An Approach to Predict Shear Modulus of Soils in the Range of 10^{-6} to 10^{-2} Strain Levels

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AN APPROACH TO PREDICT SHEAR MODULUS OF SOILS IN THE RANGE OF 10^{-6} TO 10^{-2} STRAIN LEVELS

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ABSTRACT

This paper presents the results of several resonant column tests carried out in a large variety of soils, including lateritic and saprolitic tropical soils. The data analysed are restricted to the strain dependence of shear modulus in the range of 10^{-6} to 10^{-2}.

Despite the different isotropic consolidated stresses, degrees of saturation and overconsolidation ratios applied for the different types of soil, all the test results fit very well in a previous proposed normalised curve of G/G_0 as a function of $\gamma/\gamma_{0.7}$, where $\gamma_{0.7}$ is the shear strain corresponding to a value of $G=0.7 \times G_0$. The hyperbolic stress-strain equation can conveniently describe this fitting.

Simple practical relationships are also proposed for tropical soils to the prediction of G_0 and $\gamma_{0.7}$ allowing with the normalised curve to estimate shear modulus in the strain range relevant to the service state of many civil engineering structures.

INTRODUCTION

The deformation parameters on the strain range of 10^{-6} to 10^{-2} are important design parameters. In fact, the strains operating in the ground interesting the serviceability limit state of structures are usually smaller than about 5×10^{-3} (Burland 1989, Tatsuoka et al. 1997, Biarez et al. 1999).

This strain range is too small to be determined by conventional laboratory testing; for example in routine triaxial tests, accurate measurements are only possible for strains greater than 10^{-3} and more usually 10^{-2}. To overcome this problem, Biarez and Hicher (1994), Biarez et al. (1999) proposed a method for classifying and comparing tests, that uses groupings of parameters to obtain « reference behaviours ».

It has been revealed that in the 10^{-6} to 10^{-3} strain range the soil exhibits for very small strains (generally until around 10^{-5}) a quasi-elastic behaviour, and then a hysteretic and non-linear behaviour, where the secant stiffness decreases with the increasing of strain level. In practice, it is therefore important to evaluate the stiffness of soil for the stress and strain states that are relevant for a specific geotechnical design.

In many practical dynamic analysis the equivalent linear method is used, and the stiffness degradation curve of soil (G/G_0-γ) is required. Despite the significant number of parameters that may affect shear modulus degradation curve a quasi-unique curve « reference behaviour » has been proposed by Santos (1999): the secant shear modulus for a given shear strain level (G) is normalized by the very small-strain (elastic) shear modulus (G_0 or G_{max}) and the shear strain by a reference threshold shear strain (γ_t^r) corresponding to a stiffness degradation factor (G/G_0) of 0.7 ($\gamma_{0.7}=\gamma_t^r$). This normalization follows the original hyperbolic stress-strain relationship tested successfully for sands in cyclic tests, by Teachavorasinskun et al. (1991). This will be discussed later in modelling.

In this paper the « reference behaviour » proposed by Santos (1999) is generalized for a great variety of lateritic and saprolitic tropical soils, using test results obtained with resonant column apparatus. To make this procedure practical for engineering using, some empirical relationships are proposed to obtain the parameters of normalization (G_0, $\gamma_{0.7}$), for tropical soils.

QUASI-UNIQUE STIFFNESS DEGRADATION CURVE. BACKGROUND

It is well known that the strain-dependent curves G/G_0 depend mainly on soil plasticity in cohesive soils (Vucetic and Dobry, 1991) and are affected by the mean effective stress in cohesionless soils (Ishibashi and Zhang, 1993). But it was found that the influence of these factors can be considered in a simple form when using a normalised shear strain defined by: $\gamma^*=\gamma/\gamma_t^r$, with $\gamma_t^r = \gamma_{0.7}$ (Santos, 1999; Santos and Gomes Correia, 2000).

This normalization is based on the key parameter γ_t^r which is a reference threshold shear strain. Its meaning is closely related to the concept of volumetric threshold shear strain (Vucetic, 1994) which represents the limit beyond which the soil structure starts to change irreversibly: in drained conditions permanent volume change will take place, whereas in undrained conditions pore water pressure will build up.

