International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics

1981 - First International Conference on Recent Advances in Geotechnical Earthquake Engineering & Soil Dynamics

27 Apr 1981, 2:00 pm - 5:00 pm

Progressive Strain of Sand Due to Cyclic Loading

D. H. Timmerman
University of Akron, Akron, Ohio

S. Leelanitkul
Texas A&M University, Kingsville, Texas

Recommended Citation

Timmerman, D. H. and Leelanitkul, S., "Progressive Strain of Sand Due to Cyclic Loading" (1981). *International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*. 15.
https://scholarsmine.mst.edu/icrageesd/01icrageesd/session01b/15

Proceedings: First International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil
Dynamics, April 26 - May 3, 1981, St. Louis, Missouri

Progressive Strain of Sand Due to Cyclic Loading

D. H. Timmerman
Associate Professor of Civil Engineering, University of Akron, Akron, Ohio

S. Leelanitkul
Assistant Professor of Civil Engineering, Texas A & I University, Kingsville, Texas

SYNOPSIS Progressive axial strain produced by cyclic, triaxial loading conditions was measured on Ottawa sand samples of various relative densities. The theoretical cyclic shearing strain amplitude in each sample was determined using a modified, hyperbolic shearing stress-shearing strain relationship. The data shows a linear log-log correlation between the measured progressive axial strain and the calculated cyclic shearing strain amplitude. The correlation is proposed as a general procedure for predicting progressive strains resulting from general cyclic, triaxial loading conditions.

INTRODUCTION

Progressive strain of dry or moist cohesionless soil subjected to various loading conditions has been studied by various researchers in recent years. This paper considers the deformation characteristics of dry sands under the influence of small cyclic shear strain amplitudes which are likely to occur under vibrating foundations.

Previous work by Silver and Seed (1971, 1972) has shown that progressive settlements in dry cohesionless soil deposits which are subjected to earthquake loadings can be correlated with the shearing strains produced by the horizontal, cyclic earthquake loading. This was accomplished through emperical correlations with the results of cyclic, simple shear tests used to simulate the earthquake loading. The research reported herein has generalized the concept to enable prediction of progressive deformations under general cyclic, triaxial loading conditions, such as those occurring under vibrating foundations.

CYCLIC TRIAXIAL STRESS-STRAIN RELATIONS

Superposition of stresses beneath the center of a vibrating circular footing may be represented by Fig. 1 for the case of axial symmetry. The static stresses are σ_v and σ_h produced by the static load of the foundation. Under vibration, cyclic stresses $\pm\ \Delta\sigma_v$ and $\pm\ \Delta\sigma_h$ are introduced.

The dynamic stress components may be considered as the superposition of a cyclic hydrostatic stress, $\pm\ \Delta\sigma_h$ and a cyclic deviatoric stress, $\pm\ (\Delta\sigma_v - \Delta\sigma_h)$. These two stress conditions were duplicated in triaxial test by Timmerman (1969). The stresses and the stress paths for the two cases are shown in Fig. 1 (B) and (C). The stress path diagrams show that cyclic shear stress, and hence cyclic shear strain, is only produced from the cyclic deviatoric stress component.

