_d are calculated using the Liao et al. (1988) equation:

$$r_d = 1.0 - 0.00765\,z, \text{ for } z \le 9.15 \text{ m} \qquad (2a)$$

$$r_d = 1.174 - 0.0267\,z, \text{ for } 9.15 \text{ m} < z \le 23 \text{ m} \qquad (2b)$$

The parameter MSF is calculated as:

$$MSF = (M_w/7.5)^{-2.56} \qquad (3)$$

The above formulation for MSF represents the lower bound of the range of MSF values recommended in the 1996 NCEER workshop (Youd and Idriss, 1997). According to Juang et al. (2000b), however, choice of a particular MSF formula (and r_d formulation too) is not critical to the developed Bayesian mapping function. In the present study, this theory is further examined.

The liquefaction resistance of a soil, expressed as cyclic resistance ratio (CRR), may be calculated based on the SPT-SI method (Youd and Idriss, 1997):

$$CRR_{7.5} = \frac{a + cx + ex^2 + gx^3}{1 + bx + dx^2 + fx^3 + hx^4} \qquad (4)$$

where a = 0.048, b = −0.1248, c = −0.004721, d = 0.009578, e = 0.0006136, f = −0.0003285, g = −0.00001673, and h = 0.000003741. The variable x in Equation 4 is the clean sand equivalence of the corrected SPT blow count, $(N_1)_{60,cs}$ defined in Youd and Idriss (1997).

In the CPT-RW method, CRR is calculated by the following equation:

$$CRR_{7.5} = 93(q_{c1N,cs}/1000)^3 + 0.08, \text{ if } 50 \le q_{c1N,cs} < 160 \qquad (5a)$$

$$CRR_{7.5} = 0.833(q_{c1N,cs}/1000) + 0.05, \text{ if } q_{c1N,cs} < 50 \qquad (5b)$$

where $q_{c1N,cs}$ is the clean sand equivalence of the stress-corrected cone tip resistance defined by Robertson and Wride (1998). When using Equation 5, the limiting upper value of $q_{c1N,cs}$ is 160.

LOGISTIC REGRESSION

For SPT-based logistic regression, 233 data points were used. These data are the same data as those used in the development of the SPT-based Bayesian mapping function (Juang et al., 2000b). They were taken from a database of field performance cases compiled by Fear and McRoberts (1995). Performing a logistic regression analysis of these data yields the following probability equation:

$$ln\,[P_L/(1-P_L)] = 10.1129 - 0.2572\,(N_1)_{60,cs} + 3.4825\,ln\,(CSR_{7.5}) \qquad (6)$$

where P_L is the probability of liquefaction. The Nagelkerke coefficient (equivalent to R^2) of this regression is 0.49, and the success rates in classifying liquefied and non-liquefied cases are 130/150 (or 87%) and 61/93 (or 66%), respectively. Figure 1 shows a set of probability curves defined by Equation 6. Also shown in this figure is the deterministic boundary curve defined by Equation 4 (the SPT-SI method). The deterministic boundary curve is seen to be characterized with probabilities ranging from 30% to 50%. This finding agrees well with the findings of Liao et al. (1988) and Youd and Noble (1997). However, the probabilities obtained from logistic regression are influenced by the form of the function adopted for the regression, and as such, these probabilities should be viewed with caution.

Similarly, a probability equation was established based on a logistic regression analysis of a CPT database that was used by Juang et al. (2000c) in developing their ANN-based mapping functions. Performing a logistic regression analysis of these data yields the following probability equation:

$$ln\,[P_L/(1-P_L)] = 12.4259 - 0.0498\,q_{c1N,cs} + 3.9887\,ln\,(CSR_{7.5}) \qquad (7)$$

The Nagelkerke coefficient of this regression is 0.65, and the success rates in classifying liquefied and non-liquefied cases are 107/119 (or 90%) and 62/81 (or 76%), respectively. Figure 2 shows a set of probability curves defined by Equation 7. Also shown in this figure is the deterministic boundary curve defined by Equation 5 (CPT-RW method). The deterministic boundary curve is seen to be characterized with probabilities ranging from 50% to 70%. This finding agrees quite well with the finding of Toprak et al. (1999) that characterizes the CPT-RW method with a 50% probability, based on their logistic regression analysis of about 50 field cases from the Loma Prieta, California, earthquake.

