International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics

1981 - First International Conference on Recent Advances in Geotechnical Earthquake Engineering & Soil Dynamics

28 Apr 1981, 2:00 pm - 5:00 pm

Numerical Computation of Earth-Pressures During Earthquakes

D. Aubry
Ecole Centrale des Arts et Manufactures, France

D. Chouvet
Ecole Centrale des Arts et Manufactures, France

Proceedings: First International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, April 26 - May 3, 1981, St. Louis, Missouri

Numerical Computation of Earth-Pressures During Earthquakes

D. Aubry
D. Chouvet
Ecole Centrale des Arts et Manufactures, 92290 Chatenay, France

SYNOPSIS After reviewing some important features of earth pressures against retaining wall during earthquakes the authors propose an implicit-explicit scheme for time integration of the elastoplastic equations in dynamics. Some results obtai -ned with the developed computer code are presented and discussed.

INTRODUCTION

The computation of earthpressures against retaining walls during earthquakes is not so well developed as in the sta -tic case where the engineer may use a lot of more or less sophisticated theories. Though the progress of numerical methods applied to earthquake engineering has been tremendous in general it has not been uniform. In the area of soil-structure interaction with lightly embedded foundations there exist well-known numerical techniques and computer codes from which the geotechnical engineer may pick up an acceleration spectrum at the base of the building which is of utmost importance for the structural engineer. The constitutive equations which are used are generally of the viscoelastic type with hysteretic damping so that no constraint is imposed on the stress level (yield) and no permanent displacement may be evaluated from these codes.

It is often reckognized that in the area of earth-pressure these constitutive equations are inadequate especially if the distribution of initial earthpressure is close to the active state and this is certainly the reason why the analyst usually resorts to a simplified theory such as the Mononobe-Okabe equation, whose basic hypotheses are now summarized.

The displacements of the wall are supposed to be large enough to induce a state of plastic equilibrium behind it. A failure surface exists starting from the toe of the wall Along this surface the shear stress is maximum with respect to the plastic state. The sliding wedge lying between the wall and the failure surface behaves as a rigid body with constant horizontal and vertical accelerations at the base of the wall. Finally the resultant force against the wall is assumed to act at one third of the height of the wall. Numerous discussions of the validity of these assumptions have been proposed in the literature (Seed and Whitman (1970)). Concerning the height at which the resul -tant force is acting Prakash et al. (1969) have indicated that it might well be above H/3 and sould be influenced by the wall angle of friction among other parameters.

Starting with such experimental observations many authors have tried to perform further tests (Mononobe et al. (1929), Jacobsen (1939), Ishii et al. (1960)) The conclusions of these investigations seem to confirm that the acceleration is uniform behind the wall and is equal to the acceleration at the toe. The maximum pressure was found to be slightly less than predicted by the Mononobe equation and the resultant was roughly acting at 0.35H.

The pressure distribution was found to be composed of two terms : (i) a residual pressure larger than the initial one and having a hydrostatic distribution (ii) a dynamical pressure increment having a parabolic distribution with the origin at the base of the wall explaining why the point of application of the resultant should be higher than H/3.

More recently Ishihara et al. (1973) have performed tests on shaking tables and have been able to show that if the maximum horizontal acceleration is less than 0.25g then the point of application of the resultant may be always lower or higher than H/3, but when it is larger then the point of aplication will oscillate around H/3. These authors have also shown that the value of the resultant given by the Mononobe equation is obtained experimentally when the friction between the soil and the wall is maximum and when an average horizontal displacement of the wall is equal to 0.5%H. Prakash et al. (1973,1979) have studied rigid and flexible walls and confirmed that the incremental dynamic pressure has a parabolic shape and that the point of application of the incremental resultant is at about one half of the height.

There are still two important factors which deserve a detailed study and which are not accounted for by the Mononobe equation. These are the wall inertia and flexibility. Richard et al. (1979) have studied the former and have been able to derive an equation for the weight of the wall in order to satisfy an equilibrium equation. In particular there seems to exist an upper limit to the horizontal acce leration so that the wheight of the wall under design keeps a finite value. This acceleration was found to be independant on the geometry of the wall but related to the friction between the wall and the foundation soil underneath. However the satisfaction of this equilibrium equation at any instant of time induces too heavy walls so that we are bound to accept the development of irreversible displacements. Consequently the authors have performed an analysis of the accumulated permanent displacements each time the horizontal acceleration of the erathquake is larger than the maximum value required to still satisfy the equilibrium.