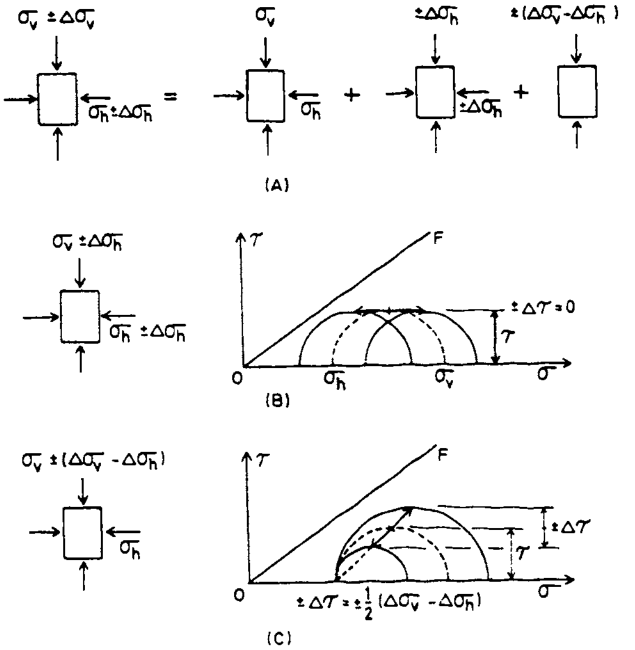

Fig. 1. (A) Stresses Beneath a Vibrating Foundation

(B) Stress Path for Cyclic Hydrostatic Loading

(C) Stress Path for Cyclic Deviatoric Loading

A detailed representation of the cyclic deviatoric stress condition in triaxial tests is shown in Fig. 2 (A). The cyclic stress $\pm\ (\Delta\sigma_v - \Delta\sigma_h)$ generates the cyclic axial strain amplitude $\pm\ \Delta\varepsilon_v$. This dynamic axial stress-strain response forms a hysteresis loop as shown in Fig. 2 (B) in which the dynamic Young's modulus, E, is defined to be the slope of the line connecting the extreme points of the loop.

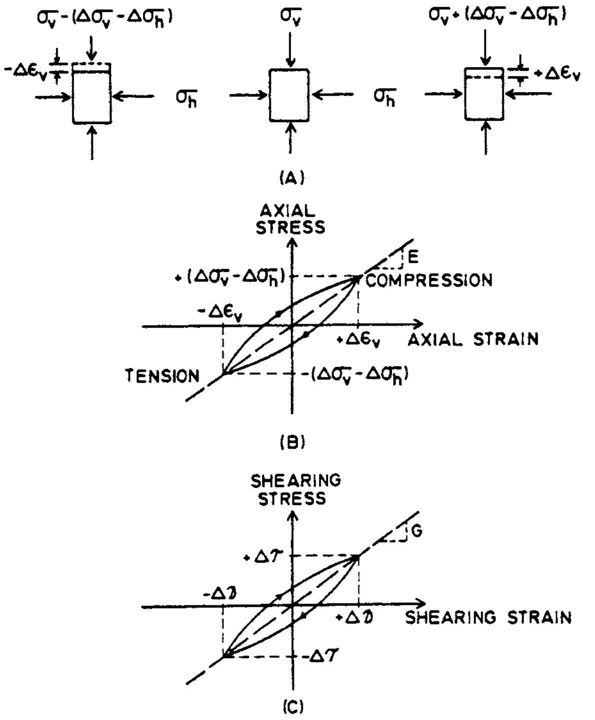

Fig. 2. Typical Response of Soil Under Triaxial Tests

(A) Cyclic Stress-Strain Condition

(B) Typical Hysteresis Loop

(C) Equivalent Shearing Hysteresis Loop

For dynamic analysis, it is often convenient to consider the shearing stress-strain response instead of the axial stress-strain response. The corresponding shearing hysteresis loop is shown in Fig. 2 (C) in which the slope of the line joining the extreme points of the loop is the dynamic shearing modulus, G. The corresponding cyclic shearing stress, ± $\Delta\tau$, can be evaluated from the cyclic deviatoric stress as

$$\pm \Delta\tau = \pm \frac{1}{2} (\Delta\sigma_v - \Delta\sigma_h) \qquad (1)$$

This cyclic shearing stress is generated in the triaxial specimen while the specimen is subjected to the mean confining pressure

$$\sigma_o = \frac{1}{3} (\sigma_v + 2\sigma_h) \qquad (2)$$

The dynamic stress-strain response can be represented as shown in Fig. 3. The curve OA in Fig. 3 represents the static shearing stress-strain curve during the application of the mean confining pressure, σ_o. Point A becomes the dynamic initial loading condition from which the dynamic initial tangent modulus can be evaluated directly from the static mean confining pressure, σ_o, and the initial void ration, e. The expressions for computing the initial tangent modulus as proposed by Hardin and Richart (1963)