In a more practical point of view, the reference threshold shear strain defines the beginning of significant stiffness degradation and also the beginning of considerable increase of hysteretic damping. These evidences suggest the idea to perform the normalisation using the reference threshold shear strain and it was shown that it is possible to define a quasi-unique strain-dependent stiffness degradation curve for sands and clays. Figure 1 shows how the results of Vucetic and Dobry (1991) and Ishibashi and Zhang (1993) can be fitted inside two simple boundary curves, for soils with different plasticity indexes (PI=NP to 50%) and subjected to confining pressure varying between 1 to 400kPa.

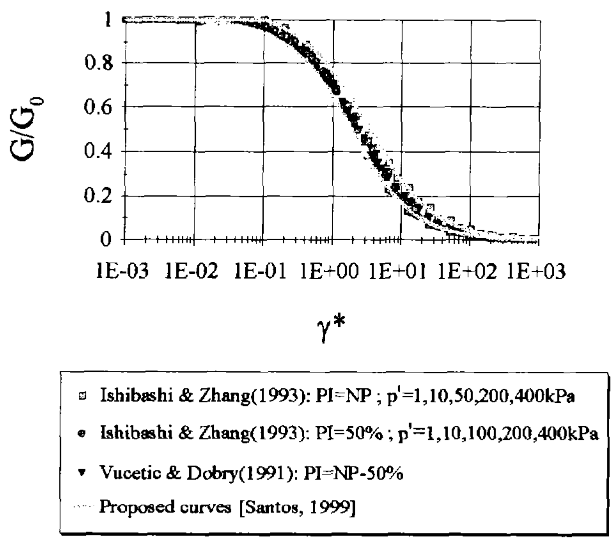

Figure 1 – Stiffness degradation curves in γ scale*

Santos (1999) proposed two simple equations to define the lower and upper bound values of G/G_0 as a function of γ^* (for $10^{-6}<\gamma<10^{-2}$):

$$\text{lower bound}\begin{cases} 1 & ,\gamma^* \le 10^{-2} \\ \dfrac{1-\tanh[0.48\cdot\ln(\gamma^*/1.9)]}{2} & ,\gamma^* > 10^{-2} \end{cases} \quad (1)$$

$$\text{upper bound}\begin{cases} 1 & ,\gamma^* \le 10^{-1} \\ \dfrac{1-\tanh[0.46\cdot\ln(\frac{\gamma^*-0.1}{3.4})]}{2} & ,\gamma^* > 10^{-1} \end{cases} \quad (2)$$

A hyperbolic equation can also be used to modelling test results in a more easy way. Teachavorasinskun *et al. (1991)* proposed the following relationship:

$$G/G_0 = 1/[1+(\gamma/\gamma_{0.5})] \quad (3)$$

where $\gamma_{0.5}$ is the shear strain at the value of G/G_0 equal to 0.5. The authors noted that if $\gamma_{0.5}$ is equal to the reference strain, $\gamma_r = \tau_{max}/G_{max}$, then, it coincides with the original stress-strain hyperbolic equation proposed by Hardin and Drnevich (1972), which is basically derived from Kondner (1963) stress-strain relationship proposed for soils and rocks under monotonic loading triaxial compression tests, which is:

$$q = \varepsilon_1/(a + b \times \varepsilon_1) \quad (4)$$

In fact, if in this equation we replace q by τ, ε_1 by γ and the coefficients a and b by $1/G_{max}$ and $1/\tau_{max}$, respectively, we obtain the equation (3).

This equation has been largely used, however, in order to fit better the experimental results, some correction factors have been used for either or both the peak strength and the initial shear modulus. The drawback of fitting approach is that these correction factors are not constant and are function of strain level. Furthermore, for a strain level corresponding to G/G_0 equal to 0.5, the soil will be submitted to significant degradation and consequently this strain level will be not appropriate as a reference strain. For this reason we adopt for modelling a more stable strain level, i.e, $\gamma_{0.7}$ (to G/G_0=0.7); the same reference strain used by Santos (1999).

TESTING PROCEDURES

Test materials

Laboratory resonant column tests were carried out on soils from ten different sites, four in São Paulo City and six in other cities of São Paulo State, Brazil.

The fundamental geotechnical properties of each sample are shown in Table 1 and 2 respectively for lateritic and saprolitic soils.

Resonant Column apparatus

The resonant column test (RCT) is one of the most accurate and repetitive ways of determining the small strain shear modulus (Lo Presti et al., 2000).

The apparatus used in the present tests takes cylindrical specimens, fixed at the basis and excited in torsion at the top. This apparatus has been described by Hardin and Music (1970). The specimens tested were solid cylinders with 3.6 cm in diameter and 8.0 cm in height. For some samples, as indicated in Table 1 and 2, the procedure used to performing the resonant column test was the multistage technique, in which different magnitudes of confining pressure are applied to the same specimen. For the remaining samples, each specimen was tested in only one confining pressure.

