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## Response of Multiple-Mass Systems to Nonvertically Incident Seismic Waves

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# Response of Multiple-Mass Systems to Nonvertically Incident Seismic Waves

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**SYNOPSIS** A variational procedure is used for calculating the response of two foundations with rectangular bases supported on a viscoelastic halfspace and subjected to horizontally and vertically incident SH-waves and Rayleigh waves. Results which include the response of massless foundations and those with mass indicate that the dynamic behavior of a rigid foundation to traveling wave excitation can be affected significantly by the presence of a neighboring foundation. The effect is most pronounced when the direction of the incoming wave is parallel to the axis of the two masses, in which case a noticeable reduction in the response of the downstream foundation is observed with respect to that of the upstream foundation.

## INTRODUCTION

Aseismic design of structures has generally been based on the assumption that the input motion consists of a pure vertical or horizontal translation which acts uniformly along the base of the foundation. This assumption is derived from the notion that the seismic waves travel in the high wave velocity base rock and are propagated vertically to the region of interest through much lower velocity layers. It is now realized, however, that nonvertically incident SH-waves, Love waves and Rayleigh waves may have wavelengths along the surface of the same order as the base dimension of a large structure. There is some experimental evidence, obtained both from records of structural response and from analyses of strong motion accelerograms (See references in Luco and Wong (1979)) to conclude that surface waves may be responsible for major contributions to the recorded motion. The most significant implications of these observations for the response of structures are that (i) Love waves and nonvertically incident SH-waves will generate a marked torsional response even in the case of symmetric structures and foundations, (ii) Rayleigh waves and nonvertically incident P and SV-waves will induce additional contributions to the rocking response, and (iii) the spatial variations of the free-field motion may lead to significant reductions of the high-frequency components of the translational response (Luco and Wong (1979), Bycroft (1980)). Structures supported on isolated foundations, such as spread footings or bridge piers, may experience, in addition, differential motions in excess of those that would occur if the ground motion were uniform.

In an effort to gain insight into the phenomena that occur in multiply interactive systems, much work has been done on two-body problems, in which rigid masses with circular or rectangular bases are attached to a halfspace or stratum model of a soil and subjected to forced excitation or vertically incident seismic waves

(See Roesset and Gonzalez (1978) for references). The situation in which nonvertically incident seismic waves excite multiple foundation or structural systems has received less scrutiny (Wong and Luco (1978), Werner et al. (1979)). In the present study we will consider the problem of two rigid masses with rectangular bases supported on a viscoelastic halfspace and subjected to traveling waves. We will study the effect of nonvertically incident waves on the response, considering, in particular, horizontally propagating SH-waves and surface Rayleigh waves, and results will be compared to those corresponding to vertically incident SH-waves. Massless foundations, whose response may be viewed as the foundation input motion to a system with mass, will be studied first, while effects of added mass will be considered subsequently. Of particular interest will be the low- to mid-range frequency response of the two adjacent foundations, and the effects of separation and type of excitation on the interaction between foundations.

## ANALYSIS OF THE SYSTEM

The foundation system to be studied initially consists of two rigid massless rectangular foundations bonded to the surface of a homogeneous, isotropic, viscoelastic halfspace, characterized by its mass density  $\rho$ , elastic shear modulus  $\mu$ , and Poisson's ratio  $\nu$  (Fig. 1). Internal soil friction is taken into consideration by letting the shear modulus of the soil be a complex quantity, i.e.,  $\bar{\mu} = \mu(1+iD)$ , where  $D$  is the material damping coefficient. The two footings are subjected to harmonic excitation which may consist of external forces and moments, base motion  $\underline{u}_g(\underline{x})$  from traveling waves as depicted in Fig. 1, or a combination of both.

The procedure employed here to determine the response of rigid foundations is similar to one

developed by Wong and Luco (1978), in which Green's functions for the elastic halfspace were used in formulating the corresponding dynamic mixed-boundary value problem. The original formulation involving a system of linear integral equations for the unknown surface tractions is used herein to derive a variational principle which then serves as the basis for a finite element procedure for evaluating the surface tractions. Once these tractions have been obtained it is straightforward to determine the behavior of complete soil-foundation-structure systems with mass as the second step of a substructuring analysis. Details of the derivation are given by Coronato (1980). Here we present only the end result which indicates that the total generalized force exerted by the foundation on the