This brief review of the existing literature on earthpressure during earthquakes shows the necessity to perform further experimental tests and to develop numerical computations taking into account several important factors. The wall inertia and flexibility must be accounted for. The possibility of relative displacements between the wall and the backfill must be included in the analysis. Finally the evaluation of the soil pressure during the cycles cannot be simulated by a linear elastic law and thus the elasto-plastic behaviour of the soil must be incorporated in the analysis. Fully nonlinear analyses are costly and if parametric are to be conducted, on one hand the usual fully implicit time integration schemes are prohibitive and on the other hand the fully explicit ones are not adapted to the somewhat low frequency content of earthquake accelerogrammes because then the time step must be chosen on the basis of stability consideration only and thus is always too small. It is the purpose of this paper to intro-duce a mixed implicit-explicit nonlinear technique to anlyse the soil-wall interaction while accounting for other factors which we have just discussed.

MIXED IMPLICIT-EXPLICIT TIME INTEGRATION SCHEMES

The analysts in fluid-structure interaction have recently developed mixed time integration schemes for dynamical loadings. Let us recall that in an explicit algorithm the time step is constrained by an upper limit which is reciprocal to the highest frequency of the finite element or diffrence mesh. Implicit algorithms do not suffer this sometimes stringent condition when they are built to be unconditionally stable. However they need much more computer storage and a skyline linear system must be solved at each time step. This is why explicit algorithms are well suited for either rapid loading or soft media while the inverse is true for implicit algorithms. When zones of highly different stiffnesses are present in an analysis it should thus be helpful to have a scheme which could take advantage of both methods.

Belytschko et al. (1977) have recently implemented and analyzed such a mixed scheme using mesh partition. As noted by Hughes et al. (1978a,1978b,1979) an element based partition should facilitate the implementation star-ting from an existing finite element code. Hughes et al. (1978a,b) have developed a mixed scheme based on Newmark method. We present here a more general scheme adapted to elastoplastic computations. For the convenience of the reader we shall divide the presentation inthree steps. In the first two steps we shall review with our notations the features of (i) a fully implicit scheme for elasto-plastic behaviour (ii) a mixed implicit-explicit technique for linear elasticity. Then we shall have the basic tools to go to the nonlinear case. In all the algorithms discussed underneath we only present what is called a semi-discretization in time which simplifiesgreatly the notations. Obviously a full discretization with eg. finite elements will always be understood.

A Fully Implicit Scheme For Elastoplastic Dynamics

Let u be the displacements ,$\varepsilon(u)$ the corresponding strains σ the stresses. V and τ will designate respectively a virtual displacemetns and stress in the variational formulation. Let f be the distrubuted forces over the domain Ω that we want to study. To simplify the notation the dot product (,) will either stands for :

$$(u,v) = \int_\Omega u_i . v_i \ dx \tag{1}$$

or for :

$$(\sigma,\tau) = \int_\Omega \sigma_{ij} . \tau_{ij} \ dx \tag{2}$$

whenever the above quantities are respectively vectors or tensors. (The summation over repeated index will always be implied). Then the variational formulation of the dynamical equilibrium equations reads :

$$(\rho\ddot{u},v) + (\sigma,\varepsilon(v)) = (f,v) \tag{3}$$

where ρ is the density. Equation (3) is clearly equivalent to the principle of virtual work. We shall assumethat the boundary conditions are of the Dirichlet type. The variational formulation of an elastoplastic equation may be written :

$$(\dot{\sigma},\tau) = (C(\sigma,\lambda) . \varepsilon(\dot{u}),\tau) \tag{4}$$

where $C(\sigma,\lambda)$ is the elastoplastic matrix depending on the stresses and the plastic multipliers λ in the case of perfect plasticity. It is important to recall that λ depends itself on the stresses but also on the rate of strains and that C has at least two different expressions according the loading-unloading criterion.