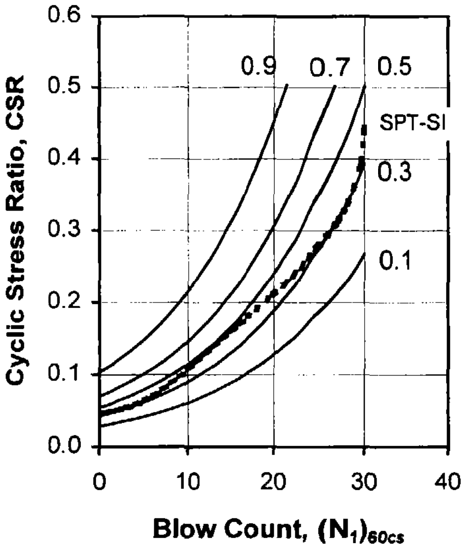

Figure 1. SPT Probability Curves - Logistic

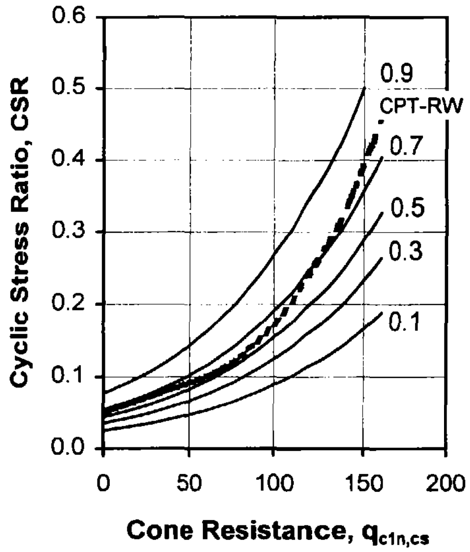

Figure 2. CPT Probability Curves - Logistic

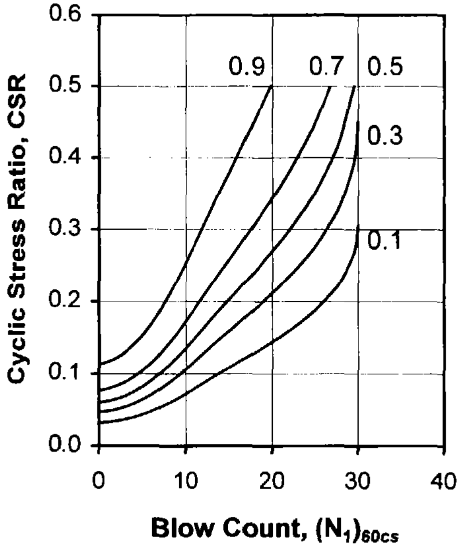

Figure 3. SPT Probability Curves - Bayesian

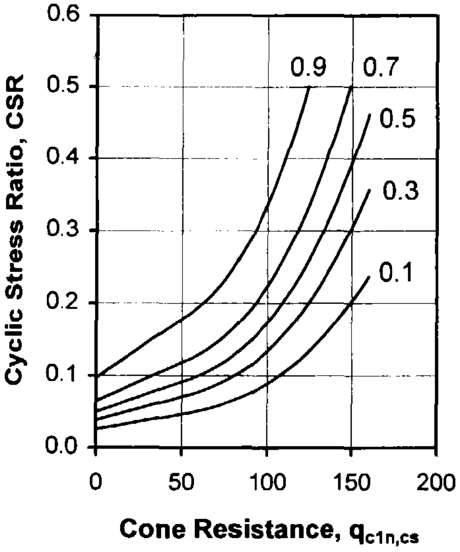

Figure 4. CPT Probability Curves - Bayesian

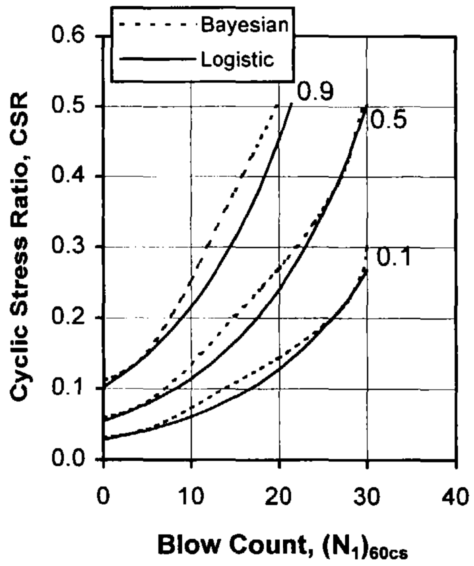

Figure 5. Comparison of SPT Probability Curves

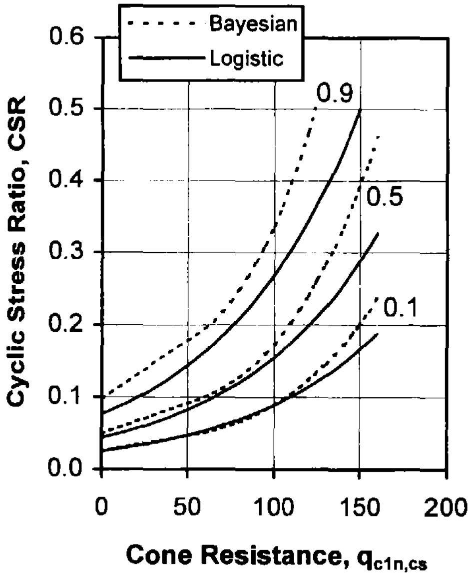

Figure 6. Comparison of CPT Probability Curves

BAYESIAN MAPPING FUNCTIONS

A Bayesian mapping function relates the factor of safety (F_S), defined as F_S = CRR/CSR, obtained from a deterministic method such as the SPT-SI method, to the probability of liquefaction (P_L) obtained by Bayesian interpretation (Juang et al., 2000b):

$$P_L = \frac{f_L(F_S)}{f_L(F_S) + f_{NL}(F_S)} \quad (8)$$

where $f_L(F_S)$ and $f_{NL}(F_S)$ are the probability density functions of the calculated F_S for the subsets of liquefied cases and non-liquefied cases, respectively. Using a set of 233 SPT-based cases (Juang et al, 2000b), the following mapping function is obtained:

$$P_L = 1/[1+(F_S/A)^B] \quad (9)$$

where the regression coefficients, A = 0.8 and B = 3.5. This Bayesian mapping function is practically the same as the one developed by Juang and Jiang (2000) using a different MSF formula, in which A = 0.77 and B = 3.25.