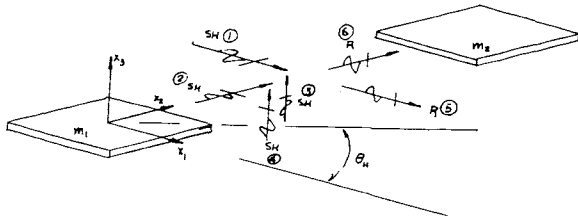


Fig. 1. Model and Excitation

soil,  $\underline{p}_s$ , a  $12 \times 1$  vector containing 3 components of force amplitudes and 3 moment amplitudes for each foundation is given by an equation of the form

$$\underline{p}_s = K\Delta + \underline{p}_e. \quad (1)$$

Equation 1 reveals that there are two sets of generalized forces acting on the foundations. The term  $K\Delta$ , where  $\Delta$  is the  $12 \times 1$  vector of rigid body translations and rotations of the foundations, represents the generalized forces that the rigid foundations exert on the soil when moving with rigid body motion  $\Delta$  in absence of seismic excitation ( $\underline{u}_g = \underline{0}$ );  $K$  is the  $12 \times 12$  impedance matrix for the foundations. The term  $\underline{p}_e$  corresponds to the forces and moments that must be applied to the foundations when the latter are held fixed while under the effects of the seismic excitation. For the case of pure seismic excitation in the absence of external loading  $\underline{p}_s$  vanishes and the resulting rigid motion is given by  $\Delta = K^{-1}\underline{p}_e$ . The resulting motion for the foundation with mass can be obtained from (1), where  $\underline{p}_s$  in this case represents the inertial forces referred to the base of each foundation. The same formulation may be used to calculate the response of a superstructure by including in  $\underline{p}_s$  both the inertial and the elastic and damping forces at the base of the superstructure. A similar formulation applies as well to the case of flexible foundations. In such case  $\Delta$  represents the vector of nodal displacements of a finite element mesh.

Numerical solutions have been obtained for the response of two square footings of sides  $2L$

subjected to vertically and horizontally incident SH-waves and Rayleigh waves, for angles of incidence  $\theta_H = 0^\circ, 90^\circ$  for the case of  $\nu = 1/3, D = 1.1$ . The response quantities of interest are the translations and rotations  $u_1, u_2, \dots, u_{12}$  of the two foundations, defined in Fig. 2. These components are complex because, in general, each foundation is out of phase with respect to the other and to the free-field surface excitation.

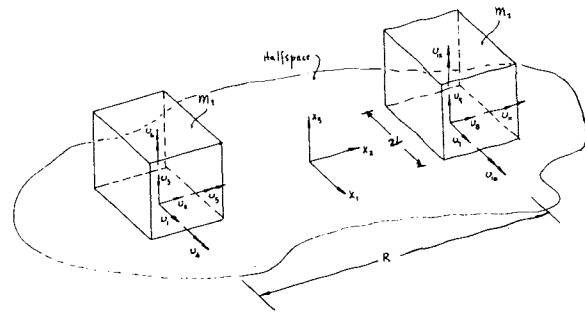
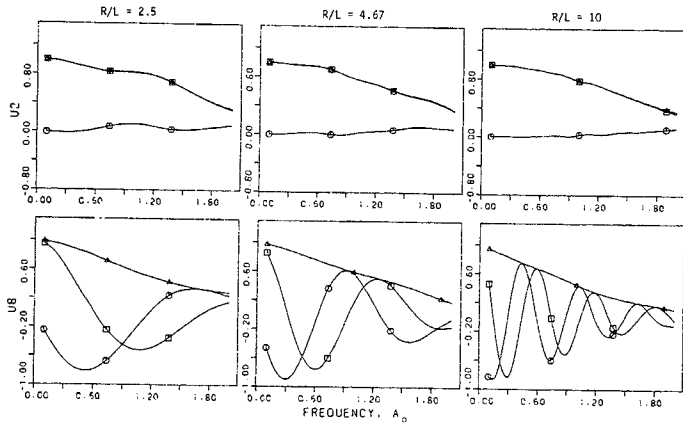