Now contrary to the linear case it is not possible to eliminate $\dot{\sigma}$ from (4) and get from (3) a nonlinear equation for u. We shall rather introduce a time discretization and solve iteratively for the displacements and the stresses using (3)(4) at each time step. Let n be the time step number, then a fully implicit scheme reads as follows : solve for u_{n+1}, σ_{n+1}, the following equations:

$$\left| \begin{array}{l} (\rho\ddot{u}_{n+1},v) + (\sigma_{n+1},\varepsilon(v)) = (f_{n+1},v) \\ (\sigma_{n+1}-\sigma_n,\tau) = (C_{n+1}.(\varepsilon(u_{n+1}-u_n)),\tau) \end{array} \right. \tag{5}$$

with u_n,σ_n previously computed.

Any given implicit scheme will provide us with a method for computing u_{n+1} from u_n, $\dot{u}_n$, $\ddot{u}_n$,$\ddot{u}_{n+1}$. Thus the Newmark algorithm gives us, with u^*_{n+1} defined as follows :

$$\left| \begin{array}{l} u^*_{n+1} = u_n + \Delta t\ u_n + \frac{\Delta t^2}{2} (1-2\beta)\ \ddot{u}_n \\ \ddot{u}_{n+1} = (u_{n+1} - u^*_{n+1}) \ / \ \beta\Delta t^2 \\ \dot{u}_{n+1} = u_n + \Delta t\ (1-\gamma)\ \ddot{u}_n + \gamma\Delta t\ \ddot{u}_{n+1} \end{array} \right. \tag{6}$$

where Δt is the current time step, β and γ are classical parameters (Newmark,1959). Now equations (5) are still nonlinear equations and some sort of iterative process is necessary for the computation of u_{n+1}, σ_{n+1}.

Let $b_n(u,v)$ denotes a symetric bilinear form to be chosen later on, and i be the iteration number, then a useful scheme is as follows :

Step 1 : Solve for u^{i+1}_{n+1}

$$b_n(u^{i+1}_{n+1},v) = b_n(u^i_{n+1},v) + (f_{n+1},v) - (\sigma^i_{n+1},\varepsilon(v)) - (\rho\ddot{u}^i_{n+1},v)$$

Step 2 : Solve for σ^{i+1}_{n+1} (7)

$$(\sigma^{i+1}_{n+1} - \sigma_n,\tau) = (C^{i+1}_{n+1} . (\varepsilon(u^{i+1}_{n+1} - u_n),\tau)$$

Step 3 : Test for convergence.

It can be shown that whenever the numerical scheme is conducted up until convergence the elastoplastic law does not destroy unconditional stability. However some loacal insta-bilities might occur which are not easily detected if the convergence criteria are not stringent enough.

Implicit-Explicit Scheme for Elastodynamics

As we already noticed in the finite element analysis of soil-structure-interaction inearthquake engineering it could be interesting to take advantage of the typically different stiffnesses of the structure and the soil especially when we have nonlinear computation in mind. Thus to do so, we split the domain into two parts Ω_I and Ω_E and we shall assume that the elements belonging to Ω_I and Ω_E will be treated respectively implicitely and explicitely respectively. Then the dot-product on Ω_k will be denoted by :

$$(u,v)_k = \int_{\Omega_k} u_i . v_i \quad dx \qquad ,k=I,E \tag{8}$$

and also when u,v are replaced by stress or strain tensors Furthermore in linear elastodynamics it is possible to eliminate the stresses with respect to the displacements and we shall write :

$$a_k(u,v) = \int_{\Omega_k} D_{ij}\, \epsilon_{ij}(u) . \mathcal{E}_{ij}(v) \quad dx \ , \ k=I,E \tag{9}$$

where D is the matrix of the elastic coefficients. Then following Hughes et al. (1978b) an implicit-explicit sche -me based on a domain partition and starting from Newmark algorithm may be written :

