27 Apr 1981, 2:00 pm - 5:00 pm

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Proceedings: First International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, April 26 - May 3, 1981, St. Louis, Missouri

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SYNOPSIS A new triaxial apparatus which can control the stress condition automatically in a constant mean principal stress under repeated loading was developed. Mechanical behavior of cohesive soils under repeated loading was investigated. Excess pore water pressure generated in loading cycles was measured and the experimental results were interpreted in terms of effective stress by using elasto-plastic models.

1. INTRODUCTION

Soil masses are often subjected to repeated or transient loads. With the increases usage of offshore structures and an increased concern for adequate seismic design, the required accuracy of dynamic soil analysis has arisen dramatically. As a result, a number of studies have been concerned with the stress and deformation responses of soil subjected to repeated loadings. Most published work on cyclic loading has been concerned with sand, however, cyclic loading of clay is equally important problem.

It has been recognized that the behavior of soils subjected to repeated cycles of loading may differ considerably from their behavior during a single loading cycle. There are many natural situations in which the duration of the series of loading cycle is such that little or no drainage of the pore water can take place during the period of the repeated loading. It is therefore useful to study the effects of repeated loading under undrained conditions in the laboratory.

Undrained repeated loading tests were performed on reconstituted saturated cohesive soil. Most experiments on repeated loading of soils have used the axisymmetric triaxial test wherein the cell pressure is held constant and the deviatoric stress changed. A new triaxial apparatus was constructed. It has a servo-mechanism and can control the stress condition automatically and precisely, so that the mean principal stress is kept constant during repeated shear loading. The excess pore water pressure generated only by dilatancy can be measured directly in this specially designed apparatus.

Measurement of the excess pore water pressure is necessary for an effective stress interpretation. In order to permit accurate measurements of pore water pressure, sufficiently slow repeated loading has to be adopted in the experiments.

It is clear that the influence of stress history is most significant in cohesive soil and that a more basic understandings of soil behavior can only be obtained by analyzing results in terms of effective stress. This paper describes the mechanical behaviors of normally consolidated cohesive soils which stay inside the state boundary surface at the time of slow repeated loading. The migration of the effective stress path towards the origin of the stress space (p, q) is demonstrated. Reduction in effective stress and the accumulation of pore pressure with continued cycling leads to the development of plastic strains inside the state boundary surface which is defined in slow monotonic loading test. These experimental results are interpreted by elasto-plastic model (Pender,1977).

2. EXPERIMENTAL PROCEDURE

2.1 Apparatus

In the triaxial test, increment of mean principal stress is represented as

$$\Delta\sigma_m = (\Delta\sigma_1+2\Delta\sigma_3)/3 \qquad (1)$$

where σ_1 is the axial stress and σ_3 is the lateral confining stress. For $\Delta\sigma_m$=0 during loading, Eq.(1) is reduced to

$$\Delta\sigma_3 = -\frac{1}{2}\Delta\sigma_1 \qquad (2)$$

Now, let u the excess pore water pressure generated by dilatancy. Then mean principal effective stress ($\Delta\sigma_m' = p$) is expressed as follows:

$$\sigma_m' = p = \sigma_m - u \qquad (3)$$

From Eq.(2), it is known that lateral stress must be decreased (or increased) in order to maintain the mean stress σ_m constant during repeated loading. This σ_m=constant condition is attained automatically and precisely by using a servo-control system, as shown in Fig.1. Data acquisition, processing and plotting are done by a microcomputer system.

2.2 Specimen and Test Procedures

Fukakusa clay was used for all tests. Physical properties of the clay are as follows; L.L.=45.5%, P.L.=22.4%, P.I.=23.1%, Gs=2.71, sand fraction 17%, silt fraction 64%, clay fraction 24%.

A slurry was prepared at a moisture content of twice the liquid limit. It was consolidated in 300mm diameter molds under one-dimensional conditions using a pressure of 0.7 kgf/cm^2. The specimens were sampled with thin-wall sampler of

