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## **A SIMPLIFIED APPROACH FOR THE EVALUATION OF KINEMATIC PILE BENDING**

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### ABSTRACT

There are two sources of loading of the pile by the earthquake: “*inertial*” loading of the pile head caused by the lateral forces imposed on the superstructure, and “*kinematic*” loading along the length of the pile caused by the lateral soil movements developed during the earthquake. Seismic codes prescribe that piles have to be designed for soil deformations arising from the passage of seismic waves which impose curvatures and thereby lateral strains on the piles along their whole length. Accepting these lines, the new Italian seismic normative (NTC, 2008) specify that kinematic effects should be taken into account in the design of pile foundations.

Pseudo-static approaches for the seismic analysis of pile foundations are attractive for practicing engineers because they are simple when compared to difficult and more complex dynamic analyses. Thus, in the paper a simplified numerical model for the analysis of the behavior of a single pile subjected to static loadings and/or to lateral soil movements based on the “*p-y*” subgrade reaction method has been adopted. The approach involves two main steps: first a *free-field* site response analysis is carried out to obtain the soil displacements along the pile; next a static load analysis is carried out for the pile subjected to the maximum *free-field* soil displacements at each node along its length and the static loading at the pile head based on the maximum ground surface acceleration.

### INTRODUCTION

The seismic evaluation and design of soil-pile interaction is an area of active research and seismic analysis should be performed to evaluate both axial and lateral loading conditions during and after a seismic event. It is known, that in the past pile groups have undergone lateral translations severe enough to cause loss of bearing support for superstructures and structural failure in the piles (Okamoto, 1983; Mizuno, 1987; Tokimatsu et al., 1996; Nikolaou et al., 2001).

Recent destructive earthquakes have highlighted the need for increased research into the revamping of design codes and building regulations to prevent catastrophic losses in terms of human life and economic assets.

European seismic code (Eurocode-8) and Italian seismic normative (NTC, 2008) state that piles shall be designed for the following two loading conditions: *a*) inertia forces in the superstructure transmitted on the heads of the piles in the form of axial and horizontal forces and bending moments; *b*) soil deformations arising from the passage of seismic waves which impose curvatures and, thereby, lateral strains on the piles along their whole length. Accepting these lines, kinematic effects should be taken into account in pile design.

While there is an ample experience in carrying out dynamic or equivalent static analyses for inertial loading (generally the inertial forces acting on the pile head are obtained from the product of mass and spectral acceleration), no universally accepted methods or procedures are available to predict deformations and bending moments from kinematic loading.

In the last years several simplified approaches for the analysis of single piles or pile groups have been developed that can be used with little computational effort (Liyanapathirana & Poulos, 2005; Poulos, 2006). These methods have given results that are often in remarkable agreement with the mathematical models (Novak, 1974; Makris & Gazetas, 1992).

To estimate the maximum internal forces on piles subjected to lateral seismic excitation, some Authors (Abghari & Chai, 1995; Tabesh & Poulos, 2001) propose an analysis in which the pile is subjected to the *free-field* soil displacements at each node along its length. These displacements are obtained from a separate *free-field* site response analysis carried out, say, by the well known SHAKE (Schnabel et al., 1972) program or similar computer codes. Then a “*p-y method*” is adopted to simulate the response of the pile to the lateral soil movements.

According to the approach proposed by Liyanapathirana & Poulos (2005), in the paper a numerical model for the analysis of

the behavior of a pile subjected to static loadings and/or to lateral soil movements based on the “*p-y*” sub-grade reaction method has been developed.

The problem is similar in principle to the problem of statically-induced ground movements (Figure 1), even if there are additional complexities that must be recognized (Poulos, 2006).

The numerical results can be presented in terms of displacement profile and bending moment distribution along the pile length and, it is shown, that the proposed numerical analysis can be successfully utilized to simulate kinematic interaction of piles.

