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Systematic Protocol for SASW Inversion

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ABSTRACT

Spectral-analysis-of-surface-waves (SASW) is a nondestructive test method for characterization of the variation with depth of the shear modulus of soils. One drawback in SASW is the need for an experienced user to conduct the inversion. Difficulty in SASW inversion arises from lack of constraint of the least squares minimization used on shear wave velocity parameters. For even simple profiles, the inversion algorithm can exhibit instability due to numerical sensitivity of the forward model calculations. The user must provide a reasonable starting profile; and then the parameters must be carefully followed and constrained to reach convergence. The inversion process was explored using a range of dispersion curves ranging from simple to complex layering systems. Three key principles were built into a new protocol to provide necessary constraints on the inversion algorithm. Dispersion data from many test sites have been inverted using the new protocol. Careful adherence to the protocol consistently produces shear wave velocity profiles indicative of site conditions. The protocol provides logic necessary for automation of the inversion process.

INTRODUCTION

The spectral-analysis-of-surface-waves (SASW) method is a testing procedure for determining shear wave velocity (shear modulus) profiles of soil systems in situ. The test is performed from the ground surface without boreholes. Measurements are made at strain levels below 0.001 percent, where elastic properties of soil are considered independent of strain amplitude. Key elements in SASW testing are the generation and measurement of Rayleigh waves. The method has been used to date for a number of applications including design of foundations for dynamic loads, nondestructive pavement evaluation, evaluation of soil liquefaction potential, evaluation of the integrity of a concrete dam, determination of elastic properties of hard-to-sample soils, and as a diagnostic tool for determining the effectiveness of soil improvement techniques. The SASW method has proven to be a valuable tool for determining shear wave velocity profiles. The ability to determine a detailed shear wave velocity profile entirely from surface measurements results in substantial time and cost savings compared to other seismic methods such as crosshole and downhole techniques.

A number of publications have described in detail the SASW method (Nazarian [1984], Hiltunen [1988]). The SASW method consists of three major components: generation and measurement of Rayleigh waves at a test site, manipulation of test data to create a dispersion relationship, and inversion of this dispersion relationship to determine the shear wave velocity profile. One drawback in SASW is the need for an experienced user to produce an accurate shear wave velocity profile. The undisputed most difficult part of SASW is the inversion. Development of a systematic inversion protocol will make SASW inversion more manageable for an inexperienced user and provide the basis for automation of this process.

INVERSION

Understanding the inversion process is necessary for development of a successful protocol. In the simplest of terms, the inversion can be described as an iterative "guessing" procedure. The inversion can be illustrated by figure 1. Field testing provides an experimental dispersion curve. In order to determine a shear wave velocity profile a user must "guess" a shear wave velocity profile that is then used to calculate a theoretical dispersion curve. The two curves are then compared to see if they match adequately, and the decision must be made to continue guessing or accept the profile. This process continues through several iterations, each building on the previous guess, until an acceptable profile is found.

The governing theory behind SASW inversion is described by Aki and Richards (1980). Inversion begins with the following equation:

$$d = Gm \tag{1}$$

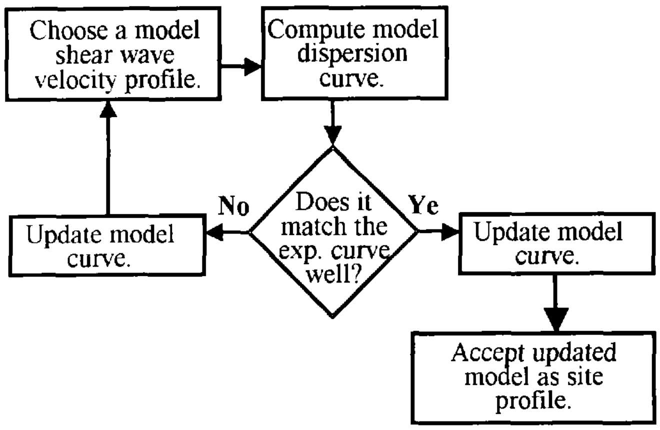

Fig. 1. SASW Inversion Procedure

where d represents a data vector, m is a vector of model parameters, and G is a matrix of first derivatives of the data points with respect to model parameters. Typically, there are fewer model parameters than data points. Manipulating equation 1 yields:

$$m = G^{-1}d \qquad (2)$$

This equation suggests that some lesser number of model parameters can be used to describe a larger number of data points given that the inverse of the first derivative matrix G^{-1} can be calculated. If the relationship in equation 2 is linear based on the model parameters, it can be solved directly.