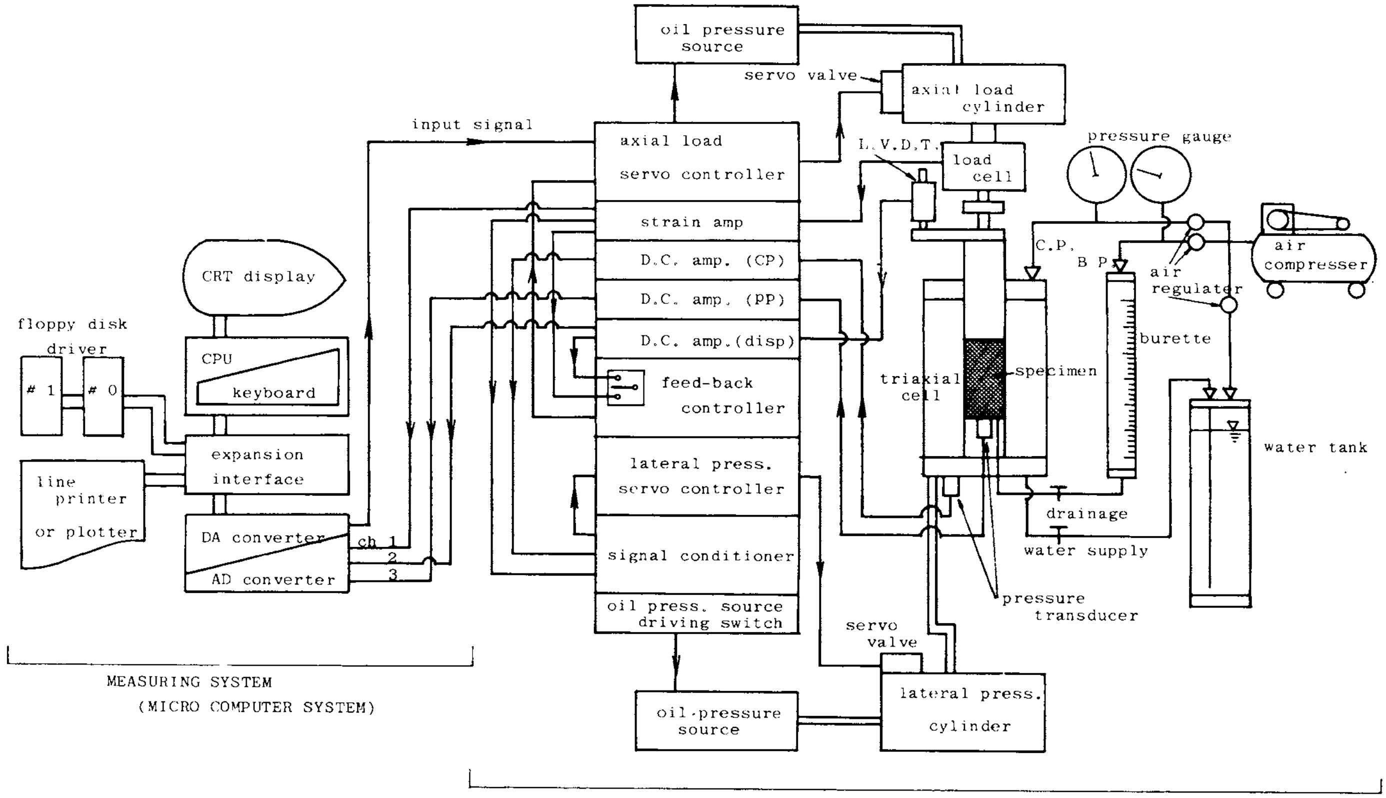

Fig.1 Cyclic Loading Test Apparatus

50mm diameter and cut to a length of 100mm. Subsequent consolidation was carried out in the triaxial cell under isotropic conditions. A back pressure of 1.0 kgf/cm^2 was always applied to ensure complete saturation of the sample. Initial effective consolidation pressure was set to 2.0 kgf/cm^2.

All tests were performed in undrained condition, so that the deviatoric strain ε is equal to axial strain ε_1. Loading was done at a fixed strain rate to exclude the rate-dependent effect of clay.

3. EXPERIMENTAL RESULTS

The accumulation of excess pore water pressure, in other words, effect of dilatancy on the effective stress path during repeated loading will be discussed. Experiments were performed by strain control and the rate of strain was 0.06 or 0.12 %/min in order to measure accurate pore water pressure.