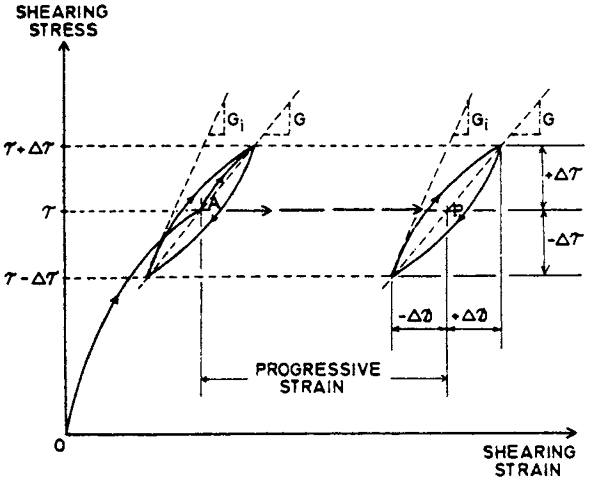

Fig. 3. Typical Progressive Shearing Stress-Strain Response

are

$$G_i = 2630 \frac{(2.170-e)^2}{1+e} \sqrt{\sigma_o} \qquad (3)$$

for round-grained sands, and

$$G_i = 1230 \frac{(2.973-e)^2}{1+e} \sqrt{\sigma_o} \qquad (4)$$

for angular-grained sands. G_i and τ_o are in psi units and e is the void ratio.

Relationships between the cyclic shearing stress and the shearing strain amplitude can be adopted from the hyperbolic shearing stress-strain relation formulated by Kondner (1963) and later modified by Hardin and Drnevich (1972). Thus, the cyclic shearing stress-strain relation is assumed to be described by the following equations:

$$\Delta\tau = \frac{\Delta\gamma}{1/G_i + \Delta\gamma/\tau_m} \qquad (5)$$

and

$$\gamma_r = \tau_m/G_i \qquad (6)$$

in which γ_r = reference strain, τ_m = maximum static shear stress based on friction angle ϕ which would exist at failure if the dynamic stress path was followed to failure in a static test and $\Delta\gamma$ = cyclic shear strain amplitude. Hardin and Drnevich (1972) combined Eqs. (5) and (6) and upon modifying for experimental deviations from theory arrived at

$$\frac{G}{G_i} = \{1 + \frac{\Delta\gamma}{\gamma_r} [1 + a \exp(-b \frac{\Delta\gamma}{\gamma_r})]\}^{-1} \qquad (7)$$

With $a = -0.5$ and $b = 0.16$, the expression can be used to relate the dynamic shearing modulus to the shearing strain amplitude for clean dry sands.

Eqs. (5) and (6) can also be transformed to the expression

$$\frac{G}{G_i} = 1 - \frac{\Delta\tau}{\tau_m} \qquad (8)$$

Equations (1) to (8) can thus be used to relate the cyclic triaxial shearing strain, $\Delta\gamma$, to the imposed shearing stress, $\Delta\tau$, as a function of the static stress state, the friction angle, ϕ, and the void ratio e, of the sand.

PROGRESSIVE STRAIN MEASUREMENTS

Air-dry Ottawa sand no. 20-60 was subjected to cyclic triaxial loadings. Cylindrical samples 1.4" diameter by approximately 2.8" high were subjected to a hydrostatic confining pressure, σ_h of 20 psi and a given static axial stress, σ_v. The samples were then subjected to a sinusoidal axial loading, $\pm \Delta\sigma_v$, at frequencies varying between 2 and 25 cps. The axial strain resulting from 10,000 cycles of loading was considered as the progressive strain from the given stress condition. The cyclic shearing strain was calculated for each sample using Eqs. (1) to (8). The progressive strain-vs-shearing strain data is shown in Fig. 4 for samples having initial relative densities of 51%, 78% and 93%.

The dynamic triaxial test results indicate that the progressive strain is only dependent on the cyclic shearing strain amplitude and the relationship is approximately linear on a log-log plot.