Step 1 : Let u_n, $\dot{u}_n$, $\ddot{u}_n$ be given

Step 2 : Compute $u^*_{n+1} = u_n + \Delta t\ u_n + \frac{\Delta t^2}{2}(1-2\beta)\ \ddot{u}_n$ (10)

Step 3 : Solve for u_{n+1}

$$1/\beta\Delta t^2\{(\rho u_{n+1},v)_E+(\rho u_{n+1},v)_I\}+a_I(u_{n+1},v)=(f_{n+1},v)+1/\beta\Delta t^2\{(\rho u^*_{n+1},v)_I+(\rho u^*_{n+1},v)_E\}-a_E(u^*_{n+1},v) \tag{11}$$

Step 4 : Compute $\ddot{u}_{n+1} = (u_{n+1} - u^*_{n+1})\ /\beta\Delta t^2$

$u_{n+1} = u_n + \Delta t(1-\gamma)\ \ddot{u}_n + \gamma\Delta t\ \ddot{u}_{n+1}$

Step 5 : n = n+1, go to 1.

The equation (10),(11) are fundamental to the mixed impli -cit-explicit scheme. u_{n+1} may be called a predictor while u_{n+1} is a corrector. What is essential to the mixed treat -ment is the fact that the part of the elastic energy re -lated to the subdomain Ω_I is implicitly treated while it is almost explicitly treated for the subdomain Ω_E .Regar -ding finite element analysis it is important to take full advantage of the expliciteness of domain Ω_E using a numerical quadrature for (8) with k=E with the integration points placed at the nodes. In this manner the mass matrix for subdomain Ω_E will be diagonal and the profile of the linear system (11) will be considerably reduced whenever Ω_E is large compared to Ω_I. This way of building an impli -cit-explicit scheme proposed by Hughes et al. (1978a) is particularly elegant and almost straightforward to be im -plemented. For the stability analysis we refer to the same paper. With these notations and above concepts we can now proceed to a non-linear mixed implicit scheme.

Implicit-Explicit Scheme for Elastoplastic Dynamics

The different steps of the algorithm are as follows:

Step 1 : Let $u_n, \dot{u}_n, \ddot{u}_n$ be known

Step 2 Compute $u^*_{n+1} = u_n + \Delta t\ u_n + \frac{\Delta t^2}{2}(1-2\beta)\ \ddot{u}_n$

Step 3 : Solve for σ^*_{n+1} in Ω_E :

$$(\sigma^*_{n+1} - \sigma_n\ , \tau)_E = (\ C^*_{n+1}(\mathcal{E}(u^*_{n+1}) - \mathcal{E}(u_n))\ , \tau\ )_E \tag{12}$$

Step 4 : Solve for σ_{n+1} in Ω_I, and for u_{n+1}

$$\left|\begin{array}{l} \frac{1}{\beta\Delta t^2}\left[(\rho u_{n+1},v)_E+(\rho u_{n+1},v)_I\right] + (\sigma_{n+1},\mathcal{E}(v))_I = (f_{n+1},v) \\ \quad + \frac{1}{\beta\Delta t^2}\left[(\rho u^*_{n+1},v)_I+(\rho u^*_{n+1},v)_E\right] - (\sigma^*_{n+1},\mathcal{E}(v))_E \\ (\sigma_{n+1}-\sigma_n,\tau)_I = (\ C_{n+1}.(\mathcal{E}(u_{n+1})-\mathcal{E}(u_n)),\tau\ )_I \end{array}\right.$$

Step 5,6 : Identical to steps 4,5 of previous algorithm.

Now step 3 of the preceeding algorithm is highly nonlinear and some kind of iterative process is necessary. Using the concept of the first algorithm the solution of step 4 is given by:

Step 4a : Let u^i_{n+1} , σ^i_{n+1} be known.

