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SYNOPSIS In this paper the dynamic deformation stages of soils under cyclic loading and the major factors affecting the shear modulus of soils are analyzed. The in-situ measurement of surface wave by larger energy source to obtain the shear modulus of the ground is described. On considering the important effect of shearing dilatancy on shear modulus of sandy soils, a modified formula is proposed. For typical soils, the axial strains under cyclic loading are determined either for estimating shear modulus or for the deformation analysis in design.

INTRODUCTION

In the dynamic analysis of the ground, foundation, and structural system, (see, for example, Hu, 1957, 1965; Hardin and Drnevich, 1972a and b; Woods, 1978) it is needed to determine the dynamic shear modulus of soils and the corresponding deformations. How to simulate the dynamic conditions and responses, and what means to determine the dynamic parameters are the problems to be solved. Various articles on this subject have been published, but some results are divergent or even controversial. So the primary factors affecting the test results for the soils should be considered accordingly. The equivalent linear dynamic parameters have often been used, but their validity within different ranges of dynamic shear strain should be specified first.

Analogous to the static analysis, the dynamic shear strains could be divided into three stages according to their characteristics.

(a) dynamic elastic strain stage $\gamma \leq 10^{-4}$ (%)
(b) dynamic elast-plastic strain stage $10^{-4} \leq \gamma \leq 10^{-2}$
(c) dynamic plastic strain stage $\gamma > 10^{-2}$

Within the first stage, the strain is very small and recoverable. That means the modulus will be independent of the number of dynamic cycles, N, suffered. But it was reported that the modulus would decrease for N > 50000 times. That might be due to fatigue effect.

In order to simulate the dynamic condition of a certain stage, it is necessary to produce the corresponding dynamic stress. If the source energy is too small, the shear modulus would be too large to be used in design. That is the reason why the magnitude of G is rather larger in the conventional $G{\simeq}\gamma$ charts. This condition was testified by Taylor and Parton (1973).

In the second stage, the modulus decreases with increasing strains and the rate of change of the modulus increases. In addition to the strain level, the modulus changes are affected by the number of cycles experienced.

On entering the third stage, the deformation develops significantly. It corresponds to the strong earthquake response. Owing to its beginning failure or flow stage, the modulus decreases monotonously with increasing rate. But in the mentioned $G{\simeq}\gamma$ chart, G decreases with diminishing rate in the range of $\gamma > 10^{-2}$. So the counter flexure point appears and each curve approaches a certain asymtote parallel to the γ -axis.

There are many factors which have influences on the magnitude of G. The in-situ measurement of wave velocity is influenced by the multi-layer conditions, the homogeneous nature within each layer, and the ground water level fluctuation. The source energy used and the method of wave measurement all have effects on the result.

In the laboratory test, there are eleven factors affecting G.(see Richart, et. al., 1976) Many reports confirmed three factors are predominant for cohesionless soils, i.e.

$$G=f(\bar{\sigma}_o, e, \gamma) \qquad (1)$$

in which $\bar{\sigma}_o$ is the average effective confining pressure, e is void ratio, and γ is the shearing strain amplitude.

In non-cohesive soils, the shearing dilatancy has been well known, but it is not yet considered quantitatively in the determination of the modulus, so it will be estimated accordingly. In saturated cohesive soils, the effect of pore water pressure will be considered for the same purpose.

INSITU MEASUREMENT OF SURFACE WAVE VELOCITY TO DETERMINE G

If the measurement of wave velocity is undertaken by using very small energy source, the induced strain will be too small while the obtained G will be larger than that of laboratory tests. Larkin and Taylor(1979) attributed it to be the effect of "disturbance factor"θ. In fact, it is the response of different strain stage. Thus, larger energy source is needed to excite the Rayleigh wave velocity, V_R.

The measurement of V_R was undertaken in-situ on the silty sand ground of a factory. The compression wave and the shear wave should be eliminated in order to obtain the slower Rayleigh wave velocity. The thickness of the soil is about 5 meters and below it is a thick stratum of gravel. The top soil has a thickness of about 0.5 m. and the ground water level is about 3.7 m. below the ground surface. The void ratio is 0.81 to 1.07 and a typical gradation is given in Table 1.