Fig. 2. Model With Mass and Displacement Components

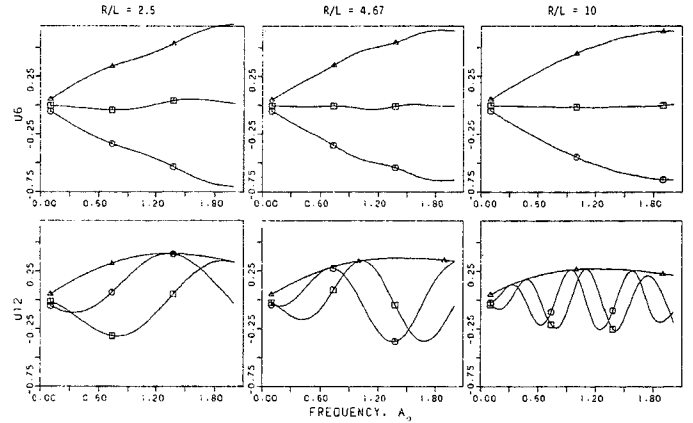
Typical curves for several components of translation and rotation for two types of incident waves, and three different separations of the foundations,  $R/L$ , are shown in Fig. 3 as a function of the dimensionless frequency  $A_0 = \omega L(\rho/\mu)^{1/2}$ , where  $\omega$  is the frequency of excitation. Both the real and the imaginary parts, and the amplitudes of the translations  $u_2, u_8, u_3, u_9$  and the torsional motion  $u_6, u_{12}$  are normalized with respect to the amplitude of the horizontal motion, and the rocking components  $u_4, u_{10}$  due to the Rayleigh incident waves are normalized with respect to the amplitude of the free-field vertical displacement, which is 1.565 times that of the corresponding horizontal free-field motion.

Some notes on the resulting displacements:

- Translation of the foundations in the direction of the soil particle motion decreases significantly with increasing frequency. In contrast, foundation rotations increase with  $A_0$ .
- The upstream foundation can have a pronounced shielding effect on the displacements of the downstream foundation. Results not shown here indicate, however, that the foundation response is not affected significantly by the presence of the other if  $\theta_H = 90^\circ$ , even for  $R/L = 2.5$ . Torsional response of the upstream foundation may reach 75% of the free-field motion. The amplitude is reduced to a peak value of 35% for the downstream foundation at small separations, and to 25% for  $R/L = 10$ .
- Both the translational and rotational displacements of the downstream foundation are out of phase with respect to those of the upstream foundation, even at relatively low frequencies and separation.
- Because of the interaction between the two masses certain displacements occur that would not arise for a single foundation, e.g.,  $u_2, u_8, u_4$  and  $u_{10}$  are nonzero for a Rayleigh incident wave traveling along the  $x_1$  axis. These components are, however, small--of the order of

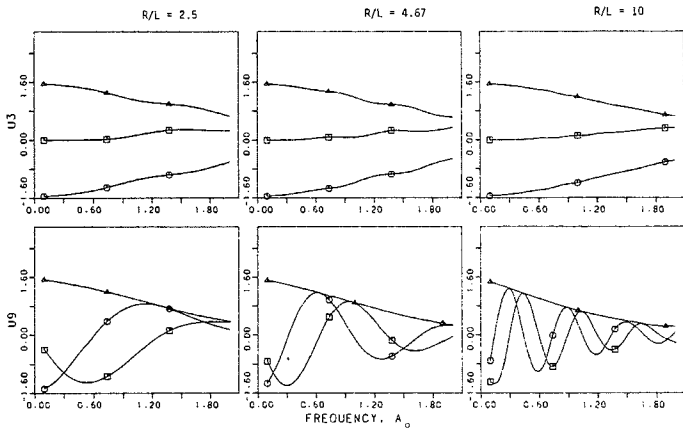


(a) Rayleigh wave excitation,  $\theta_H=0^\circ$  (No. 6)

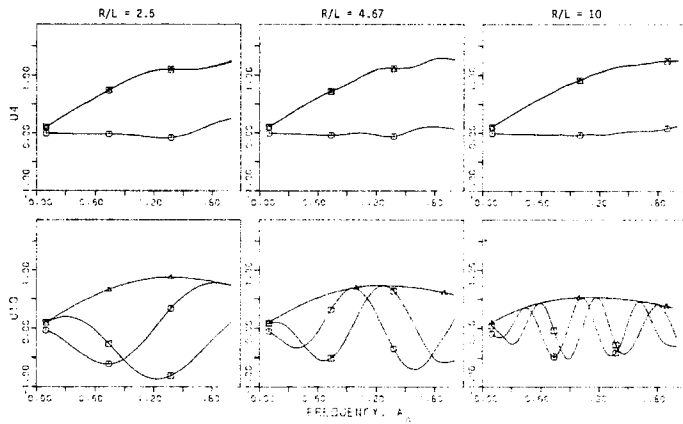


(d) SH-wave excitation,  $\theta_H=0^\circ$  (No. 2)

Fig. 3. (Continued)



(b) Rayleigh wave excitation,  $\theta_H=0^\circ$  (No. 6)



(c) Rayleigh wave excitation,  $\theta_H=0^\circ$  (No. 6)

Fig. 3. Displacements of massless foundations to various incident seismic waves. ( $\square$  = real part,  $\circ$  = imaginary part,  $\Delta$  = amplitude)

4% for  $R/L=2.5$ --and decrease with separation.