Step 4b : Solve for u^i_{n+1} (14)

$$b_n(u^{i+1}_{n+1}, v) = b_n(u^i_{n+1},v)+(f_{n+1},v) - \frac{1}{\beta\Delta t^2}\Big[(\rho(u^i_{n+1}-u^*_{n+1}),v)_I + (\rho(u^i_{n+1}-u^*_{n+1}),v)_E\Big] - (\sigma^*_{n+1},\mathcal{E}(v))_E - (\sigma^i_{n+1},\mathcal{E}(v))_I$$

Step 4c : Solve for σ^{i+1}_{n+1}

$$(\sigma^{i+1}_{n+1}-\sigma_n\ ,\tau\ )_I = (\ C_{n+1}.(\mathcal{E}(u_{n+1})-\mathcal{E}(u_n)),\tau\ )_I \tag{15}$$

Step 4d : Check for convergence tolerance on u^{i+1}_{n+1} , σ^{i+1}_{n+1}.

To take advantage of the mixed implicit-explicit it is of utmost importance to choose a wellsuited bilinear form b(u,v). The bilinear form must consequently be split into two parts to preserve a reduce profile :

$$b(u,v) = b_I(u,v) + b_E(u,v) \tag{16}$$

The author's present choice for b_I and b_E is :

$$\begin{array}{l} b_I\ (u,v) = \frac{1}{\beta\Delta t^2}(\rho\, u,v)_I + a_I\ (u,v) \\ b_E\ (u,v) = \frac{1}{\beta\Delta t^2}(\rho\, u,v)_E \end{array} \tag{17}$$

where $a_I(u,v)$ is the elastic energy of the implicit subdo -main. This choice seems to perform well but other possi -bilies are currently under study. If the finite element matrix corresponding to $b_E(u,v)$ is diagonal then the right -hand side contribution in (14) of elements strictly inclu -ded in the explicit subdomain will be constant and so for their nodes. However for explicit elements there will be a force contribution when the stresses are adjusted so that the interface nodes will be iteratively corrected ac -cording to the correction of the stresses in the implicit elements although the stresses in the explicit element are in a locked-on status.

The stability analysis of the proposed scheme and some mo -difications of it will be studied elsewhere in a forthco -ming paper. Numerous numerical tests have been performed and the efficiency of such mixed scheme has been shown to be particularly helpful in the area of elastoplastic soil-structure interaction. In the subsequent section we shall proceed to the analysis of a soil-wall interaction with such a scheme and show some results on earth-pressures computations.

COMPUTATIONS OF SOIL-WALL INTERACTION

We shall describe in this final section some recent results obtained with a finite element computer code included the mixed implicit-explicit nonlinear scheme described above. We have cho-sen an actual wall 9.50m high with a variable cross-section (from 1.35 m^2 at the toe to .55 m^2 at the top) and variable inertia (.236 m^4 at the toe to .014 m^4 at the top). The density of the wall is 2500 kg/m^3, Young's modulus 4. 10^{10} N/m^2 and Poisson's ratio 0.2. In order to account accurately for the flexibility of the wall classical beam elements were incorporated into the finite element mesh. The mechanical properties of the soil were the following: $E = 10^9\ N/m^2$; $\nu = 0.46$; $\rho = 2270\ kg/m^3$. The soil was a sand with zero cohesion and 35° of angle of friction. Its behaviour was assumed to be described by the Drücker-Prager elastoplastic law with associated flow rule. The inherent shortcomings of this law when applied to sands is well-known but it was considered to be sufficient for the first tests. The initial state of stress was assumed to correspond to the active state throughout the whole soil mesh. This last assumption is also oversimplified but its main advantage was that it did not preclude the computation of the initial state of stress during the construction of the wall and the backfill. Special interface elements taking into account a Coulomb law of friction between the soil and the wall have been included in the analysis. Finally the wall and the soil are assumed to lay directly on a rigid bedrock submitted to an horizontal acceleration. We shall now proceed to discuss some of the most interesting observations that can be extracted from the computation.

Horizontal Stresses in the Soil behind the Wall

The evolution of the stresses for the first 0.01 s is shown at differents heights. It is clearly seen that especially at the top the stresses cannot decrease below the active pressure (the vertical stresses are almost constant there) so that at each cycle the soil gives a larger reaction when the wall moves towards it than when it moves apart. This phenomenon is not predicted by an elastic law in which we have found that the stresses were oscillating symetrically around the static values.