For each confining pressure, a torsional excitation of very low amplitude was applied at the top of the specimen and the shear modulus was determined at logarithmic time intervals up to 1,000 minutes, or up to 10,000 min in some cases. Subsequently the torsional excitation was gradually increased and the variations of the shear modulus with the shear strain amplitude were determined.

When the multistage technique was used, for the next stages of the test, a similar procedure was repeated. Before starting each new stage, the regain of modulus was observed as recommended by Anderson et al. (1978).

STIFFNESS DEGRADATION CURVES FOR LATERITIC AND SAPROLITIC SOILS

Tropical soils exhibit peculiar characteristics in relation to sedimentary soils. The expression "tropical soil" includes saprolitic and lateritic soils. The former are necessarily residual soils and are characterized by exhibiting the relict rock structure. The latter can be residual or transported and are distinguished by the process of laterization suffered in nature. The behaviour of tropical soils is strongly associated with their frequent unsaturated natural condition and with the occurrence of particle cementation. They usually present some inter-particle bonding. In saprolitic soils, this bonding is inherited from the parent rock whereas in lateritic soils, is due to pedologic evolution.

With respect to stiffness degradation curve, Barros (1997) observed that tropical soils show a more dramatic reduction of modulus with strain than that indicated in the literature (Ishibashi and Zhang, 1993, Vucetic and Dobry, 1991), especially saprolitic soils. Figure 2 shows an example how much degradable is a saprolitic soil compared with a lateritic one for approximately the same confining pressure.

The stiffness degradation curves obtained by Barros (1997) will be used in this paper to validate the "reference behaviour" proposed by Santos (1999) and for modelling.

Table 1. Lateritic soils investigated

Site	Depth (m)	Geological Origin	PI (%)	σ_c (kPa)	ρ (g/cm^3)	w (%)	e	Sr (%)	Confining pressure with determination of stiffness degradation curve (kPa)
Bela Vista	2.0	Red clays -	39	250	1.47	39.1	1.57	68	24.5 – 49 –98 - 245 – 490
(São Paulo)	8.0	São Paulo	34	450	1.69	31.6	1.14	76	24.5 – 49 –98 - 245 – 490
Moema	1.5	Sedimentary	37	210	1.39	36.6	1.67	60	588
(São Paulo)	4.6	Basin	17	1250	1.74	36.1	1.14	87	588
Sorocaba	6.5	residual	11	200	1.53	25.4	1.24	56	49 - 98 - 196 (*)
	11.5	(claystone)	13	840	1.89	22.7	0.78	80	49 - 98 - 196 (*)
Guaíra	1.5	residual (basalt)	23	60	1.37	33.1	2.03	51	24.5 - 49 – 98 – 245
Unicamp – Campinas	2.75	residual (diabase)	16	100	1.38	23.2	1.69	41	49 - 98 - 245 – 490
Bauru	0.5	residual	4	95	1.64	9.1	0.76	32	19.6 - 49 – 98 (*)
	4.8	(sandstone)	5	140	1.70	10.4	0.72	38	49 - 98 - 196 (*)
	8.85		7	120	1.79	8.3	0.61	36	49 - 98 - 196 (*)
São Carlos	1.4	cenozoic	13	38	1.48	14.6	1.13	36	19.6 - 49 – 98 (*)
	4.6	sediment	13	68	1.69	18.0	0.90	54	49 - 98 - 196 (*)
Campinas	0.95	residual	20	60	1.36	25.1	1.73	43	19.6 - 49 – 98 (*)
	4.5	(diabase)	11	85	1.37	23.3	1.80	40	49 - 98 - 196 (*)
	6.8		14	120	1.56	28.0	1.56	56	49 - 98 - 196 (*)

(*) multistage technique

Table 2. Saprolitic soils investigated

Site	Depth (m)	Geological Origin	PI (%)	σ_c (kPa)	ρ (g/cm^3)	w (%)	e	Sr (%)	Confining pressure with determination of stiffness degradation curve (kPa)
C. Universit.	7.1	residual	15	960	1.95	21.0	0.68	83	98 - 196 – 294 (*)
(São Paulo)	8.5	(migmatite)	18	980	1.99	19.5	0.65	82	98 - 196 – 294 (*)
Caxingui	11.7	residual	19	135	2.03	17.4	0.54	86	196 – 392 (*)
(São Paulo)	15.0	(gneiss)	18	110	1.82	18.5	0.72	68	294 – 392 (*)
São Carlos	8.2	resid. (sandstone)	16	190	1.89	13.1	0.59	59	49 - 98 - 196 (*)
Campinas	7.8	residual (diabase)	16	120	1.52	27.7	1.32	58	49 - 98 - 196 (*)