Similarly, using the 225 CPT-based cases reported in Juang et al. (2000c), a Bayesian mapping function that relates F_S determined from the CPT-RW method to P_L is obtained. The form of the mapping function is the same as that shown in Equation 9. This mapping function is defined with A = 1.0 and B =3.3, and is practically the same as the one developed by Juang and Jiang (2000) using a different MSF formula, in which A = 1.0 and B = 3.34.

Figures 3 and 4 show the SPT- and CPT-based probability curves, respectively. The SPT-based curves are developed using the mapping function with A = 0.8 and B =3.5. This is done by rearranging Equation 9 (noting F_S = CRR/CSR and CRR $= f[(N_1)_{60,cs}]$, defined in Equation 4):

$$f[(N_1)_{60,cs}] / CSR_{7.5} = A\ [(1/P_L) - 1]^{1/B} \quad (10)$$

The CPT-based probability curves are developed in the same way using the CPT-based Bayesian mapping function with A = 1.0 and B =3.3. Note that in Figures 3 and 4, the boundary curves for the deterministic methods are not shown. The SPT-SI boundary curve coincides with the 30% probability curve in Figure 3, and the CPT-RW boundary curve coincides with the 50% probability curve in Figure 4.

It is noted that unlike the probability curves developed from the results of logistic regression, which are independent of the deterministic methods, the probability curves based on Bayesian mapping functions are *specific* to the deterministic methods adopted. Thus, the probability of liquefaction may be readily inferred based on a factor of safety calculated by a specific deterministic method (in this case, the SPT-SI method or the CPT-RW method). This approach greatly facilitates risk-based decisions on design against liquefaction using the traditional deterministic methods.