Using the same system and excitation as before, masses are now added to the foundations. Three different mass ratio combinations,  $M_i$ , are considered, where  $M_i = m_i / 8\rho L^3$ , and  $m_i$  is the mass

of foundation  $i$ . The center of mass is taken to be along the vertical axis of symmetry at a height  $L$  above the halfspace surface. Amplitudes of several displacement components normalized with respect to the pertinent horizontal or vertical component of the free-field motion are plotted in Fig. 4 for each of the three mass combinations and separations, as functions of frequency, for several types of excitation. Note the following about these figures:

- Coupling of response at close range is evidenced both by the change in amplitude with respect to the response of a single footing, and by the shifting in response frequencies. The latter are, however, quite small.
- Dynamic interaction is more pronounced for foundations with different mass ratios as the larger mass tends to 'drive' the smaller one.
- Response due to Rayleigh waves exceeds that due to SH-waves at small frequencies of excitation. At higher frequencies the response due to Rayleigh wave incidence and horizontally incident SH-waves is significantly smaller than that under vertically incident SH-waves.
- Significant vertical translations can be generated by the rocking coupling of the foundations even for cases in which the free-field particle motion is horizontal. Results obtained by Coronato (1980) indicate that the maximum amplitude can be of the order of 40% of the horizontal free-field motion for  $R/L=2.5$  and decreases rapidly with separation.

CONCLUSIONS

Results of this study show that even though present seismic design methods that ignore out of phase response of foundations and interaction between separate structures are adequate for many problems, the effects of traveling wave