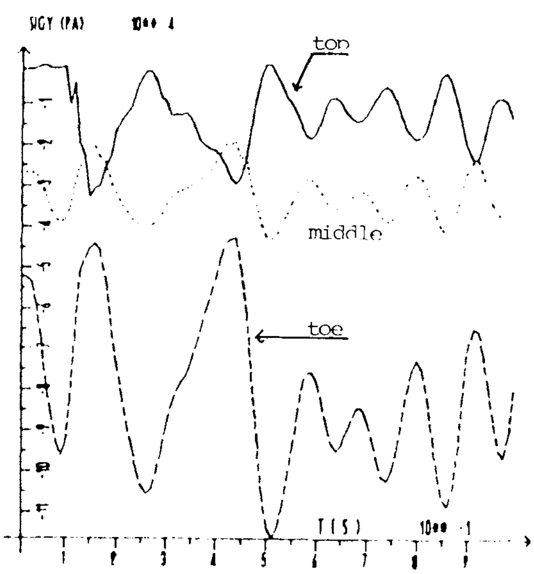

Fig. 1 : Stress history behind the wall

Resultant of Soil Stresses Against the Wall

The hotizontal and vertical components of the force exerted by the soil on the wall are shown on figure 2. The horizontal component is seen to be increasing above the static value which corresponds to an active state.

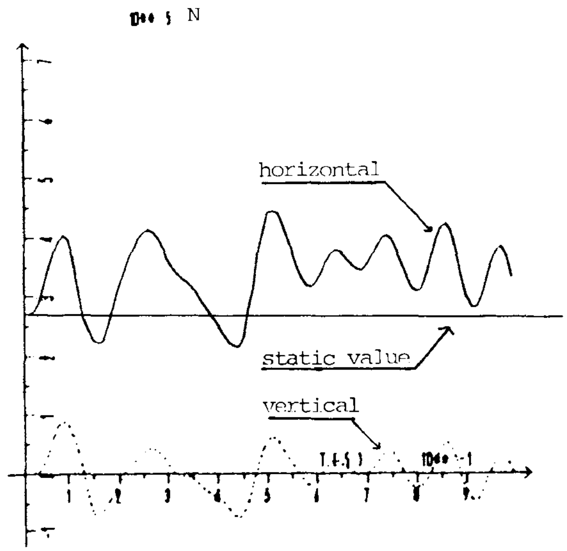

Fig. 2 : Resultant force history (elastoplastic case)

The average value is below the value predicted by the Mononobe equation but the duration of the analysisis clearly too small and further computations are obviously needed. However the trend is clearly indicated.

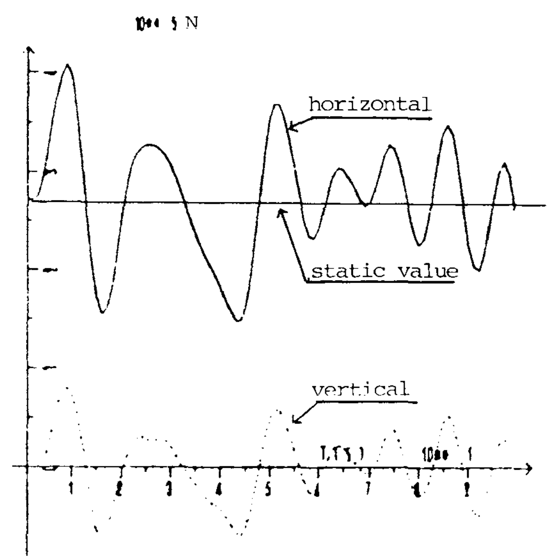

Fig. 3 : Resultant force history (elastic case)

On figure 3 the same run was performed but with an elastic law. The horizontal component is seen to be much more symetric around the static value. Also the vertical component (shear force) is much higher in the elastic case than in the elastoplastic case.
On figure 4, we show the variation of the angle between the resultant force and the wall. The minimum value obtained is about 70 degrees which corresponds to an inclination of 20 degrees with respect to the normal to the wall. This value is not far from $\Phi/2$ obtained in experimental tests.