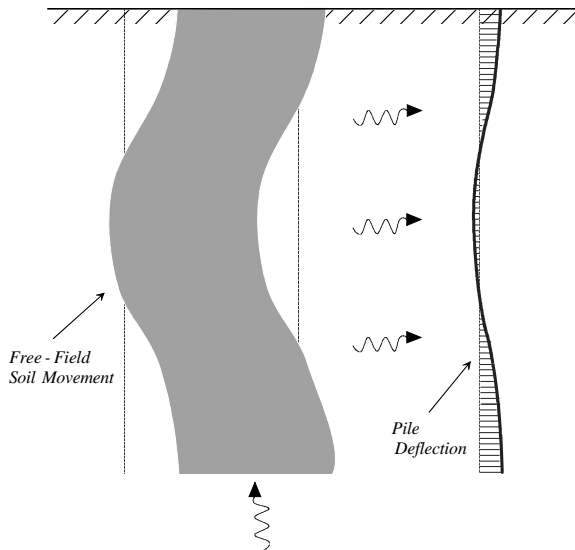


Fig. 1. Profile of external soil movement caused by earthquake.

### BASIC ASSUMPTIONS

The passage of seismic waves through soft soil during strong earthquakes may cause significant strains develop in the soil. In the presence of embedded piles, curvatures will be imposed to the piles by the vibrating soil which, in turn, will generate bending moments. These moments will develop even in the absence of a superstructure and are referred to as “*kinematic*” moments, to be distinguished from moments generated by lateral loads imposed at the pile head (so-called “*inertial*” moments).

Kinematic pile bending tends to be amplified in the vicinity of interfaces between soft and stiff soil layers (Figure 2). The reason is, soil shear strain is discontinuous across interfaces because of the different shear modulus between the soil layers and, thereby, the associated soil curvature (the derivative of strain) is infinite. In fact most of the pile damage observed deep below the soil surface is concentrated close to such discontinuities (Mylonakis, 2007).

To take into account these two sources of loading, Abghari & Chai (1995) proposed an analysis in which the pile was subjected to the *free-field* soil displacements at each node along its length. These displacements were obtained from a separate *free-field* site response analysis. The inertial forces acting on the pile were obtained from the product of superstructure mass and spectral

acceleration. These forces were applied to the heads of the piles as static forces.

Similarly Tabesh & Poulos (2001) proposed a simple approximate methodology for estimating the maximum internal forces of piles subjected to lateral seismic excitation. The method requires the evaluation of the maximum *free-field* soil movements caused by earthquake and the analysis of the response of the pile to the maximum *free-field* soil static movements plus a static loading at the pile head (Figure 3), which depends on the computed maximum spectral acceleration of the structure being supported.

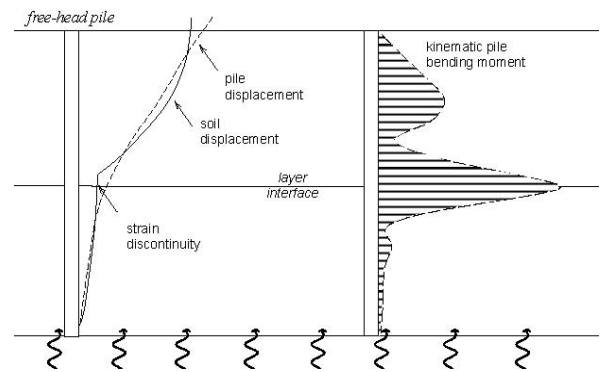


Fig. 2. Kinematic bending of a free-head pile in a two layer soil profile (Castelli et al., 2008).

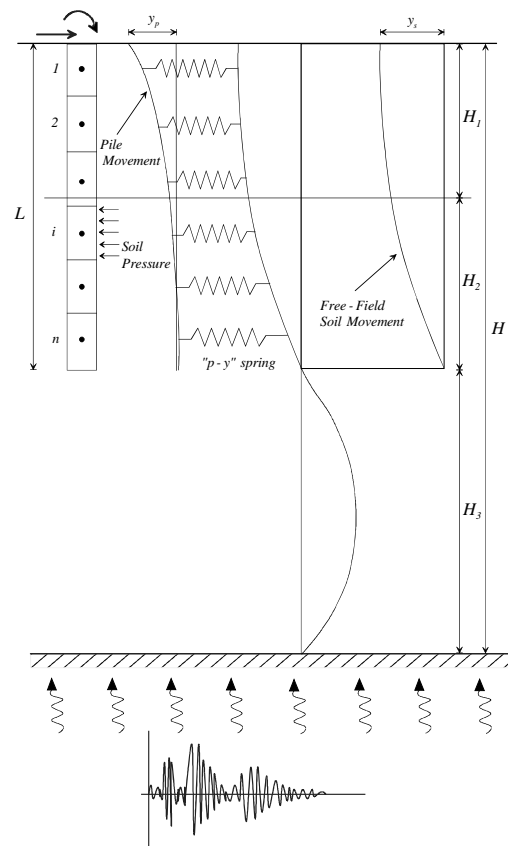


Fig. 3. Pile model for pseudo-static seismic analysis.