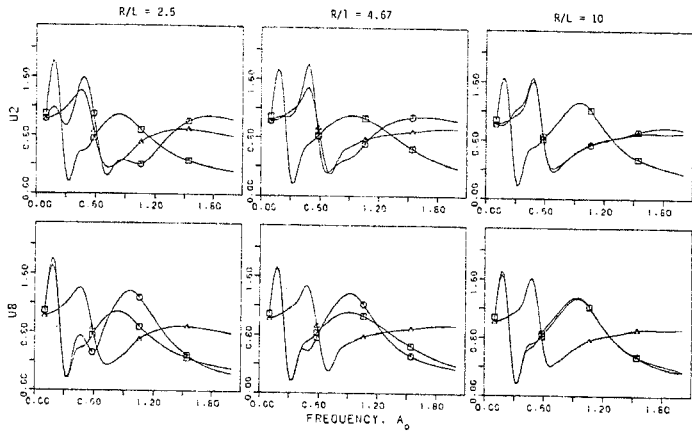
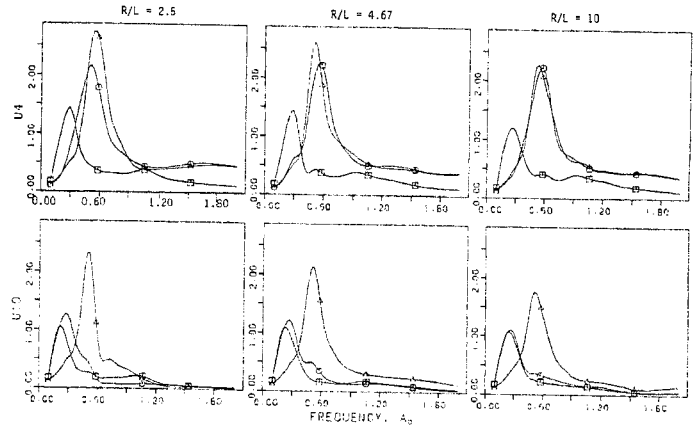
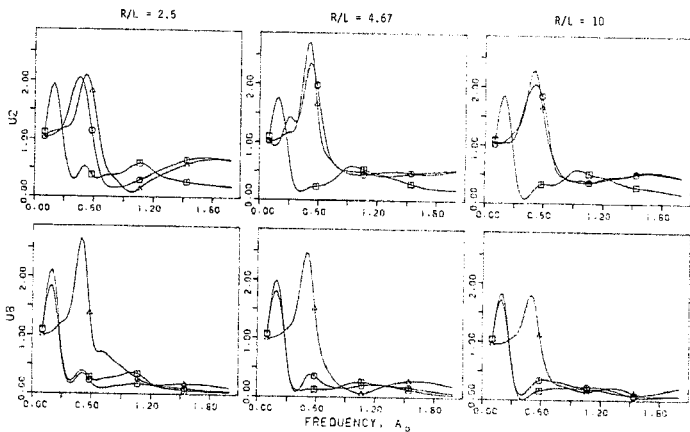
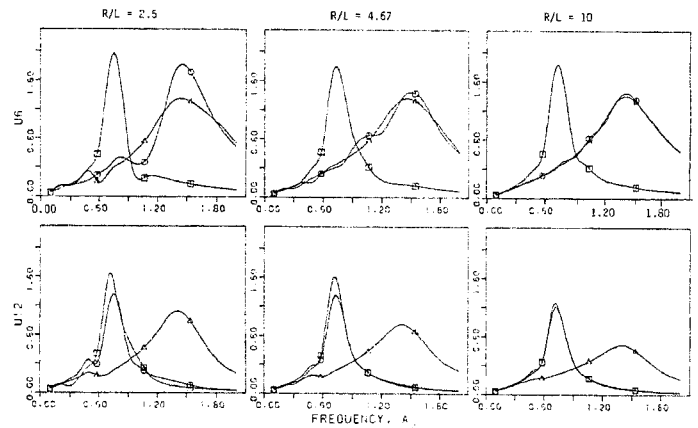
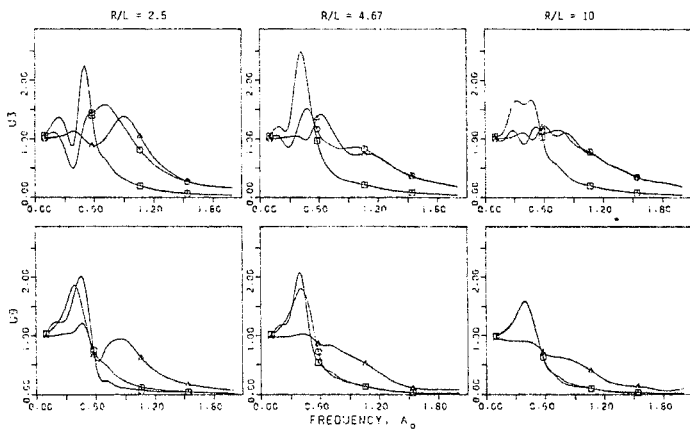
(a) Vertically incident SH-waves,  $\theta_H = 0^\circ$  (No. 3)(d) Rayleigh wave excitation,  $\theta_H = 0^\circ$  (No. 6)(b) Rayleigh wave excitation,  $\theta_H = 0^\circ$  (No. 6)(e) Horizontally incident SH-waves,  $\theta_H = 0^\circ$  (No. 2)(c) Rayleigh wave excitation,  $\theta_H = 0^\circ$  (No. 6)

Fig. 4. Displacements of foundations with mass to various incident seismic waves.  
 ( $\Delta$ :  $M_1=1, M_2=1$ ;  $\square$ :  $M_1=5, M_2=5$ ;  $\circ$ :  
 $M_1=1, M_2=5$ )

Wong, H. L. and J. E. Luco (1978), "Dynamic Response of Rectangular Foundations to Obliquely Incident Seismic Waves", *Earthqu. Eng. Struct. Dyn.*, Vol. 6, 3-16.

Fig. 4. (Continued)

excitation and phase differences between the impedance functions at different locations can be an important consideration for the design of closely spaced structures, and for individual structures on large foundations, including those supported on mat foundations or spread footings.

## REFERENCES

- Bycroft, G. (1980), "Soil-Foundation Interaction and Differential Ground Motions", *Earthqu. Eng. Struct. Dyn.*, Vol. 8, 397-404.
- Coronato, J.A. (1980), "Seismic Response of Multiple Mass Systems on a Viscoelastic Foundation", Thesis submitted for the degree of Master of Science, Carnegie-Mellon University, Pittsburgh, Pennsylvania
- Luco, J. E. and H. L. Wong (1979), "Response of Structures to Nonvertically Incident Seismic Waves", Report of the Dept. of AMES, Univ. of California, San Diego.
- Roesset, J. M. and J. J. Gonzalez (1978), "Dynamic Interaction Between Adjacent Structures", *Proc. of DMSR 77*, Vol. 1, 127-166, Karlsruhe, Germany.