In the case of SASW inversion, the vector m contains the proposed shear wave velocity profile parameters, while the vector d contains the data points from the experimental dispersion curve. There is no linear relationship between the model parameters and the data. The solution of equation 2 then requires an iterative process to find the best profile parameters.

A single proposed profile inversion will be detailed to further illustrate this process. A proposed shear wave velocity profile will contain the model parameters of layer thickness, shear wave velocity, Poisson's ratio, and mass density for each layer. Thomson-Haskell matrix propagation (as adjusted by Knopoff (1964)) is used as a forward model to create a theoretical dispersion curve from these profile parameters. The fit, or match, of this curve with the experimental dispersion curve is evaluated by the chi-squared (χ^2) measure of fit. The Levenberg-Marquardt non-linear least squares technique is used to perturb only the shear wave velocities and a new dispersion curve is calculated. The χ^2 fit of this new curve with the experimental data is then evaluated. Typically, these perturbations continue until the χ^2 is minimized. The shear wave velocities that best model the experimental data in light of the given layering are returned. To complete the single proposed profile trial, the decision must be made if the profile returned by the inversion is representative of the site conditions. If the profile is not accepted, the layer thicknesses and shear wave velocities are altered and the process repeated.

The difficulty in SASW inversion arises from the lack of constraint of the least squares minimization on the shear wave velocity parameters. The inversion algorithm is a complex and delicate numerical process. For even simple profiles, it can exhibit instability due to numerical sensitivity of the forward model calculations to the input shear wave velocity profile. The user must provide a reasonable starting profile; and the layer thickness and shear wave velocity parameters must be carefully followed and constrained to reach convergence. The protocol presented here will provide a systematic means to create and alter shear wave velocity profiles.

RATIONALE OF THE INVERSION PROTOCOL

The concept behind the new inversion protocol is to systematically provide additional constraints on shear wave velocity necessary for successful inversion. The essence is to build a profile starting from the most simple and progressively add complexity. The process is based upon the following principles:

- Use of dispersion data to guide initial selection of shear wave velocities and letting trends in dispersion data guide inversion results. This will help ensure that shear wave velocity values are realistic and representative of site conditions.

- Make small changes in layering that progressively add depth. Also, do not add complexity to a profile until all simple models have been evaluated and the best simple profile found. This will help maintain stability of the inversion algorithm.

- Layering changes between profile trials should be small, systematic, and limited to a single layer. This will also maintain stability of the inversion algorithm.

Given these additional constraints the Levenberg-Marquardt least squares minimization is effectively prevented from returning unreasonable shear wave velocity profiles.

To develop the protocol, the inversion process was explored using a range of dispersion curves generated analytically from known profiles. The benefit of using this type of "synthetic" dispersion curve is the ability to know when the correct profile has been achieved. Beginning with simple layering systems, various methodologies to perform inversion iterations were tested by trial and error.

The first layering system investigated was a one layer plus half space system. The half space is always included as the final layer in a profile. It represents the infinite mass of soil beyond the depth characterized. This simple model consists of one soil layer over a half space. A series of systems were inverted from which a systematic approach to successfully complete the inversion was developed.

Synthetic profiles with greater complexity were then used to expand the protocol to encompass systems that are more intricate. In a similar fashion, various two layer plus half space models were used to determine how to successfully invert two layer systems. This process continued with synthetic profiles of increasing complexity until the protocol was systematic and able to deliver an accurate and consistent profile. The protocol guidelines were then tested on several real dispersion data sets to ensure success when using real data. These real cases included geologic profiles and some irregular profiles. Geologic profiles exhibit a continual increase in shear wave velocity with increasing depth while irregular profiles will contain deviations from this increasing trend.