(*) multistage technique

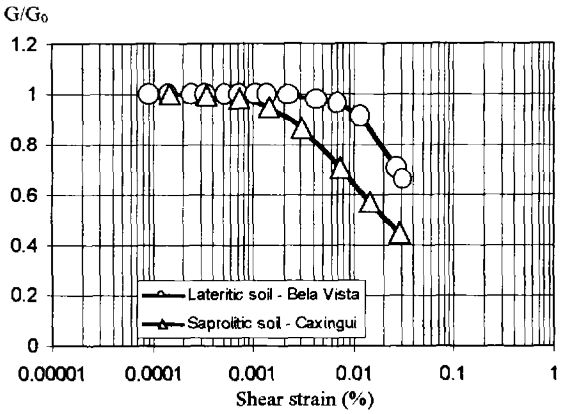

Figure 2. Comparison of stiffness degradation curves for lateritic and saprolitic soils

GENERALISATION OF QUASI-UNIQUE STIFFNESS DEGRADATION CURVE FOR LATERITIC AND SAPROLITIC SOILS

Validation of stiffness degradation curve for lateritic and saprolitic soils and modelling

Figure 3 shows the results of 37 tests of lateritic and saprolitic soils represented in the normalized scale proposed in Figure 1. The reason of using only 37 test results is because in only these tests the degradation level of $G/G_0=0.7$ was achieved. It can be seen that for the range of shear strain tested, the relationship between G/G_0 and $\gamma^*=\gamma/\gamma_t^r$ is scarcely affected by the kind of soils (temperate or tropical soils), plasticity index, confining pressure, degree of saturation and overconsolidate ratio. This validates and generalizes this type of relationship proposed by Santos (1999) as a "reference behaviour" curve (stiffness degradation reference curve) for soils.

The average "reference behaviour" for tropical soils can be approximately modelled by using the hyperbolic relationship in equation (5) with a correction factor for the normalized shear strain $\gamma/\gamma_{0.7}$. This relationship is represented in Figure 3, which equation is:

$$G/G_0 = 1/\{1+[0.35\times\gamma/(\gamma_{0.7})]\} \qquad (5)$$

Empirical rules to obtain small strain shear modulus and reference shear strain

As shown by Barros (1997), lateritic and saprolitic soils show very distinct dynamic properties, which can not usually be predicted by the correlations determined from temperate zone soils.

Figure 4 shows a comparison of experimental data of Brazilian lateritic tropical soils with correlations between G_0 from crosshole tests and N_{SPT} obtained by some investigators (Ohsaki and Iwasaki, 1973, Imai and Tonouchi, 1982, Seed et al., 1983) in temperate zone soils. As can be seen, for lateritic soils, those expressions underestimate in a significant way the G_0 value.

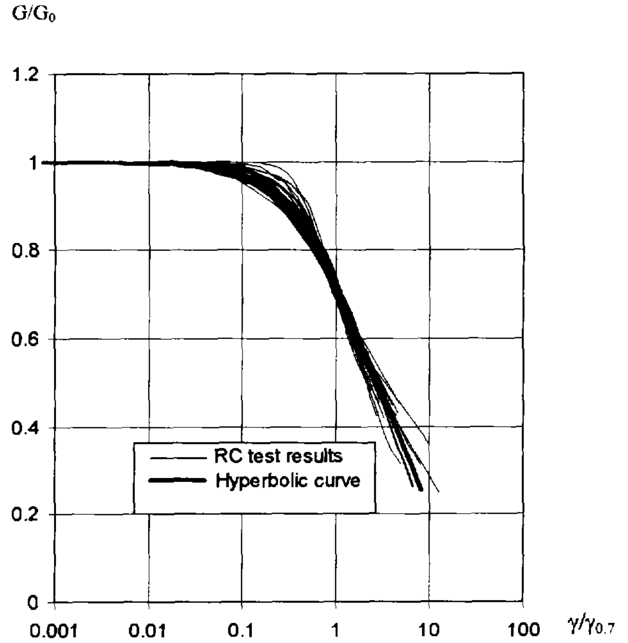

Figure 3. Relationship between normalized secant shear modulus and normalized shear strain $(\gamma/\gamma_{0.7})$ and hyperbolic fitting

