



Missouri University of Science and Technology
Scholars' Mine

International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics 1981 - First International Conference on Recent Advances in Geotechnical Earthquake Engineering & Soil Dynamics

29 Apr 1981, 9:00 am - 12:30 pm

Soil-Pile Interaction Parameters in Vertical and Torsional Vibrations

N. R. Krishnaswamy
I.I.T., Madras, India

B. K. Sharas Chandra
I.I.T., Madras, India

Aftab A. Salam
I.I.T., Madras, India

R. Daniel
I.I.T., Madras, India

Follow this and additional works at: <https://scholarsmine.mst.edu/icrageesd>



Part of the [Geotechnical Engineering Commons](#)

Recommended Citation

Krishnaswamy, N. R.; Chandra, B. K. Sharas; Salam, Aftab A.; and Daniel, R., "Soil-Pile Interaction Parameters in Vertical and Torsional Vibrations" (1981). *International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*. 4.

<https://scholarsmine.mst.edu/icrageesd/01icrageesd/session04/4>

This Article - Conference proceedings is brought to you for free and open access by Scholars' Mine. It has been accepted for inclusion in International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics by an authorized administrator of Scholars' Mine. This work is protected by U. S. Copyright Law. Unauthorized use including reproduction for redistribution requires the permission of the copyright holder. For more information, please contact scholarsmine@mst.edu.



Soil-Pile Interaction Parameters in Vertical and Torsional Vibrations

N. R. Krishnaswamy, Asst. Professor, B. K. Sharas Chandra, Research Scholar,

Aftab A. Salam and Smt. R. Daniel

Department of Civil Engineering, I.I.T., Madras, India.

SYNOPSIS In this paper, two lumped parameter analogues with the inclusion of constant friction damping are suggested to explain the dynamic behaviour of pile supported footings during vertical as well as torsional modes of vibrations. Simple theoretical procedures have been described by which the constant frictional force and the constant frictional moment of the friction damping can be evaluated during vertical and torsional vibrations respectively. The suggested procedures take into consideration the relevant physical characteristics of the interface between the pile and the soil, as also the length to diameter (L/d) ratio of the pile.

Field vibratory tests have been carried out on model pile supported footings under steady state vertical and torsional excitation. The test data obtained are compared with the values predicted by the proposed theory and the agreement between the two is found to be satisfactory.

INTRODUCTION

The dynamic analysis of pile supported foundations find their applications in machine foundations and other types of structures exposed to vibratory loads from moving machinery and dynamic loads such as due to wind, earthquake, or waves. During vibrations the development of slip in the regions of high shear is quite likely. The existing theories of analysis overestimate the real response by neglecting the damping forces which are inevitable as a result of slip at the interface of pile and soil.

The mathematical models suggested by Novak (1977) and Novak and Howell (1977) to study the dynamic response of piles in vertical and torsional excitation respectively, are taken as the basis for the proposed analytical models in this paper.

PROPOSED THEORETICAL MODELS

During vertical and torsional vibrations of a pile supported foundation, there is a likelihood of slip in the region of high shear and friction can be developed along the soil pile interface. The friction so developed is included as lumped parameter in the analogue models. The lumping of the soil-pile interaction parameters with the corresponding parameters for the footing is done in the same manner as attempted by Krishnaswamy (1972, 1976).

Then, the differential equations of motion of a pile supported footing during vertical and torsional vibrations, respectively, can be written as

$$M\ddot{Z} + [C_{ZF} + C_{ZP}]\dot{Z} + [K_{ZF} + K_{ZP}]Z = F = Q(t) \quad (1)$$

$$I_0\ddot{\theta} + [C_{\theta F} + C_{\theta P}]\dot{\theta} + [K_{\theta F} + K_{\theta P}]\theta = M_{\theta F} = T(t) \quad (2)$$

where

- Z = vertical translation
- θ = torsional rotation
- M = total mass of the footing and the machine together with the mass of pile
- I_0 = mass moment of inertia of the footing about the centroidal axis of rotation
- F = frictional force mobilised during vertical vibration
- $M_{\theta F}$ = frictional moment mobilised during torsional vibration
- $Q(t)$ = exciting force = $m_0 e w^2 \sin wt$
- $T(t)$ = exciting torque = $\dot{m}_0 e l w^2 \sin wt$

where

- m_0 = eccentric rotating mass
- e = eccentricity of the mass
- l = lever arm
- w = circular frequency of excitation