3.1 Effective Stress Path

(a) Strain amplitude controlled test

A strain amplitude controlled test is a repeated loading test in which the amplitude of strain (maximum and minimum strain) is fixed. Fig.2(a) shows an effective stress path (p-q space: q is defined as stress difference σ_1-σ_3) of the test with $\dot{\varepsilon}$=0.12 %/min, ε_{max}=0.78 % and ε_{min}=0 %. Numbers shown in the figure designate the number of loading cycles. It is known from the figure that the excess pore pressure is accumulated as the loading cycle proceeds and the peak values of the deviatoric stress q at

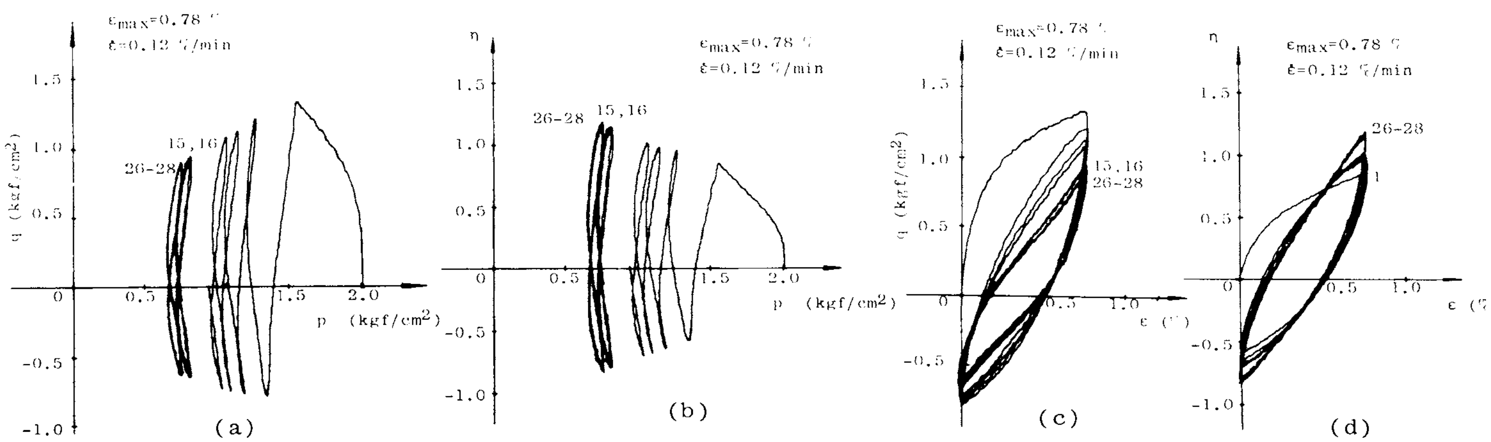

Fig.2 Strain Amplitude Controlled Test

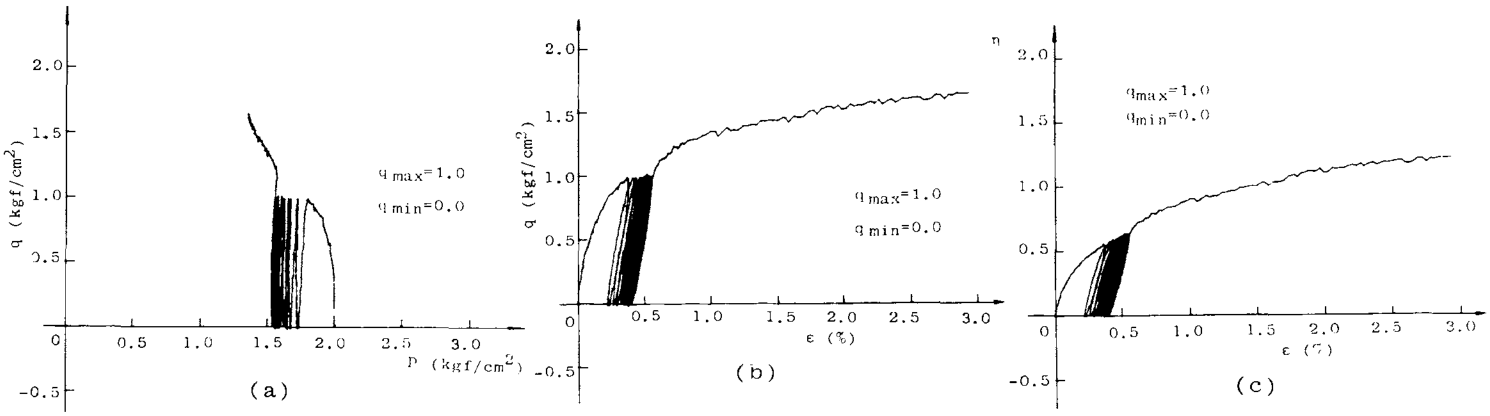

Fig.3 Stress Amplitude Controlled Test

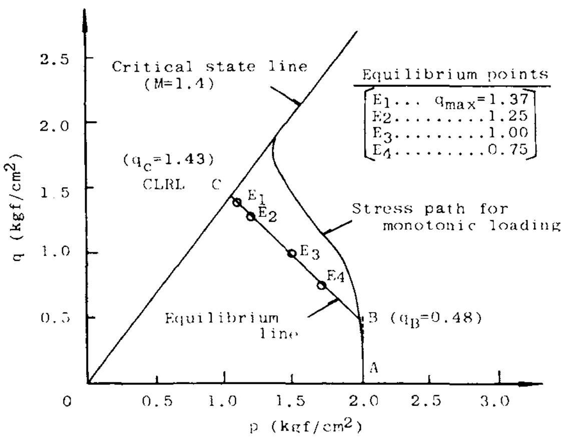

Fig.4 Equilibrium Line

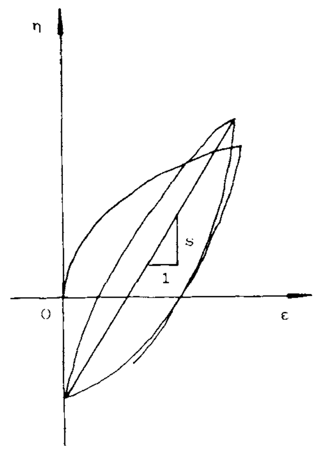

Fig.5 Definition of S

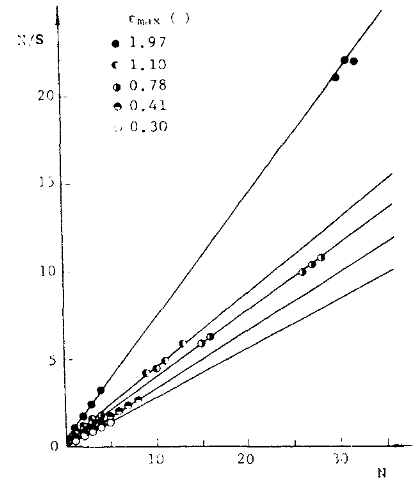

Fig.6 Hyperbolic Relation between N and S

each cycle decreases gradually in spite of constant magnitude of strain amplitude. The effective stress path follows the state boundary surface at the first loading cycle, then migrates into the surface at the subsequent unloading and loading. Since the large magnitude of $|q|$ ($q<0$) is necessary to bring the plastic strain back to zero, fairly large value of $|q|$ is shown in the figure at $\varepsilon=0$.

Fig.2(a) can be rewritten in terms of stress ratio (q/p) as shown in Fig.2(b), where the maximum and also minimum value of η increases as the loading cycle proceeds with a constant strain amplitude. This behavior is also noticed in the other series of tests, since it is the result of strain-hardening of cohesive clay under repeated loadings.