For purposes of comparison, the data of Fig. 4 is summarized in Fig. 5 (A) along with similar data from cyclic simple shear tests conducted by Seed and Silver (1971, 1972) in Fig. 5 (B). The simple shear tests were conducted on angular, crushed quartz sand no. 20-40 having relative densities of 45%, 60% and 80% and for 10 and 300 loading cycles. Although the soil types and loading stress paths are different, the similarities in behavior are evident.

Further analysis of the observed progressive strain behavior and the equations controlling the shearing strain amplitude suggest that the response can be divided into three approximate categories:

(1) When the shear strain amplitude is less than 10^{-5} in./in.

(2) When the amplitude is between 10^{-5} and 10^{-2} in./in. and

(3) When the amplitude is larger than 10^{-2} in./in.

For the first category with shearing strain amplitudes less than 10^{-5} in./in., no significant deformation occurs and the soil behaves essentially like an elastic medium. When the shear strain amplitude is between 10^{-5} and 10^{-2} in./in., progressive deformation of the soil occurs. The progressive deformation is produced primarily by the dynamic shear strain with only insignificant volume changes occurring; therefore the deformation can be correlated with the shear strain amplitude with reasonable accuracy. When the shear strain amplitude exceeds 10^{-2} in./in., large volume changes occur and the resulting deformation is considerably larger than that predicted by the shear strain correlations. A strain amplitude of 10^{-2} in./in. corresponds approximately to the theoretical limit of $\Delta\tau = \tau_m$ for the sand. Therefore, when $\Delta\tau > \tau_m$ or $\Delta\gamma > 10^{-2}$ in./in., grain to grain slippage would be expected resulting in potential large volume charges.