NEW INVERSION PROTOCOL

A general overview of the new protocol includes the following phases. These phases are directly derived from the principles set forth in the previous section. First, the experimental dispersion curve is examined for observable dispersion characteristics such as irregularities in phase velocity and approximate depth of characterization. Next, the minimum and maximum phase velocities are located to determine two initial shear wave velocities. These two shear wave velocities are used to create a proposed one layer over a half space profile. Using the protocol guidelines, the most representative one layer over half space profile is determined. This best one layer profile is then used to develop the most representative two layer over a half space profile. The process continues with each profile building upon the previous solution until the best model of the shear wave velocity profile is developed. The details behind these phases are in the following sections. While an unskilled user will still find SASW inversion challenging, the new protocol eliminates much of the intuitive input previously required.

Initial Observable Dispersion Characteristics

An essential part of the new inversion protocol is to let dispersion data guide the construction of shear wave velocity profiles. The objective is to develop a profile to as great a depth and detail that given dispersion data can accurately support. Prior to any inversion work being conducted two pieces of information can be derived from inspection of a dispersion curve. First, potential irregularities can be detected. A reversal for instance, is a decrease in phase velocity with decreasing frequency as opposed to the typical continuous increase with decreasing frequency as expected from geologic profiles. This would represent a layer of soil at some depth within the profile with a lesser shear wave velocity than the layers above and below. Prior knowledge of the potential for this type of behavior is beneficial in the inversion process since relative changes in phase velocity translate directly into relative changes in shear wave velocity and this atypical behavior is then anticipated.

The second piece of information available from perusal of the dispersion data is approximate depth of characterization. The traditional approximation of depth characterized by a SASW test has been one third of the longest recorded wavelength. The authors have found through repeated experimental trials that this depth is more realistically one fifth of the longest recorded wavelength. Some a priori estimate of this depth, to be used as a guide, is helpful in development of the shear wave velocity profile. Once these initial characteristics have been evaluated, the inversion begins with methodical development of the shear wave velocity profile from simple trial profiles building up to profiles that are more complex.

Development of One Layer Plus Half Space Profiles

The protocol developed in this study can be considered a top down process. Prospective shear wave velocity profiles are created by adding depth and detail to the profile from the surface down. The first profile to be determined is a one layer over a half space profile. The starting profile as shown in figure 2 will consist of a one foot layer (~0.3 m) above the half space. The initial shear wave velocities used for this profile are determined from the dispersion data. The minimum and maximum phase velocities are increased by 7% to provide the soil layer and half space shear wave velocities, respectively, as shear wave velocity has been shown approximately 7% greater than Rayleigh wave velocity at Poisson's ratios typical for soils (Richart, Hall and Woods, 1970). The thickness of the top layer is then increased by one-foot (~0.3 m) increments until the χ^2 fit of this profile is minimized. In essence, the best one layer plus half space model is found.

The layer thickness increment chosen for use in the protocol is an increment of one foot (~0.3 m). This increment could be any small increase consistent with the system of units of the dispersion data. An increment of one foot (~0.3 m) is a sufficiently small increase in layer thickness when compared with the total depth of characterization, which is typically on the order of 15 ft (4.5 m) for a hand held source and greater for mechanical sources. A final issue needs to be considered in determining the best one layer plus half space profile: the potential for a false minimum χ^2. When increasing the thickness of the surface layer it will appear that a minimum χ^2