The increment of accumulation of excess pore water pressure in each step of loading cycle decreases gradually and attains to zero at 26th load cycle. This is an equilibrium state of stress for a given initial test condition. From other test results, it was found that the equilibrium state is dependent upon the strain amplitude. The state of equilibrium shown herein is very similar to the equilibrium line below the critical level of repeated loading (CLRL) given by Sangrey,et.al.(1969), although his results are obtained in stress controlled repeated loading tests.

The accumulated excess pore water pressure u_σ can be expressed in the relation with the number of loading cycles N as follows (Akai, et.al.,1979):

$$u_\sigma = N/(a+bN)$$

where a and b are material constants depending on $\dot{\varepsilon}$ and ε_{max}. The ultimate value of the excess pore water pressure at equilibrium state is obtained by letting $N \to \infty$ as $(u_\sigma)_{ult} = 1/b$.

(b) Stress amplitude controlled test

A stress amplitude controlled test is a repeated loading test in which Fig.3(a) shows an effective stress path (p-q space) of the test under the condition of $\dot{\varepsilon}$=0.06 %/min, q_{max}=1.0 kgf/cm^2 and q_{min}=0.0 kgf/cm^2. Similar to Fig.2(a), the stress path migrates into the state boundary surface which was determined from monotonic loading tests. The excess pore water pressure is also accumulated and eventually the stress condition reaches nonfailure equilibrium without any further measurable changes in strain or pore water pressure as reported by Sangrey,et.al.(1969).