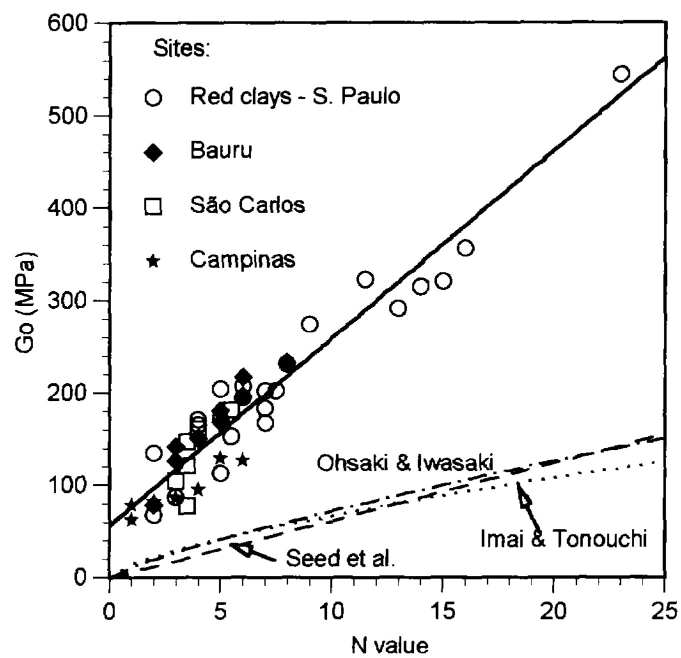

Figure 4. Experimental data for lateritic soils and correlations obtained from temperate zone soils (Barros, 1997a)

For example, the relationship between values of G_0 measured and G_0 calculated by Ohsaki and Iwasaki expression varied between 2.5 to 6.8, with an average value of 4.0. On the other hand, for saprolitic soils, the measured values of G_0 are higher than the predicted values for low values of N, occurring the opposite for high values of N. The following correlations were obtained by Barros (1997, 1997a):

for lateritic soils:

$$G_0 = 56 + 20{,}3\ N\ (r = 0{,}954) \quad (6)$$

and for saprolitic soils:

$$G_0 = 94 + 2{,}3\ N\ (r = 0{,}932) \quad (7)$$

where G_0 is in MPa.

Similar conclusions can be obtained from laboratory tests. Figures 5 and 6 show the comparison between G_0 values calculated by the well-known Hardin expression (Hardin, 1978) with experimental data obtained in resonant column tests. Hardin equation underestimates the G_0 value for all lateritic soils tested. In a total of 148 determinations, it was obtained an average value of 2.4 for the relationship between G_0 measured to G_0 calculated by Hardin equation. New correlations were proposed for these soils (Barros, 1997). For the saprolitic soils, their behaviour is more similar to temperate zone soils, and the amount of data available is smaller. Therefore, it is suggested that Hardin expression be used for preliminary estimates of G_0 in saprolitic soils.

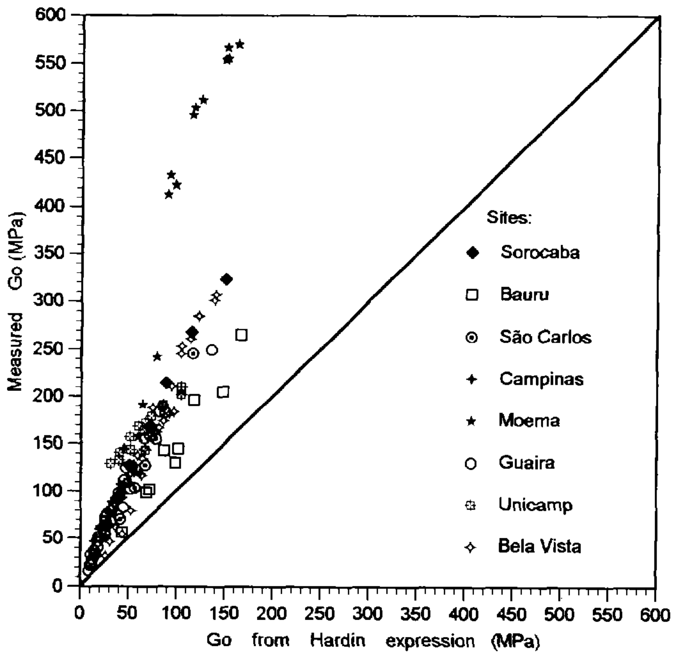

Figure 5. Comparison of calculated and measured values of Go in resonant column tests - Lateritic soils