COMPARISON OF CALCULATED PROBABILITIES

Figure 5 shows the comparison of the SPT-based probability curves obtained by the logistic regression and the Bayesian mapping function approach. Both sets of the SPT-based probability curves are quite comparable to each other, except that in the range of $10<(N_1)_{60,cs}<25$, the curves obtained from the Bayesian mapping function inherit the "shape" from the deterministic boundary curve (defined in Equation 4). From a design standpoint, the curves based on the Bayesian mapping function are more conservative than those based on logistic regression, because the former requires a smaller $(N_1)_{60,cs}$ value than does the latter to assure of a specified risk level (e.g., P_L = 20%) at a given seismic load (CSR).

Figure 6 shows the comparison of the CPT-based probability curves. At lower probabilities, both sets of the CPT-based probability curves are quite comparable to each other. At higher probabilities, the probability curves based on the Bayesian mapping function are more conservative than those based on logistic regression. Again, the curves obtained from the Bayesian mapping function inherit the "shape" from the deterministic boundary curve (defined in Equation 5).

Another way to compare the two approaches (Bayesian interpretation versus logistic regression) is to develop a mapping function based on the logistic regression equation. Here, the probability of liquefaction (P_L) of each case is calculated by the logistic regression equation, while the factor of safety (F_S) is calculated from the deterministic method (Equation 4 for SPT-based and Equation 5 for CPT-based evaluation). The mapping function is then obtained by curve fitting the set of (P_L,F_S) data points obtained. Table 1 compares the mapping function parameters A and B for the two different approaches.

Table 1. Mapping Function Parameters

Parameter	SPT-based Function		CPT-based Function	
	Bayesian	Logistic	Bayesian	Logistic
A	0.8	0.9	1.0	1.1
B	3.5	3.7	3.3	3.5

Figure 7 shows the comparison of these mapping functions. The logistic mapping functions are found to be identical in "shape" to the Bayesian mapping functions, and both sets of mapping functions are quite comparable to each other. Table 2 compares the probabilities of liquefaction for F_S equal to 1.2 and 1.5 obtained from these mapping functions. If a site is designed with a factor of safety of 1.2 using the SPT-SI method (Equation 4 for CRR and Equation 1 for CSR), the probability of liquefaction is expected to be in the range of 19% (based on Bayesian mapping) to 26% (based on logistic regression). Both probabilities fall in the range of 15% to 35%, within which the likelihood of liquefaction is described

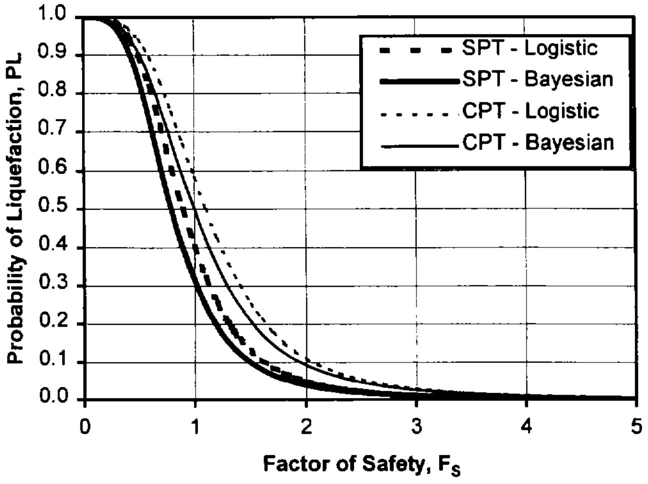

Figure 7. Comparison of Mapping Functions

Table 2. Expected Probability of Liquefaction

Method	Design F_S	Probability of Liquefaction	
		Bayesian	Logistic
SPT-based	1.2	19%	26%
	1.5	10%	13%
CPT-based	1.2	35%	42%
	1.5	21%	25%

Table 3. Liquefaction Likelihood Classification

Class	Probability of liquefaction (P_L)	Description of likelihood
5	$P_L \geq 0.85$	Almost certain that it will liquefy
4	$0.65 \leq P_L < 0.85$	Very likely to liquefy
3	$0.35 \leq P_L < 0.65$	Liquefaction and no liquefaction are equally likely
2	$0.15 \leq P_L < 0.35$	Unlikely to liquefy
1	$P_L < 0.15$	Almost certain that it will not liquefy

as "unlikely" (class 2 as defined in Table 3). If the site is designed using the SPT method with F_S = 1.5, the expected probability of liquefaction will be in the range of 10% to 13%. Both probabilities fall in the range of 0 to 15%, within which the likelihood of liquefaction is described as "almost certain that it will not liquefy" (class 1).