Obviously, subjecting the pile to the envelope of the maximum *free-field* soil displacements at each depth does not guarantee maximum bending and the time-history of the *free-field* soil deformations should be taken into consideration. By the way, many time-history displacement profiles must be considered, and even in this case the results obtained are approximate due to the model adopted for the site response analysis.

Nevertheless, it can be noted that the bending moment distribution obtained by a pseudo-static push over analysis and/or subjecting the pile to the envelope of the *free-field* soil displacements is very close to that obtained by a dynamic analysis in the time domain, assuming as input motion an acceleration time history (Castelli et al., 2008). Thus, for the evaluation of the bending moment profile due to the kinematic interaction, it seems reasonable to assume the envelope of the maximum *free-field* soil displacements and, thereby, to evaluate maximum bending independently of the time history.

## MODELING PROCEDURE

Several approaches are currently available to analyze the behavior of piles subjected to lateral loads ranging from complex models, as non linear dynamic analysis and 2D or 3D finite element methods, to the use of simplified approaches as limit equilibrium (LE) approach and *p-y* analysis approach.

The *p-y* approach for analyzing the response of laterally loaded piles is essentially a modification of the basic Winkler model, where *p* is the soil pressure per unit length of pile and *y* is the pile deflection. The soil is represented by a series of nonlinear *p-y* curves that vary with depth and soil type.

Current research based on results of field tests on full scale piles, both in cohesive than in cohesionless soil, suggests to employ non linear *p-y* relationship (Reese et al., 1974; Reese & Welch, 1975; Reese et al., 2000; Juimmarongrit & Ashford, 2006), thus in the proposed approach a hyperbolic *p-y* relationship has been adopted:

$$p(z) = \frac{y_p(z)}{\frac{1}{E_{si}(z)} + \frac{y_p(z)}{p_{lim}(z)}} \quad (1)$$

where  $E_{si}$  is the initial modulus of horizontal subgrade reaction,  $y_p$  is the pile lateral deflection [L],  $p$  and  $p_{lim}$  are the mobilized and the ultimate horizontal soil reaction per unit length of pile [ $FL^{-1}$ ] respectively.

Therefore, the units of  $E_{si}$  are in  $FL^{-2}$ . If  $p$  and  $p_{lim}$  are expressed as the mobilized and the ultimate horizontal soil resistance [ $FL^{-2}$ ], then  $k_{si} = E_{si}/D$  is the coefficient of horizontal subgrade reaction (force per unit volume [ $FL^{-3}$ ]) being  $D$  the pile diameter (Prakash & Kumar, 1996).

The hyperbolic *p-y* relationship (1) is defined by the two parameters  $p_{lim}$  and  $E_{si}$ . By this approach a situation of layered soil can be easily analyzed assuming for these parameters different values along the pile length. The numerical model is based on an iterative procedure taking into account decrease in model stiffness with an increase in the applied horizontal load

(Castelli et al., 1995; Castelli & Maugeri, 1999; Castelli, 2002; 2006).

To evaluate kinematic pile bending, the simplified numerical approach proposed in the paper is based on the idea of coupling and evaluating the effects due to the applied load at the pile head and lateral movements along the pile length by a series of independent “*p-y* curves”, relating soil reaction and relative soil - pile movements.

Since these relations are nonlinear, the equivalent linear procedure using secant modulus is normally used to establish a law relating a set of discrete interaction forces at the soil/pile interfaces to a corresponding set of discrete relative motions between piles and *free-field* soil.

The implementation of the method involves imposing a known *free-field* soil movement profile. When the expected *free-field* movement is large enough to cause the ultimate pressure of laterally spreading soils to be fully mobilized, the ultimate pressure, instead of *free-field* soil movement, may be used.