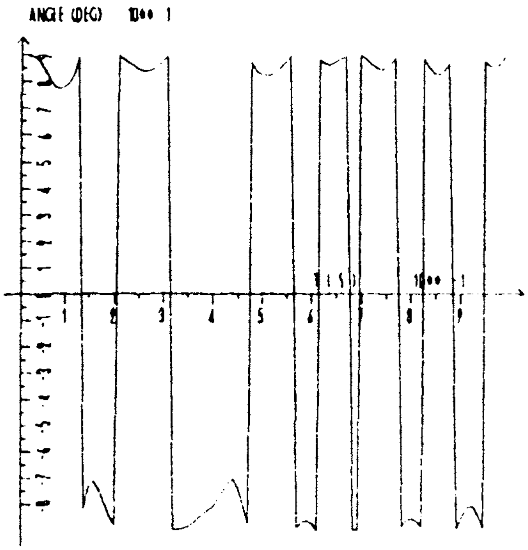

Fig. 4 : Resultant force. Inclination vs time

Influence of Wall Flexibility

Larger horizontal displacements have obviously been obtai -ned in that case. But more interesting is the fact that the horizontal component of the resultant force is decrea -sed which has been already confirmed by experiments. Also the shear force is highly increased due to the high -er vertical movements of the wall with respect to the soil.

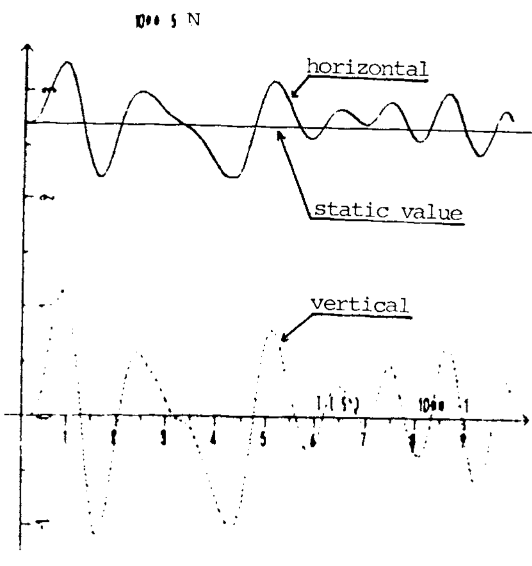

Fig.5 : Resultant force history
(Flexible wall, elastic case)

Permanent Displacement

On figure 6, are shown the horizontal displacements at the top of the wall. It is demonstrated that permanent displa -cements are well accumulated (seisme duration: 5.s) and seemingly tending to an asymptotic value of about .7 mm. This phenomenon may also be explained by the non-symetric response of the soil starting from an almost ac -tive state of stress and taking into account the plasti -city of the soil.

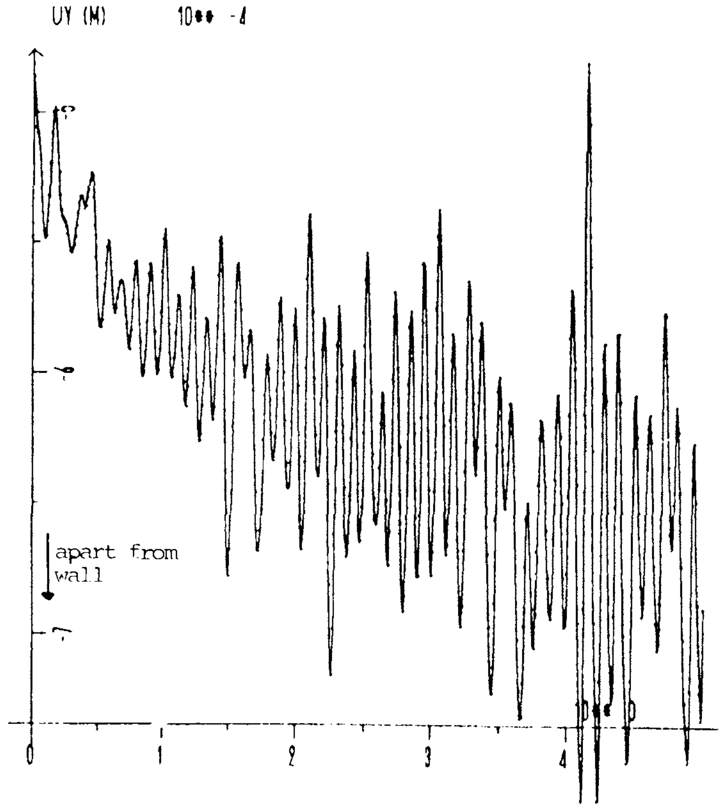

Fig. 6 :Top of the wall. Displacement vs time

It is to be noted that the low value of the obtained dis -placements is due to the very low value of the maximum acceleration of the seisme (0.02 g)