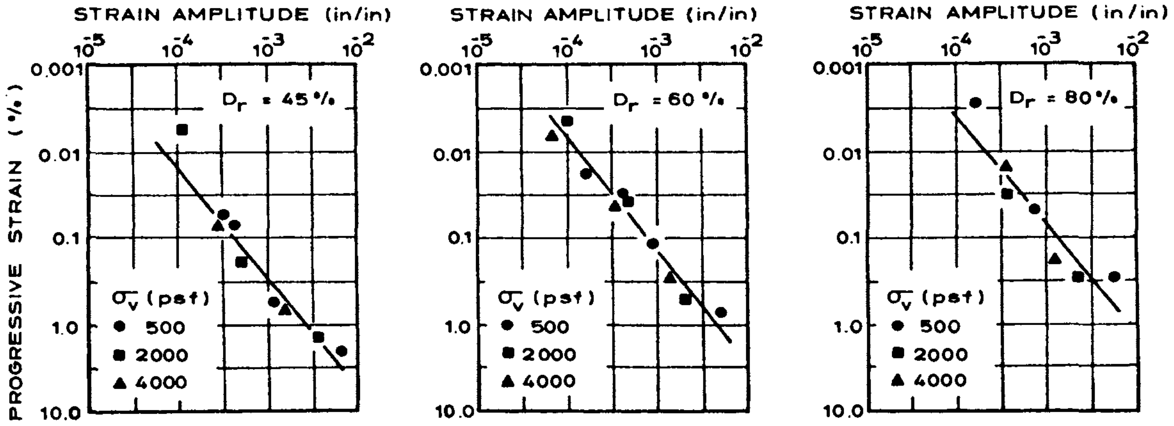

Fig. 4. Progressive Strain-vs-Shearing Strain Amplitude

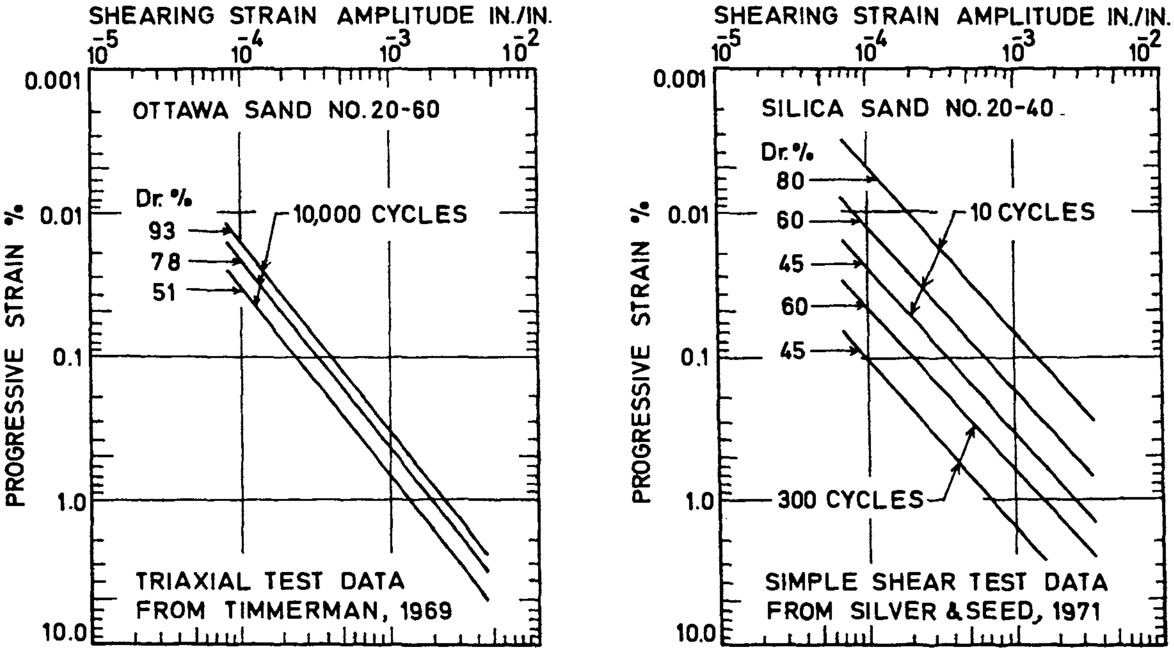

Fig. 5. Progressive Strain-vs-Shearing Strain Amplitude

(A) Triaxial Test

(B) Simple Shear Test

CONCLUSIONS

For general cyclic triaxial loading conditions with shearing strain amplitudes between 10^{-5} and 10^{-2} in./in., the resulting progressive deformations of sands can be predicted by (1) calculating the static and dynamic imposed stresses using elastic theory, (2) calculating the strain dependent cyclic shear modulus using a modified, hyperbolic shearing stress-strain relationship, (3) calculating the resulting shear strain using the imposed cyclic shear stresses and the cyclic shear modulus and (4) determining the resulting deformation using emperical shear strain-progressive deformation relationships from triaxial laboratory data.

Predicted deformations using the general triaxial model agree well with data from other research using cyclic simple shear data.

REFERENCES

Hardin, B. O. and V.P. Drnevich (1972), "Shear Modulus and Damping in Soils: Measurement and Parameter Effects", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 98, No. SM6.

Hardin, B.O. and V. P. Drnevich (1972), "Shear Modulus and Damping in Soils: Design Equations and Curves", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 98, No. SM7.

Hardin, B. O. and F. E. Richart, Jr. (1963), "Elastic Wave Velocities in Granular Soils", Journal of the Soil Mechanics and Foundations Division, ASCE Vol. 89, No. SM1.

Kondner, R.L. (1963), "Hyperbolic Stress-Strain Response: Cohesive Soils", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 89, No. SM1.

Leelanitkul, S. (1980), "Progressive Settlements of Foundations on Cohesionless Soils Subjected to Vertical Vibratory Loadings", Thesis submitted for the degree Doctor of Philosophy in Engineering, University of Akron, Akron, Ohio.

Seed, H.B. and M. L. Silver (1972), "Settlement of Dry Sands During Earthquakes", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 98, No. SM4.

Silver, M. L. and H. B. Seed (1971), "Deformation Characteristics of Sands Under Cyclic Loading", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 97, No. SM8.

Silver, M. L. and H. B. Seed (1971), "Volume Changes in Sands During Cyclic Loading", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 97, No. SM9.