The value of spring constant for the footing in vertical and torsional vibration respectively is given by

$$K_{ZF} = 4Gr_0 / (1 - \mu) \quad (3)$$

$$K_{\theta F} = (16/3) G r_0^3 \quad (4)$$

where

- G = dynamic shear modulus of the soil
- μ = Poisson's ratio of the soil
- r_0 = equivalent radius of footing

Damping constant for the footing is to be calculated from the expressions

$$D_{ZF} = C_{ZF}/2\sqrt{(K_{ZF} M)} = 0.425/\sqrt{B_{ZF}} \quad (5)$$

$$D_{OF} = C_{OF}/2\sqrt{(K_{OF} I_0)} = 0.5/(1 + 2B_{OF}) \quad (6)$$

$$B_{ZF} = (1 - \mu) M/(4 \rho r_0^3) \quad (7)$$

$$B_{OF} = \text{Mass ratio} = I_0/\rho r_0^5 \quad (8)$$

where

$$\rho = \text{mass density of soil}$$

The natural frequency w_n of the pile-supported footing is given by

$$w_{nz} = \sqrt{[(K_{ZF} + K_{ZP})/M]} \quad (9)$$

$$w_{n\theta} = \sqrt{[(K_{OF} + K_{OP})/I_0]} \quad (10)$$

where

$$K_{ZF} + K_{ZP} = \text{combined spring constant of the footing and pile in vertical vibration}$$

$$K_{OF} + K_{OP} = \text{combined spring constant of the footing and pile in torsional vibration}$$

Also, stiffness and damping constants for the pile during vertical and torsional vibrations can be evaluated as described below.

Novak(1977) and Novak and Howell(1977) have reported that following dimensionless parameters govern the soil-pile interaction in vertical and torsional vibrations: shear wave velocity v_p/v_s and v_b/v_s ; slenderness ratio, l'/r_0' ; mass ratio, ρ/ρ_p ; dimensionless frequency, a_0 ; and material damping ratio, $\tan \delta$. Here,

$$v_b = \text{shear wave velocity of soil below pile tip}$$

$$v_p = \text{shear wave velocity of pile}$$

$$v_s = \text{shear wave velocity of soil above pile tip}$$

$$l' = \text{length of pile}$$

$$r_0' = \text{effective radius of pile}$$

$$\rho = \text{mass density of soil}$$

$$\rho_p = \text{mass density of pile}$$

$$a_0 = r_0' w \sqrt{(\rho/G)}$$

$$G = \text{shear modulus of soil}$$

$$w = \text{angular frequency}$$

Even though the stiffness and damping of piles vary with the above listed factors, their weak dependence on frequency is fortunate since it simplifies the parametric study and makes it possible to choose approximately constant values of stiffness and damping terms for design purposes.

From an examination of theoretical results reported by Novak (1977), it can be observed that in the vertical mode of vibrations, the

stiffness and damping parameters, f_{w1} and f_{w2} can be approximated to constant values for piles of slenderness ratio ranging between 20 and 100. Constant values of these parameters can be chosen from corresponding graphical solutions presented for various values of ρ/ρ_p , v_b/v_s and end conditions of these piles. Since these parameters have been so chosen as to be frequency independent, the stiffness and damping of piles evaluated from these parameters can be lumped with the corresponding terms of the footing or pile cap to form single equivalent spring and dashpot terms.

From an examination of theoretical results reported by Novak and Howell (1977), it can be observed that for the torsional mode of vibrations, the stiffness and damping parameters for reinforced concrete piles can be approximated to constant, frequency independent values within the range of frequencies, $0.5 < a_0 < 1.5$, for given values of ρ/ρ_p and $\tan \delta$. Hence, the stiffness and damping of piles, evaluated from parameters $f_{T,1}$ and $f_{T,2}$ can be lumped with the corresponding terms for the footing or pile cap to form single equivalent spring and dashpot terms respectively.

The solutions of Eqns. (1) and (2) for the case of vibrations under frequency dependent sinusoidal excitation can be obtained by a similar procedure reported by Anandakrishnan and Krishnaswamy (1973) for vertical vibrations and Krishnaswamy (1976) for torsional vibrations. The theoretical solutions are presented in nondimensional graphical form in Figs. 1 and 2.