If the soil mass moves and the pile movement  $y_p$  is less than the soil movement  $y_s$ , the soil exerts a driving force on the pile. However, if the pile movement  $y_p$  is greater than the soil movement  $y_s$ , the soil provides the resistance force  $p_{lim}$  to the pile. Thus, the response of the pile can then obtained by solving the following governing differential equation:

$$EI \frac{d^4 y}{dz^4} - p(y_p - y_s) = 0 \quad (2)$$

where  $EI$  = pile stiffness,  $p$  = soil reaction per unit pile length and  $z$  = depth.

This equation can be solved by a numerical procedure based on a pile finite-element discretization, in which the pile load (force per unit area [ $FL^{-2}$ ]) due to relative pile-soil movement ( $y_p - y_s$ ), can be represented by a series of *p-y* curves on both sides of the pile shaft along its length (Figure 4).

As a first approximation an idealized elastic *p-y* relationship could be used:

$$p(z) = \frac{E_{si}(z)}{D} [y_p(z) - y_s(z)] \quad (3)$$

where  $D$  is the pile diameter.

As mentioned earlier, to take into account that the lateral pile response to static or dynamic loading is non linear, the following *p-y* relationship could be adopted:

$$p(z) = \frac{[y_p(z) - y_s(z)]}{\frac{1}{E_{si}(z)} + \frac{[y_p(z) - y_s(z)]}{p_{lim}(z)}} \quad (4)$$

in which  $E_{si}$ ,  $y_p$ ,  $p$  and  $p_{lim}$  are defined as in equation (1).

Construction of *p-y* relationships at each specified depth location depends on soil strength parameters, i.e., the friction angle for sands and the cohesion for clays. The main difficulty of applying the *p-y* curve approach is the appropriate evaluation of the functions parameters for a realistic estimation of single pile and/or pile group performance.

The adoption of hyperbolic  $p$ - $y$  curves, in particular, requires the determination of the ultimate horizontal soil resistance  $p_{lim}$  and initial modulus of horizontal subgrade reaction  $E_{si}$ .

In a single pile-soil interaction, the ultimate horizontal soil resistance  $p_{lim}$  can be evaluated according to the well known formulas existing in literature (Matlock, 1970) both for cohesive (Broms, 1964<sup>a</sup>) than for cohesionless soils (Broms, 1964<sup>b</sup>). As concern the initial modulus of horizontal subgrade reaction  $E_{si}$ , it can be evaluated according to the relationships proposed by Matlock (1970), Welch & Reese (1972), Robertson et al., (1989) for hyperbolic  $p$ - $y$  curves.

An alternative term that is often used in place of  $E_{si}$ , is the coefficient of horizontal subgrade reaction  $k_{si}$ , which is expressed in units of force per unit volume, even if  $E_{si}$  is a more fundamental soil property because it not involve the pile diameter.

The simplified non linear approach proposed in the paper can be considered a semi-empirical design method. Often the design of single pile and pile groups for lateral capacity has tended to rely on semi-empirical approaches, that make it difficult for practical designers to select the appropriate procedure.

An improvement of these approaches can be achieved if more reliable and relevant soil properties are adopted in defining the input parameters even if the design method is empirical or semi-empirical. The shear modulus  $G_o$ , as example, can be considered essential parameters for the estimation of the soil response, especially in the case of dynamic loading, and it is generally determined in situ by measurements relating it to the propagation velocity of seismic shear waves  $V_s$  and the soil density  $\rho$ , thought the well known relationship:  $G_o = \rho V_s^2$ .

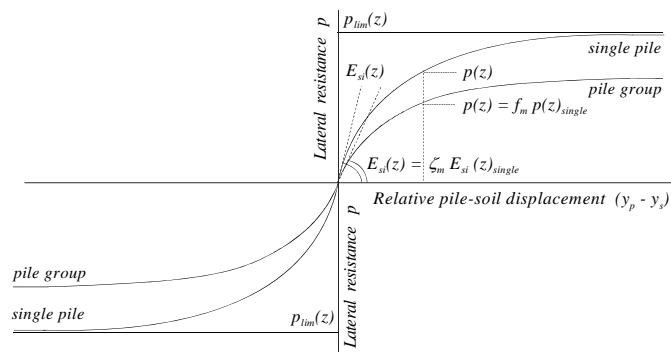


Fig. 4.  $p$ - $y$  curves to model loading due to lateral movements.