Table 1. Particle Size Gradation for Silty Sand

Gravel(mm)			Sand(mm)				
			coarse	medium	fine	l.f	m.f.
10	10-4	4-2	2-1	1-0.5	0.5-0.25	0.25-0.1	<0.1
34%		36%	16%		11%	2%	1%

A forge hammer of 2 tons was used as an energy source, and pick-up points were set along the ground at spacings of 20 m. From 15 series of measurements the average V_{R2} = 144.5 m./sec was obtained. Shearing strains developed at the distances of 2.5, 5.0, and 7.5 m. from the 2 top hammer energy source were 1.1 X 10^{-4}, 6.0 X 10^{-5}, and 3.1 X 10^{-5} respectively. With ν_d=0.33, ρ =0.143 $tsec^2/m^4$, and the measure values, a dynamic shear modulus of G_{d2}=3470t/m^2 was obtained for a depth of 1 m. The average G_d within this sand deposit, calculated by integration, will increase about 49% of G_{d2}.

A free hammer of 3 tons was then used and the pick-up points were of 25 m. spacing. From 16 series of measurements, the value of V_{R3}=150 m/sec was obtained. The shearing strains at distances of 2.5, 5.0, and 7.5 m. from the hammer energy source were 1.3 X 10^{-4}, 7.4 X 10^{-5}, and 3.9 X 10^{-5} respectively. From this information a value of G_{d3}=3720 t/m^2 was obtained for a 1 m. depth. Both of these values are smaller than those measured under small strain conditions.

It is suggested to use more in-situ measurement of surface wave velocity so as to eliminate the limitations of laboratory tests and to reflect the original state of the ground.

THE EFFECT OF SHEAR DILATANCY ON G OF SANDY SOILS

It is well known that medium to dense sands will dilate during the shearing process, the higher the relative density, the more dilatancy occurs.

The expression for the increase of shearing strength is dilatancy is:

$$\tau_d = \sigma \tan\phi_i + c_i \qquad (2)$$

In which σ is the normal stress on the sliding surface, ϕ_i is the equivalent friction angle in dilatency, c_i is the interlocking force in shear ($c_i = \tan\phi_o$, and ϕ_o is the equivalent friction angle due to interlocking action.) This increase is induced by the volume expansion in the shear process against the volume compression σ as it would happen, therefore

$$\tau_d = \sigma \frac{\delta V}{\delta \gamma} = \sigma \eta \qquad (3)$$

in which δV is the volumetric strain increment during dilatancy, $\delta\gamma$ is the shearing strain increment during dilatancy, and η is the dilatancy coefficient. Then

$$\eta = (D_r - 0.40) \tan(45^o + \phi/2) \qquad (4)$$

in which D_r is the initial relative density of the sand stratum, and ϕ is the internal friction angle of the sand.

This coefficient of dilatancy reflects the tendency of particle structure to resist any change of initial volume during shear. It is a function of coarseness of particles, and the sand's textrue and denseness of original compaction. The dilatancy coefficient induces a variation of its original normal pressure on the sliding plane and it has a nature of passive resistance of soil to counteract the shearing. The coarser and denser the particles, the greater the dilatancy effect.

Two types of empirical expressions were proposed for G_d of sandy soils with rounded particles ($0.3<e<0.8$) and of coarse granular soils respectively. In these formulas, the three main factors were considered. But the effect of dilatancy was not included, so we propose an important modification for dilatancy.

For sands with rounded particles.

$$G = \frac{m_1 (m_2 - e)^2}{1 + e} (\bar{\sigma}_o (1 + \eta))^{0.5} \qquad (5)$$

in which m_1, m_2 are test constants from regional soils, e is initial void ratio, $\bar{\sigma}_o$ is the average effective confining pressure, and η is the dilatancy coefficient.

THE DEFORMATION OF SOILS UNDER CYCLIC LOADING

In the laboratory condition, the deformation process under cyclic loading could be better measured and analyzed. Whether it be non-cohesive or cohesive soils, the recoverable and irrecoverable deformations occur under cyclic loading. For any soil under each cycle of dynamic loading, the compression and elongation are certainly not equal, so it belongs to a kind of Bauschinger effect, though the Masing is often used to treat it. Anyhow the residual deformation is induced after the intended cycles of loading are applied.

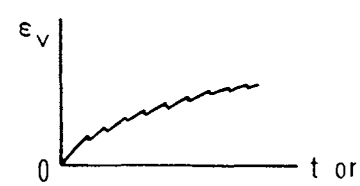

Fig.1. Total residual compression strain after Nth cyclic loading.