In the series of our repeated loading tests (q_{max}=1.37, 1.25, 1.00, 0.75 kgf/cm^2), all of the specimens reached the nonfailure equilibrium and the locus of the points representing the stress peaks of the equilibrium hysteresis loops are shown in Fig.4 as a line BC. q_{max}=1.43 (point C) seems to correspond with the critical level of repeated loading (CLRL). For the repeated loading at $q_{max} \geqq 1.43$, soil specimens will fall into the failure state (i.e. cyclic mobility) due to the large amount of accumulated excess pore pressure and loss of effective stress.

Sangrey et.al.(1969) reported that the locus

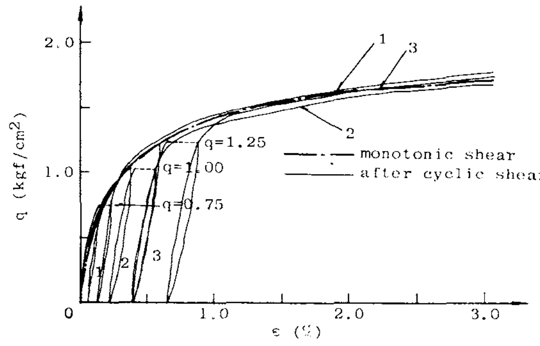

Fig.7 Stress-strain Relationship

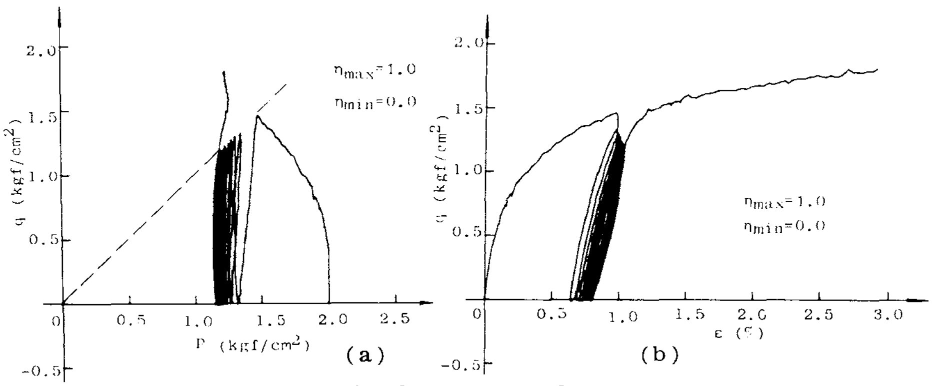

Fig.8 η controlled Test

passes through point A, but it was found in our test results that the equilibrium line intersected with $p=2.0$ kgf/cm^2 line at $q=0.48$ (point B). This means that no excess pore water pressure will be generated in repeated loading if $q_{max} \leq 0.48$. The q value of 0.48 is the lower bound of the stress amplitude above which the repeated loading causes yielding of cohesive soil. A similar result has been reported by Matsui et.al. (1977) in a cyclic triaxial test with a high frequency.

3.2 Deviatoric Stress-Strain Relations

(a) Strain amplitude controlled test

Figs.2(c) and 2(d) show ε-q and ε-η relationships respectively under the condition of $\dot{\varepsilon}=0.12$ %/min, $\varepsilon_{max}=0.78$ % and $\varepsilon_{min}=0.0$ %. An overall picture of the relationships between deformations and stress level may be obtained from these figures. As shown in Fig.2(c), the peak stress difference q_p at each loading cycle gradually decreases in both compression and extension sides and the stress-strain relation approaches to the equilibrium hysteresis loops. This phenomenon is more clearly shown in Fig.2(a).

It has been known that shear strains in soils are controlled by the stress ratio $\eta(=q/p)$ and it is therefore interesting to examine the deformation in terms of this ratio. Fig.2(d) shows that the peak value of η (η_p) is enlarged as repeated loading cycle proceeds. This is because of strain hardening of soil subjected to repeated loading. The increase rate of η_p lessens gradually and a equilibrium state is obtained. The relationship with number of cycle (N) is expressed hyperbolically.

The deformability of soil under repeated loading may be represented by a value of S indicated in Fig.5. S is defined as the slope of the straight line which connects the maximum and minimum points of strain in stress-strain loops. S and N relationship is also hyperbolic as shown in Fig.6 for the case of $\dot{\varepsilon}=0.12$ %/min.

(b) Stress amplitude controlled test

Fig.3(b) is a stress-strain relationship of the cohesive soil tested with the condition of $\dot{\varepsilon}=0.06$ %/min, $q_{max}=1.0$ kgf/cm^2 and $q_{min}=0.$ kgf/cm^2. At the first unloading, a large amount of residual strains when q=0. Most of the cases, the residual strain is defined as a plastic component and the rebounded strain as a elastic one. We will discuss about these definitions later.