Evaluation of Friction Parameters

For a pile embedded in a C- ϕ soil, the total amount of interfacial friction that would be mobilised during vibrations can be written as

$$F = [0.5 k_0 \gamma l'^2 \mu_f + C_a l'] L \quad (11)$$

where

$$k_0 = \text{lateral earth pressure coefficient at rest}$$

$$\mu_f = \text{coefficient of kinematic friction}$$

$$C_a = \text{wall adhesion under dynamic condition}$$

$$L = 2\pi r_0' = \text{perimeter length of embedded pile}$$

$$\gamma = \text{density of soil}$$

For a pile embedded in a C- ϕ soil, the total frictional moment can be written as

$$M_{FO} = F r_0' \quad (12)$$

where F is obtained from Eqn. (11).

The above equations have been developed by adopting the same assumptions as Anandakrishnan and Krishnaswamy (1973) and Krishnaswamy (1976). F and M_{FO} can be obtained from Fig. 3 for various pile slenderness ratios.

EXPERIMENTAL INVESTIGATION

Field vibratory tests on two piles of reinforced concrete, were conducted to obtain data to check the validity of the theoretical approaches mentioned above.

The experimental program and the test procedure have been described in detail by Rosamma (1980) and hence not reported here. The results of the tests for vertical and torsional vibrations are given in Tables 1 and 2.

SUMMARY AND CONCLUSIONS

The field test data obtained from the present experimental investigation are compared with the values predicted by the proposed theoretical models using Figs. 1 and 2. From an examination of the results tabulated in Tables 1 and 2, it is evident that there is good agreement between the behaviour of the proposed theoretical models and observed results.

REFERENCES

Anandakrishnan, M., and Krishnaswamy, N.R. (1973) 'Response of Embedded Footing to Vertical Vibrations', Jnl. of Soil Mech. and Foun. Div., ASCE, Vol. 90, No. SM10, pp.863-883.

Krishnaswamy, N.R. (1972): 'A Study of the effect of Embedment and Related Factors on the Dynamic Behaviour of Footings; Ph.D. thesis, IIT, Kanpur, India, p. 293.

Krishnaswamy, N.R. (1976): 'Torsional Vibrations of Embedded Footings', Geotechnical Engg., Vol. 1, Jun., pp. 47-55.

Novak, M. (1977): 'Vertical Vibrations of Floating Piles', Jnl. of the Engg. Mech. Div., ASCE, Vol. 103, No. EML, Proc. Paper 12747, Feb., pp. 153-168.

Novak, M., and Howell, J.F. (1977): 'Torsional Vibrations of Pile Foundations', Jnl. of Geo. Engg. Div., ASCE, Vol. 103, No. GT 4, Proc. paper 12850, Apr., pp. 271-285.

Rosamma, D. (1980): 'Dynamic Response of a Footing Supported by a Friction Pile', M.Tech. dissertation, IIT, Madras, India, p. 107.

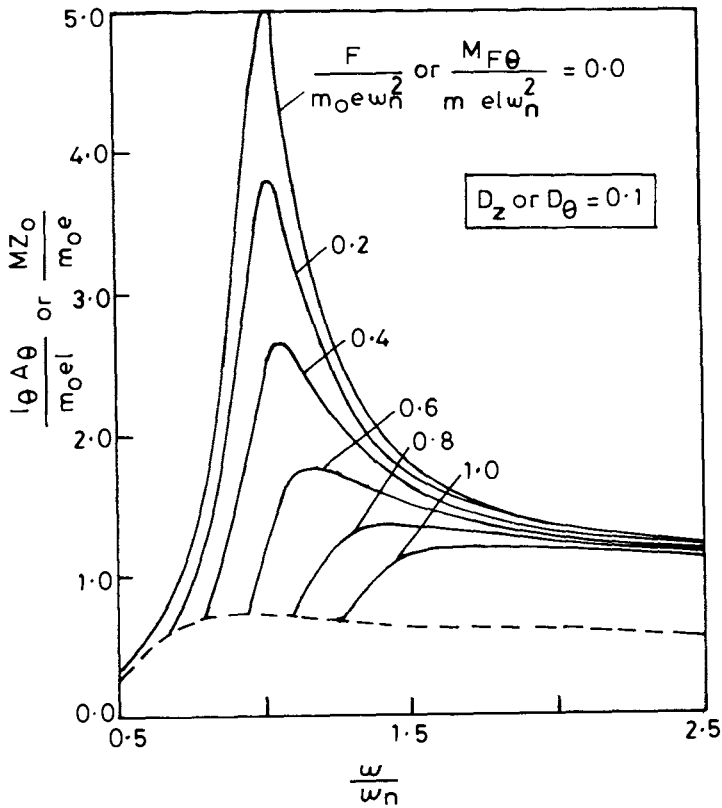


Fig.1 Typical Theoretical Response Curves.