(1)Dynamic Axial Strain of Unsaturated Dry Sands

For the non-cohesive soils with low water content, the dynamic strain includes two parts: the strain induced by average normal stress and that induced by shear stress. During each cycle of loading, the axial srain increment of loading portion in the compressive half cycle is:

$$\Delta\varepsilon_{v1} = \alpha_1 \Delta\sigma - \eta\delta\gamma\ \Delta\tau \qquad (6)$$

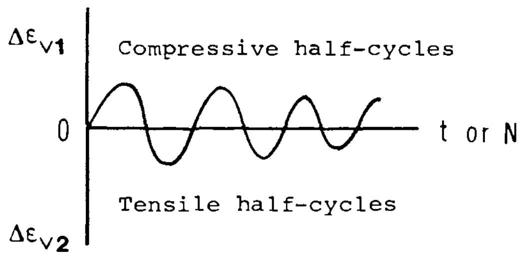

Fig.2. Axial strain increments in compressive and tensile half-cycles.

in which $\Delta\sigma$ is the average compressive stress increment, and α_1 is the volumetric compression coefficient in the compressive half cycle. In Eq. (6),

$$\alpha_1 = \alpha_o \frac{1}{N_i} \qquad (7)$$

in which α_o is the volumetric compression coefficient in the first compressive half cycle, and N_i and i-th dynamic cycle.

Similarly, the axial strain increment of unloading portion in compressive half cycle is:

$$\Delta\varepsilon_{\nu 2} = -\beta_1 \Delta\sigma - \eta\delta\gamma\ \Delta\tau \qquad (8)$$

in which β_1 is the volumetric rebound coefficient of the compressive half cycle. In Eq. (8),

$$\beta_1 = \beta_o \frac{1}{N_i} \qquad (9)$$

in which β_o is the volumetric rebound coefficient in the first compressive half cycle. Then the residual compressive strain in compressive half cycle will be

$$(\alpha_1 - \beta_1)\Delta\sigma - 2\eta\cdot\delta\gamma\cdot\delta\tau \qquad (10)$$

Similarly, the residual tensile strain in tensile half cycle will be

$$(\alpha_2 - \beta_2)\Delta\sigma - 2\eta\cdot\delta\gamma\cdot\Delta\tau \qquad (11)$$

in which α_2 is the volumetric tensile coefficient in tensile half cycle, β_2 is the volumetric rebound coefficient in tensile half cycle while

$$\alpha_2 = \frac{\alpha_o}{L} \frac{1}{N_i} \qquad (12)$$

$$\beta_2 = \frac{\beta_o}{L} \frac{1}{N_i} \qquad (13)$$

in which L is the ratio of volumetric compressive coefficient to volumetric tensile coefficient in first cycle, $L>1$.

Thus, the total residual compressive strain of sands after the Nth cyclic loading will be

$$\varepsilon_\nu^{(s)} = \sum_{N_i=1}^{N} [(\alpha_1 - \beta_1)\Delta\sigma - (\alpha_2 - \beta_2)\Delta\sigma - 4\eta\delta\gamma\Delta\tau]$$

$$= \sum_{N_i=1}^{N} [(\alpha_o - \beta_o)\frac{1}{N_i}(1 - \frac{1}{L})\Delta\sigma - 4\delta\gamma\Delta\tau] \qquad (14)$$

From this expression, it implies that the dilatancy of sands has strong effect on the volumetric change as well as the total deformation. For sandy soils with medium to dense state, η is positive and increases with D_r. Therefore the total compressive strain of the sand decreases significantly.

However, for the loose sands, η is negative and still less with the decrease of D_r. It makes the total compressive strain of loose sands to increase quickly. When N_i and Δ_τ increase, it induces very large deformation rapidly.

We can use the dynamic triaxial apparatus to measure all the parameters, whereas N_i is adopted according to the dynamic condition to be simulated.

(2) Dynamic Axial Strain of Saturated Cohesive Soils

For normally consolidated cohesive soils, when it sustains cyclic stress under saturated undrained condition, its volumetric change is induced by pore water pressure variation. The cohesive soil does not possess the dilatancy property. Thus, we could deduce the total residual compressive strain of cohesive soil after the Nth cyclic loading to be

$$\varepsilon_\nu^{(c)} = \sum_{N_i=1}^{N} \left[(\alpha_o - \beta_o) \frac{1}{N_i} \left(1 - \frac{1}{L} \right) \Delta u \right. \qquad (15)$$

in which Δu is the pore pressure increment.

Similarly, this expression could be used for dynamic deformation analysis and calculation of E_{od} and G_d of saturated cohesive soils.

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