Residual strain at q=0 hyperbolically increases as loading cycle proceeds and a equilibrium state is obtained below the critical stress level of repeated loading. The phenomenon is equivalent to the result of the strain amplitude controlled test, in which the excess pore water pressure was accumulated. A relationship between ε and η is shown in Fig.3(c), which indicates the increase of slope S, i.e. stiffness of the cohesive soil.

In this series of tests, loading was continued until 3% of strain after completing the repeated loading cycle. Undrained stress-strain curves return to the original monotonic loading curve as shown in Fig.7 and an apparent change of undrained strength of the cohesive soil could not be found.

4. INTERPRETATION OF RESULTS

4.1 Elasto-plasticity

Many studies have been done in order to construct the constitutive equation of saturated cohesive soils. Most of them were concerned with the theory of plasticity. Most famous model among the many proposed models which try to represent the mechanical behavior of soil is "Cam clay" or "Cambridge" model which considers plastic yield, taking into account the concept of state boundary surface. New models are subsequently presented in the framework of this concept. For example, Pender (1977) proposed a new model in which soil exhibits plastic yielding whenever the value of η changes. He made an assumption in his model that constant stress ratio lines are yield loci inside a state boundary surface, i.e. yield loci $f=q-\eta_i p=0$ where η_i is a value of η for a particular yield locus. And he developed the stress-strain relationship using non-associated flow rule, assuming the shape of effective stress path under an undrained condition. Although the idea that yield loci consist of η constant lines has already been proposed by Poorooshasb,et.al. (1966,1967) especially for sand, Pender introduced a new concept that yielding occurs even when η decreases and it is the introduction of kinematic hardening. Kinematic hardening is an important concept when the stress-strain relationships under repeated loading are investigated. Later Prévost (1977) and Mroz,et.al. (1978) presented new models using the concept of "field of plastic moduli", extending the formulation of kinematic hardening.

Here we will discuss about yield function f, plastic potential g and hardening function h that are main parameters in defining the mechanical behavior of cohesive soil under repeated loading by using the theory of plasticity, in which