In which concerns the reference shear strain, $\gamma_{0.7}$, there is not a unique relationship for this parameter for the tropical soils. The best correlation found is against the consolidation pressure, p'_0 (Fig. 7). It can be observed, both for lateritic and saprolitic soils, an increase of $\gamma_{0.7}$ with the increase of p'_0. However, the saprolitic soils exhibits for the same p'_0 a lower $\gamma_{0.7}$, despite the slope of the curves are approximately the same. This shows that the saprolitic soils have a lower volumetric threshold shear strain compared with the lateritic soils, and consequently the beginning of stiffness degradation for much smaller shear strain levels, let say around 7×10^{-5}.

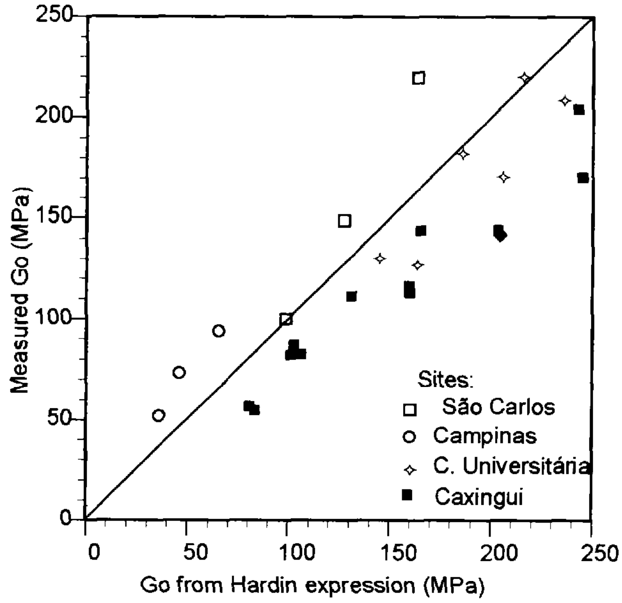

Figure 6. Comparison of calculated and measured values of Go in resonant column tests - Saprolitic soils

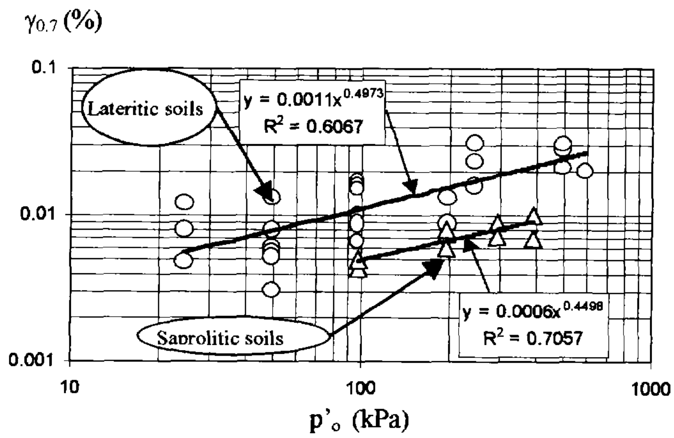

Figure 7. Relationships between $\gamma_{0.7}$ and p'_o for lateritic and saprolitic soils

CONCLUSIONS

Tropical soils exhibit peculiar stiffness behaviour compared with sedimentary and remoulded soils. They show a significant reduction of stiffness with strain, mainly the saprolitic soils.

Based on an important number of tests performed with resonant column apparatus in tropical soils it was possible to generalize the quasi-unique stiffness degradation curve established for a great variety of soils, which could be considered as a "reference behaviour" curve for soils. For the range of strains tested it is scarcely affected by the type of soils, plasticity index, confining pressure, degree of saturation and overconsolidation ratio. This "reference behaviour", expressed by the relationship between the normalized shear modulus, G/G_0, and the normalized shear strain, $\gamma/\gamma_{0.7}$, can be modeled by an average curve, fitted by an hyperbolic equation with a correction factor for the normalized shear strain. To make this applicable in engineering practice, mainly in dynamic analysis, empirical rules have been proposed to obtain the normalized parameters, G_0 and $\gamma_{0.7}$, for tropical soils.

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