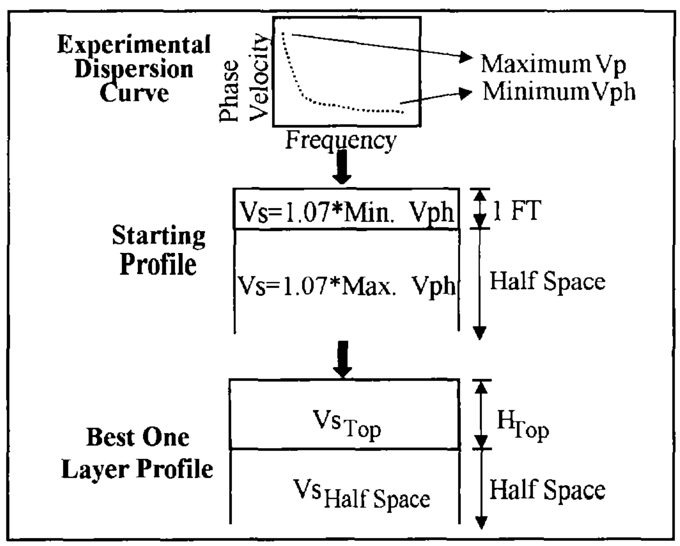

Fig. 2. Development of One Layer Plus Half Space Profile

fit will be determined. Closer inspection of the returned profile will reveal layer velocities that are greatly different from the proposed profile velocities and seem unrealistic. By increasing the thickness of the top layer further, the χ^2 fit will initially increase and then decrease to a new minimum with a more believable profile being returned. This type of false minimum is often seen when inverting data that characterizes to great depth.

Development of Two Layer Plus Half Space Profiles

After the best one layer plus half space model has been determined, a two layer plus half space model is then attempted. This two layer profile is based on the previously determined one layer plus half space profile. Figure 3 illustrates the procedure for using the one layer profile to find the next profile. First, the location of the additional layer must be determined. Two choices of where to insert a new one-foot layer exist: above the half space, Profile 1, or above the top layer, Profile 2. For these two profiles, three initial layer velocities are suggested. In Profile 1, the new one-foot layer inserted between the two previously established layers can have one of the three proposed new velocities Vs. In Profile 2, the new one-foot layer is assigned the velocity developed for the top layer in the best one layer profile. The resulting middle layer will have one of the three proposed new velocities Vs. Thus, six different profiles can be tested. Each profile. If a trial does reduce the χ^2, the resulting shear wave of these profiles are investigated until a configuration decreases the χ^2 from the minimum found for the one layer velocities are used to update the profile.

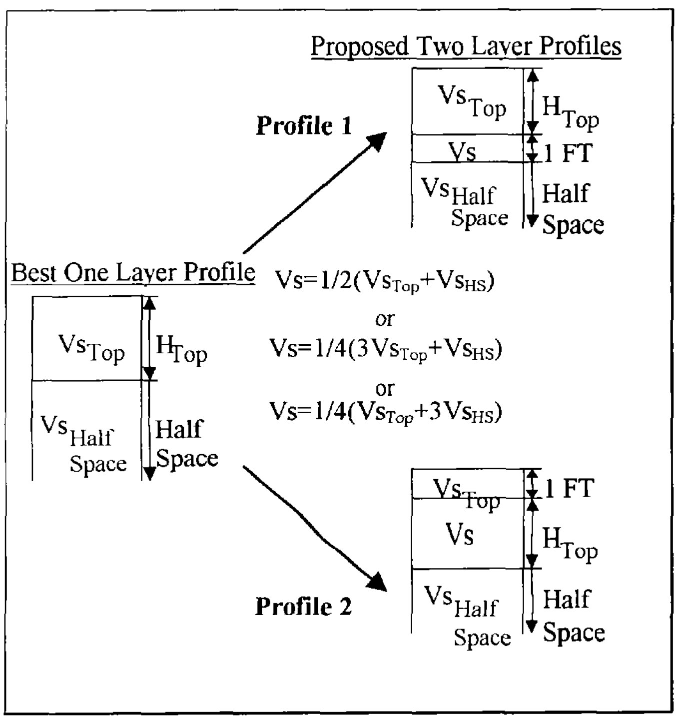

Fig. 3. Trial Two Layer Plus Half Space Shear Wave Velocity Profiles

Potentially, none of the six two layer models will improve the χ^2. In this case, the process is repeated with the addition of a one-foot (~0.3 m) layer as seen before, but the original surface layer is reduced in thickness by one foot (~0.3 m). If this fails to improve the χ^2 fit the inserted layer is increased to a two-foot (~0.6 m) layer and the process repeated.