When using the CPT-RW method (Equation 5 for CRR and Equation 1 for CSR) for liquefaction evaluation, the meaning of factor of safety is quite different. If a site is designed with F_S = 1.2, the expected probability will be in the range of 35% (based on Bayesian mapping) to 42% (based on logistic regression). Both probabilities fall in the range of 35% to 65%, within which the likelihood of liquefaction is described as "equally likely" as the likelihood of no liquefaction (class 3). If a site is designed with F_S = 1.5, the expected probability will be in the range of 21% to 25%. Both probabilities fall in the range of 15% to 35%, within which the likelihood of liquefaction is described as "unlikely" (class 2).

From the above discussion, if a site is designed for a specified factor of safety using a deterministic approach, it is more conservative (or safer) to use the SPT-SI method than the CPT-RW method. This conclusion may also be interpreted based on the results presented in Table 4 where the *required* factor of safety for a design to assure that the probability of liquefaction is not greater than a specified level. For example, if a risk (probability) level of 20% is specified for a design, the required F_S values are in the range of 1.2 (based on Bayesian mapping) to 1.3 (based on logistic regression) when using the SPT-SI method in the design, while the required F_S values are in the range of 1.5 to 1.6 when using the CPT-RW method. Since it requires a smaller FS in the design to assure of the same risk level, the SPT-SI method is more conservative than the CPT-RW method.

Table 4. Required Factors of Safety for Specified Risks

Specified Probability (Risk)	Required Factor of Safety (F_S)			
	SPT-based		CPT-based	
	Bayesian	Logistic	Bayesian	Logistic
10%	1.5	1.6	1.9	2.1
20%	1.2	1.3	1.5	1.6
30%	1.0	1.1	1.3	1.4

It is noted that the discussion of which deterministic method is more conservative is meaningful *only* if a design decision is made purely based on a specified factor of safety. As discussed above, the design based on the CPT-RW method is more likely to liquefied than that based on the SPT-SI method if the same F_S is used. Thus, the latter is judged to be more conservative than the former. However, if the risk-based design decision is adopted in practice, the design based on the CPT-RW method can be as conservative (i.e., achieving the same risk level) as the one based on the SPT-SI method. For example, adopting a factor of safety of 1.5 when using the CPT-RW method will in principle result in a site that has the same likelihood of liquefaction as the one that is designed using the SPT-SI method with F_S = 1.2.

Finally, it is important to note that the term factor of safety discussed in this paper is treated as a fixed variable. Possible variations in the input parameters are not considered in the calculation of CSR and CRR (and thus F_S). If the calculated F_S has to be described as a range, then the inferred probability of liquefaction should be reported as a range. Likewise, if the calculated F_S is treated as a random variable (described by a particular distribution), then the inferred probability should be reported as a distribution. In general, it is not necessary to consider the variation in the calculated F_S when interpreting the probability of liquefaction from the developed mapping functions. However, if the variation of the input parameters for the calculation of F_S is too great to be ignored, reliability index may be calculated, considering the parameter uncertainty. Then, a mapping function that relates the calculated reliability index to the probability of liquefaction should be used (Juang et al., 2000a).

Risk-Based Design

Use of a deterministic method for liquefaction potential evaluation is still preferred by most geotechnical engineers. The developed mapping functions provide a critical link with which risk-based decisions for design against liquefaction may be made using the traditional, deterministic methods.

CONCLUSIONS

1. When adopting the deterministic approach to evaluating liquefaction potential, the SPT-SI method is shown to be more conservative than the CPT-RW method.
2. Probability of liquefaction interpreted based on logistic regression is comparable with that interpreted based on Bayesian mapping function.
3. The developed mapping functions provide a critical link for making risk-based design decisions using the traditional, deterministic methods.

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