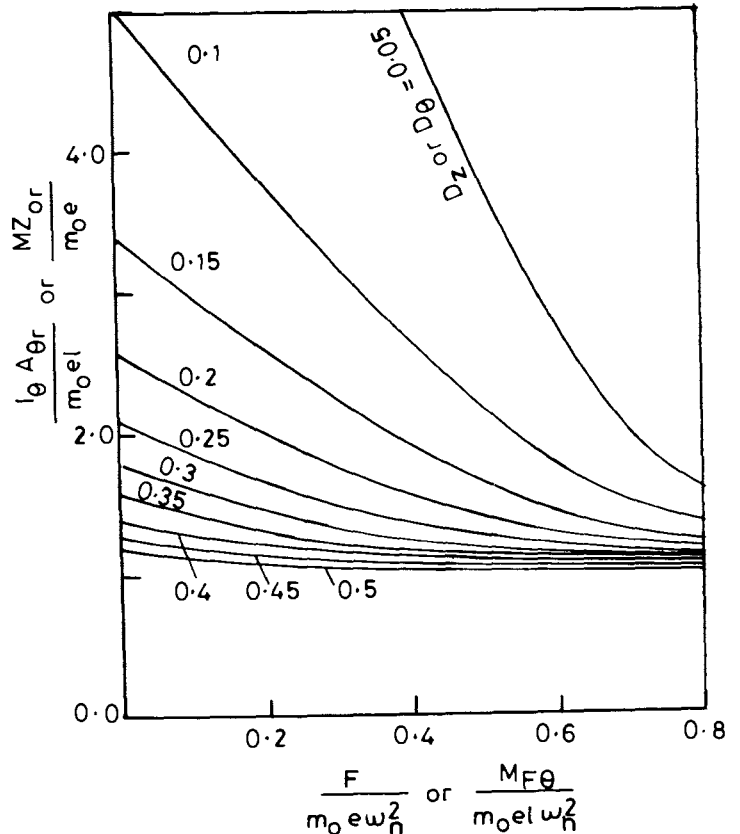


Fig.2 Decrease of Maximum Amplitude at Resonance with Friction Factor.

TABLE I: Vertical Response - Prediction and Comparison.

Addl. wei-ghts. added kg	Friction factor $F/m_0 e w_n^2$	D_z	Predicted values		Experimentally observed value	
			X_{or} mm	w_r rad/sec	X_{or} mm	w_r rad/sec
Pile length = 100 cms.						
223.0	0.488	0.0415	1.327	98.40	0.530	226
327.0	0.614	0.0400	0.618	91.20	0.485	218
425.0	0.733	0.0394	0.384	91.07	0.455	211
Pile length = 125 cms.						
223.0	0.593	0.0425	0.855	114.00	0.545	222
327.0	0.745	0.0412	0.408	113.36	0.490	214
425.0	0.888	0.0402	0.266	117.77	0.420	214

Weight of Vibrator + eccentric mass = 40.6 kg
 Weight of base plate = 32.0 kg
 Weight of 100 cm length pile = 18.9 kg
 Weight of 125 cm length pile = 23.6 kg
 Weight of footing = 180.0 kg