Once the location of the new layer has been decided, the thickness of the layer must be determined. Development of the new layer is attempted by increasing its thickness in one-foot (~0.3 m) increments and monitoring the χ^2 for improvements. The thickness is increased until the χ^2 ceases to decrease. Once the third layer is established, the other layers provided by the two-layer model are then checked for adjustments, by increasing and decreasing the thicknesses of these layers individually and checking for improvements in the χ^2. If either of the two layers is changed, the new layer is also checked for adjustments to the thickness. Once these adjustments are accomplished, and changing the thickness of any of the three layers no longer results in an improvement to the χ^2, the best two layer plus half space model has been determined.

There are instances when the χ^2 of a trial profile will improve but the resulting profile will be unacceptable. If returned shear wave velocities contain a reversal and none is expected from perusal of dispersion data a profile should not be accepted. Also, if the inversion returns negative shear wave velocities a profile must not ever be accepted.

An artifact of the protocol is the occasional inclusion of one-foot (~0.3 m) layers deep in the profile. If the contrast in layer velocities between two successive layers is large, as is often the case between the final layer and the half space, the inversion may include a thin layer to ease the transition in velocities. The inclusion of these thin layers will improve the χ^2 fit dramatically. When they are reported in the profile, it should be recognized that they do not represent an actual distinct layer in the soil system but a transition in the shear wave velocity.

Development of Three Layer Plus Half Space Profiles and Additional Profiles

To proceed to a three layer plus half space profile from the two layer profile, three positions become apparent for insertion of a new layer (figure 4). The trial velocities for the newly inserted layer are developed in a similar fashion to those in figure 3. In a similar fashion as for the two layer plus half space profile, the velocities and layer thicknesses are adjusted to find the best three layer plus half space model. Profiles consisting of four, five and higher layers plus a half space are developed in a similar fashion to the adjustment process detailed for the two and three layer profiles. Eventually adding complexity to the profile will fail to improve the χ^2 fit significantly. Once this occurs, the best-fit model prior to the attempted changes is accepted as the site profile.

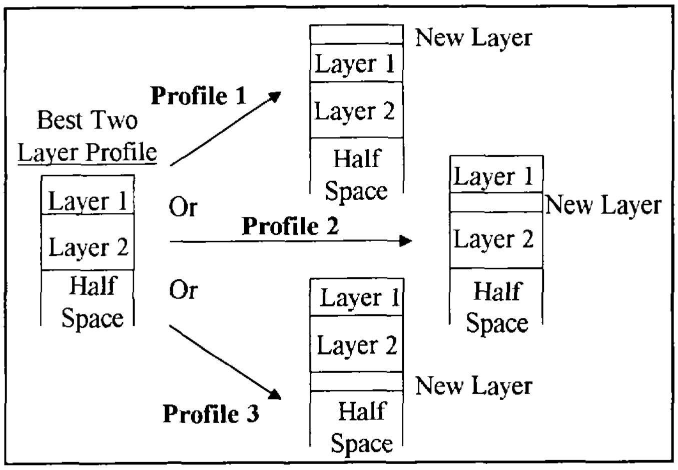

Fig. 4. Trial Three Layer Plus Half Space Shear Wave Velocity Profiles

FINDINGS

The inversion protocol has been used on numerous dispersion curves determined from a wide variety of test sites. Careful adherence to the protocol has consistently produced shear wave velocity profiles whose predicted dispersion data match well with experimental data. Further, the shear wave velocity profiles appear to be indicative of site conditions. In several cases, results from crosshole testing were available to further verify the SASW results. The systematic nature of the protocol makes SASW inversion more manageable for an inexperienced user, and also provides the logic necessary for automation of the inversion process.

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