CONCLUSIONS

In reviewing some important factors contributing to the building of earthpressures during earthquakes we have tried to show that the incorporation of truly elastoplas -tic behaviour of the soil was not solely desirable but necessary. However the cost of nonlinear finite computa -tion is well known to be still high and thus it was found to be efficient to develop a nonlinear implicit-ex -plicit scheme where the stiff wall could be treated im -plicitely while the somewhat softer soil could be analy -sed by an explicit process. A new type of such a mixed algorithm has been thoroughly detailed and in a final section we have presented the first results that were ob -tained in a particular case. It is obviously too early to make definite conclusions but some experimentally ob -served results were found to be caught by our computa -tions. It seems now important to go to more quantitative informations, and a series of experimental tests on sca -led models will soon be conducted in order to validate the results by comparison.

ACKNOWLEDGMENT This research work has been undertaken under contract with Electricité de France /SEPTEN. The authors very kindly acknowledge Mr Betbeder for helpful discussions.

REFERENCES

Belytschko, T. and R. Mullen (1977), "Mesh Partitions of Explicit-Implicit Tim Integration", in Formulations and Computational Algorithms in Finite Element Analysis. Ed. J. Bathe et al. , MIT Press.

Ichihara, M. and H. Matsuzawa (1973), "Earth Presure during Earthquake": Soils and Foundations (13), 4, 75-86.

Ishii, Y. et al. (1960), "Lateral Earthe-Presure in an Earthquake", Proc. 2 nd World Conference on Earthquake Engrg. Tokyo.

Jacobsen, L.S. (1939), described in Appendix D of the Kentucky Project, Tech. Report N°13, Tennesse Valley Authority, 1951.

Hughes, T.J.R., and W.K. Liu (1978 a), "Implicit-Explicit Finite Element in Transient Analysis : Stability Theory". ASME Journal of Applied Mechanics (45), June 1978, 371-374.

Hughes, T.J.R., and W.K. Liu (1978 b), "Implicit-Explicit Finite Element in Transient Analysis : Implementation and Numerical Examples". ASME Journal of Applied Mechanics (45), June 78, 375-378.

Mononobe, N., and H. Matsuo (1929), "On the Determination of Earth Pressures during Earthquakes". Proc. World Engineering Congress, 9, 176.

Newmark, N.M. (1959), "A Method of Computation for Structural Dynamics". ASCE Journal of the Engineering Mechanical Division, (85), 3, 63-94.

Prakash, S. and B.M. Basavanna, (1969), "Earth Pressure Distribution behind Retaining wall during Earthquake". Proc. 4th World Conference on Earthquake Engrg. Santiago, Chile, A5, 133-148.

Prakash, S. and P. Nandakumaran, (1973), "Dynamic Earth-Pressure Distribution on Rigid Walls", Proc. Symposium Earth and Earthstructures Subjected to Earthquakes and other Dynamic Loads, Roorkee, India, I, II-16.

Prakash, S. and P. Nandakumaran, (1979), "Earth Pressure during Earthquakes", Proc. 2 nd National US Conference on Earthquake Engrg., 613-622.

Richards, R. and D.G. Elms, (1979), "Seismic Behaviour of Gravity Retaining Walls", ASCE Journal of the Geotechnical Engineering Division, 105, (4), 449-464.

Seed, H.B., and R.V. Whitman, (1970), "Design of Earth Retaining Structures for Dynamic Loads". Proc. Speciality Conference on Lateral Stresses and Earth Retaining Structures, Soil Mechanics and Foundations Division - ASCE.