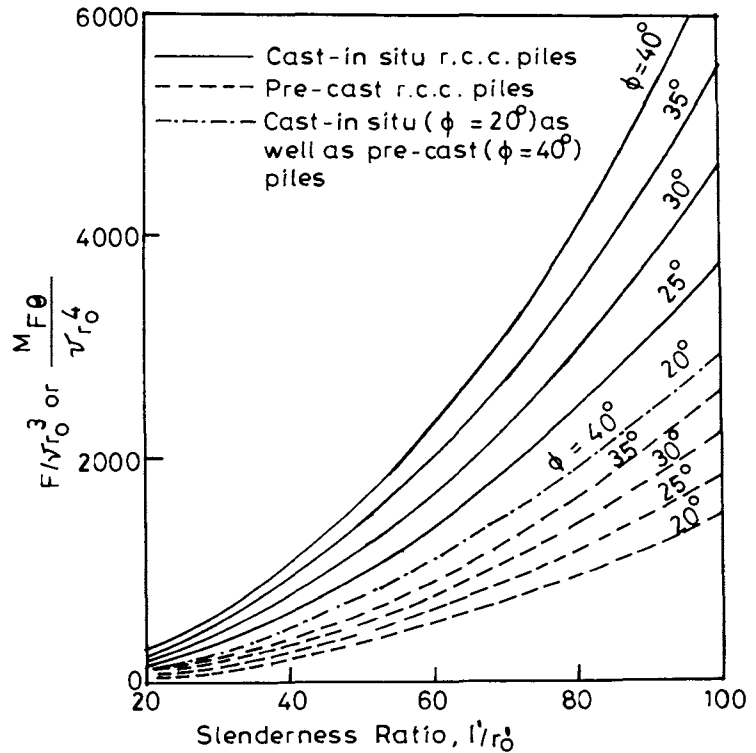


Fig.3 Evaluation of Frictional Force or Moment for Piles in Cohesionless Soils

TABLE II: Torsional Response - Prediction and Comparison.

Addl. wei-ghts. added kg	D_0	With mat. damp- ing	With- out mat. damp- ing	w_n rad/ sec	M_{F0} kg. cm	$M_{F0} / (m_0 e l w_n^2)$	Experimentally observed values w_r rad/ sec	Predicted results				
								A_{Or} (dimension- less)	w_r (rad/ sec) with mat. damp- ing	w_r (rad/ sec) without mat. damp- ing	A_{Or} (dimension- less) with mat. damp- ing	A_{Or} (dimension- less) without mat. damp- ing
Pile length = 100 cms.												
113.5	0.078	0.048	317.5	1606.5	0.301	176	3.71	317.5	317.5	4.1	6.4	0.0529
223.0	0.064	0.034	263.3	2009.0	0.550	163	4.05	283.1	263.3	2.6	4.3	
327.0	0.057	0.027	231.3	2393.4	0.850	201	4.50	312.2	283.3	1.5	1.7	
113.5	0.078	0.048	317.5	1606.5	0.230	176	4.60	317.5	317.5	5.0	7.3	0.0706
223.0	0.064	0.034	263.3	2009.0	0.410	163	4.50	263.3	263.3	3.8	7.1	
327.0	0.057	0.027	231.3	2393.4	0.630	157	4.30	259.0	238.2	2.4	3.8	
113.5	0.078	0.048	317.5	1606.5	0.162	176	2.58	317.5	317.5	5.8	8.3	0.0979
223.0	0.064	0.034	263.3	2009.0	0.296	157	2.70	263.3	263.3	5.2	9.2	
327.0	0.057	0.027	231.3	2393.4	0.457	138	2.72	231.3	231.3	4.0	8.0	
425.0	0.053	0.023	209.8	2750.2	0.638	144	2.89	237.1	218.2	2.2	4.3	
Pile length = 125 cms.												
113.5	0.078	0.048	316.6	1681.5	0.317	100	3.60	316.6	316.6	4.0	6.2	0.0529
223.0	0.064	0.034	262.6	2084.1	0.572	88	3.13	286.2	262.6	2.5	4.1	
327.0	0.057	0.027	230.6	2468.3	0.877	82	2.70	315.9	288.3	1.4	1.5	
113.5	0.078	0.048	316.6	1681.5	0.238	106	3.37	316.6	316.6	5.0	7.2	0.0706
223.0	0.064	0.034	262.6	2084.1	0.428	100	3.07	265.2	262.6	3.5	6.6	
327.0	0.057	0.027	230.6	2468.3	0.657	94	2.95	265.2	241.0	2.2	3.6	
425.0	0.052	0.022	209.2	2825.0	0.914	88	2.84	297.1	272.0	1.3	1.5	
113.5	0.078	0.048	316.6	1681.5	0.171	106	2.76	316.6	316.6	5.6	8.2	0.0979
223.0	0.064	0.034	262.6	2084.1	0.309	94	2.75	316.6	316.6	5.1	8.9	
327.0	0.057	0.027	230.6	2468.3	0.474	82	2.28	234.1	230.6	3.8	7.7	