$$d\varepsilon_{ij}^{p} = h\frac{\partial g}{\partial \sigma_{ij}}df$$

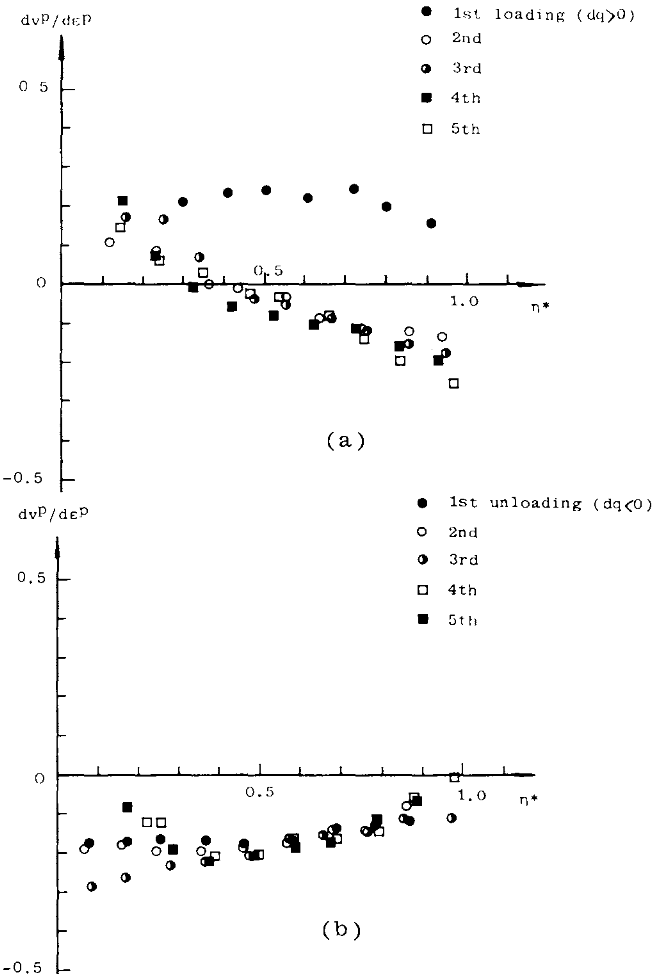

Fig.9 $dv^P/d\varepsilon^P$ and η^* relationship

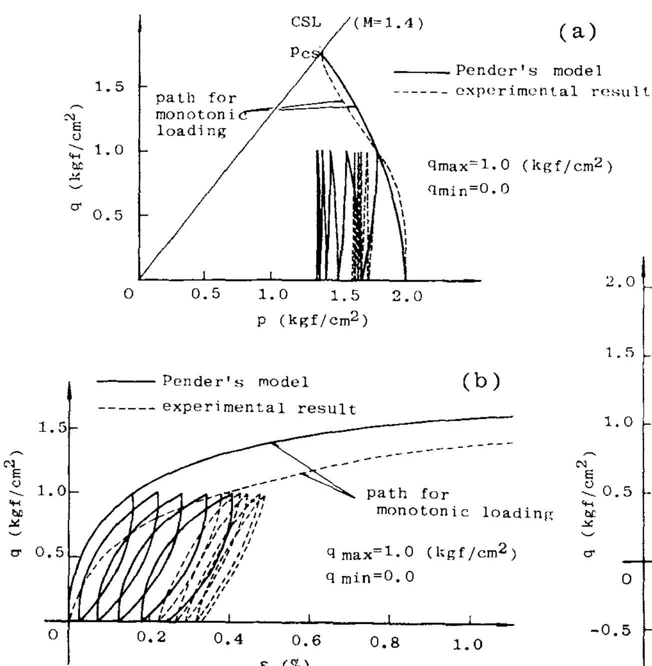

Fig.10 Stress Amplitude Controlled Test

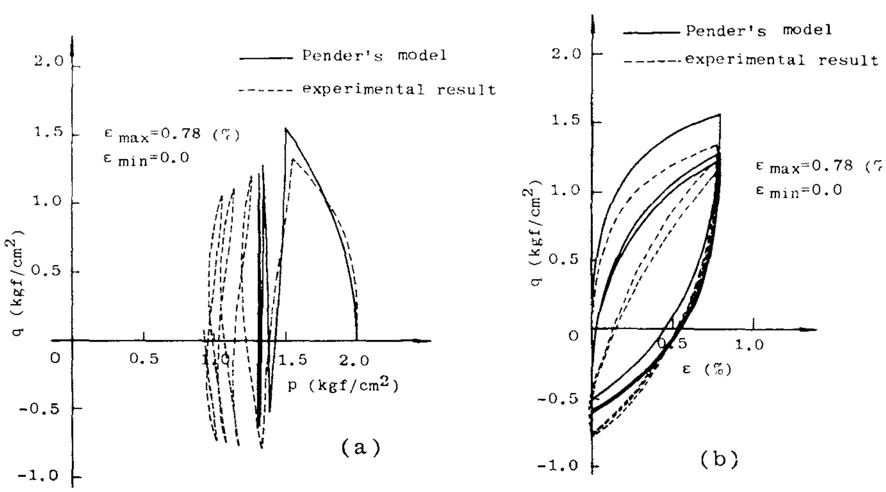

Fig.11 Strain Amplitude Controlled Test

(a) Yield condition

In the stress-strain curve of Fig.3(b), the amount of deviatoric strain caused by repeated loading is small and negligible compared with that of initial loading or beyond the prefixed stress amplitude. And also in the effective stress path of Fig.3(a), the generated excess pore water pressure (e.g. volumetric plastic strain) after second cycle of loading is small compared to that of the first cycle of loading. If we adopt the idea that unloading process is elastic, the constitutive model is very much simplified. However, repeated loading at in-situ ground usually has strong stress intensity and amplitude even at second or subsequent cycles. So we consider that there is no elastic deviatoric strain in unloading and therefore the plastic strain starts occuring at the point of change of loading direction. This coincides with the Pender model in which η=constant lines are yield loci. The validity of this assumption is proved by the test result of η controlled test that is performed under the condition in which the peak value of η of each loading cycle is fixed. Figs.8(a) and 8(b) show respectively the effective stress path and the deviatoric stress-strain relationship with $\eta_{max}=\eta_{peak}=1.0$, $\eta_{min}=0$. If the yield condition proposed by Poorooshasb, et.al. is valid, plastic strains do not occur after second cycle of loading in η controlled test. However, as seen in the figure, the plastic strain occured in unloading and this suggests the necessity of introduction of kinematic hardening and validity of Pender model.

(b) Plastic potential

A plastic potential surface is defined as the surface which is normal to the direction of a plastic increment vector. Determination of the plastic strain incremental ratio $dv^P/d\varepsilon^P$ will derive the partial derivative of g ($\partial g/\partial p$, $\partial g/\partial q$) with some informations. The relationship between $dv^P/d\varepsilon^P$ and $\eta^*=|\eta-\eta_0|$ for the tests are shown in Figs.9(a) and 9(b). It was found that there is a linear relationship between them as follows:

$$dv^P/d\varepsilon^P = c - \alpha\eta^*$$

where c and α are constants which depend on the value of η at turning point (η_0), volumetric plastic strain v^P and consolidation histories. Most extensive research has to be done to clarify these relationships qualitatively and quantitatively.

(c) Hardening function

A hardening function determines the magnitude of the plastic strain increment. In order to get a hardening function for shear component, a test in which yielding due only to shear occurs (no volumetric yield) must be performed. Sometimes, ε-η relationships often assumed to be hyperbolic to determine the hardening function.

In case of volumetric yield, isotropic consolidation test is the one for the hardening function, since there is no shear strain in the test.

4.2 Comparison with Pender Model

The behavior of the clay is so complicated that only the outline of the differences between the Pender model and the test results are discussed here and some comments for the development of a new model will be stated.

(a) Effective stress path

Fig.10(a) shows the computed result of effective stress path for the repeated loading with the condition q_{max}=1.0 kgf/cm^2, q_{min}=0 by Pender model. The broken line in the figure shows the test result. Though the theoretical curve and the experimental curve agree well for the first loading, the theory overestimates the pore water pressure for first unloading and subsequent cycles. After five cycles of loading/unloading, predicted pore water pressure comes up to one and half times of the experimental result, but after that, very little pressure will be generated, according to the model. This may be caused by the assumption in the model that effective stress path always converges to p_{cs} (the value of p at the critical state). Hence, effective stress path which starts in the 'wet' side never goes into the 'dry' side, or vice versa. Also, effective stress path always reaches to the point of p_{cs} irrespective of stress amplitudes. Then the equilibrium state mentioned above is never predicted in this model.

The comparison in the case of ε_{max}=0.78 %, ε_{min}=0 is shown in Fig.11(a). In this case, prediction by Pender model agrees well with test result at the first cycle, as well. However, actual effective stress path goes into 'dry' side passing p_{cs} (1.3 kgf/cm^2 in Fig.2(a)), and this behavior is not predicted by the model. For the values of deviatoric stress q at the peak (turning), the decreasing trend in the compression side is represented well, but in the extension side, the model predicts the increasing trend of the absolute value of q against the test result.

The prediction of pore water pressure is required not only because the decrement of effective stress causes the loss of stabilization of the soil but also because the dissipation of accumulated pore water pressure in the succeeding static loading causes the settlement of the ground. However, the above analytical results show that the dilatancy characteristic of the clay is not always predicted well. The accurate prediction of pore water pressure will be attained by the correct estimation of the loading history and constitutive relation as mentioned in Section 4.1 (b).

(b) Deviatoric stress-strain relationship

A comparison for the deviatoric stress-strain relationship is shown in Fig.10(b) for the same test as Fig.10(a). Pender model predicts the larger value of q for the same strain than the result of test. The behavior of the model in repeated loading generally resembles the actual one, but the rate of increase in plastic strain at each cycle stays constant and the equilibrium state is not attained in this model, whereas in the actual test plastic strains are accumulated in a decreasing rate and a equilibrium state occurs.

Fig.11(b) shows the stress-strain relationship for the strain amplitude controlled test as Fig.11(a). As the first loading cycle, Pender model predicts the higher value of q than the actual case. The q-ε curves of the test after second cycle have steeper slopes than the one at the first cycle as indicated by broken lines in the figure, but the curvature of ε-q relationship predicted by the model is same in any cycle of loading. Also the decreasing trend of q at peak is not predicted well.

It seems that accurate prediction of the mechanical behavior of cohesive soils is very difficult. However, the accumulation of experimental data and the development of more general constitutive equation will overcome these difficulties in near future.

5. CONCLUSION

In order to clarify the mechanical behavior of cohesive soils under repeated loading, a new triaxial test system was developed. Many experimental data have shown that the accurate prediction of the mechanical behavior of cohesive soil under repeated loading is still difficult by using the advanced elasto-plastic models. Development of more general constitutive law and experimental results under different boundary conditions are necessary to analyze the actual field problems.

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