## NUMERICAL ANALYSIS

The proposed approach was implemented in a novel computer code developed by the authors. Such code allows the assessment of the pile lateral response (deflection, moment and shear force distribution). The pile can be considered subjected to the simultaneous application of a lateral force and/or moment at its head and/or a free-field soil movement profile along its length.

Validation of the proposed approach for kinematic loading along the length of the pile caused by the lateral ground movements developed during the earthquake has been carried out comparing

the computed results with those obtained from other existing analytical and numerical methods.

In particular, the numerical simulation regards the results presented and discussed during the workshop on “Recent advances in Codes” held in Thessaloniki (2007) during the 4<sup>th</sup> International Conference on Earthquake Geotechnical Engineering. The aim of the workshop was to investigate on the state of the art in geotechnical earthquake engineering codes.

One of the benchmark problems was the evaluation of the kinematically induced bending moments in a pile, drilled through a very soft clay layer, and subjected to a strong and long duration shaking.

Numerical analyses were performed for a free-head pile having a length of 28.5 m. It is a hollow-cylinder of 40 cm external diameter and 30 cm internal and it is made of pre-stressed high strength concrete. The soil deposit consists of 20 meters very soft clay and peat, underlain by a 10 m thick, much stiffer sandy silt layer. Dense gravel exists between the depths of 30 to 132 m, where rock is encountered. The gravel is interrupted by a thick layer of mudstone between the depths of 40 to 52 m.

The site stratigraphy, soil properties and shear wave velocity profile were used as provided in the problem statement (Table 1). More information’s are reported in the proceedings of the conference and/or in *Summary of Workshop 3* available on the web site of the 4<sup>th</sup> ICEGE.

An earthquake ground motion recorded at a depth of 153 m (in the sand stone bedrock) in Atsuma by the Kik-net array, was used as input motion. It was shaken by the 2003 Tokachi-oki Earthquake. Figure 5 shows the acceleration time history.

In the following, the results of the numerical simulations carried out by Fugro West Inc. & AMC (2007) and Muroso & Hatanak (2007) are taken into consideration. These results are obtained by two approaches in which a separate *free field* site response analysis is carried out (in the first case using the code SHAKE), and the *free field* displacement profile (Figure 6) is applied to model the kinematic loading induced by the propagation of the seismic waves.

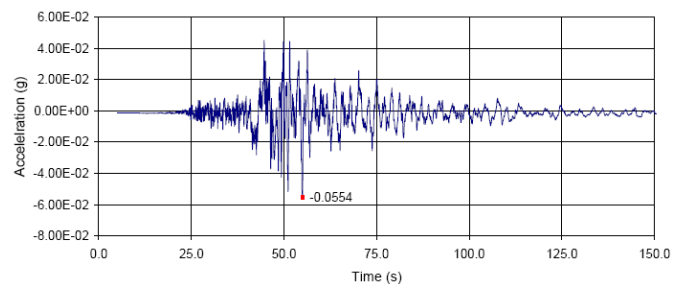


Fig. 5. Input ground motion: acceleration time history.

Table 1. Soil properties derived by laboratory tests.

Depth (m)	Soil	$V_s$ (m/s)	Density ( $\text{kN/m}^3$ )
0-6	Peat	60	13
6-20	Clay	90	15
20-30	Sandy Silt	190	18
30-40	Gravel	320	20
40-52	Mudstone	210	22
52-76	Gravel	310	20
76-132	Gravel	430	20
132-153	Sandstone	520	23

The numerical simulation has been carried out and the results obtained are reported in Figure 7 in terms of bending moment distribution along the pile length.

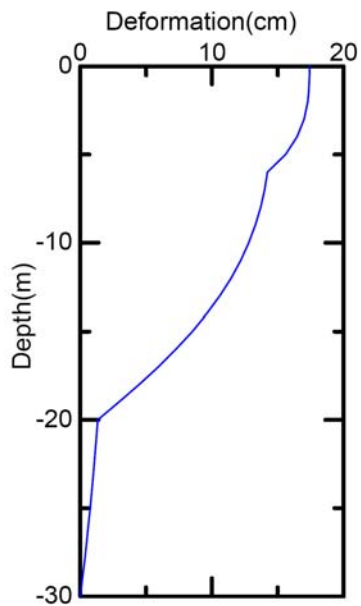


Fig. 6. Seismic ground deformation relative to the toe of the pile (Murono & Hatanak, 2007).

The properties of the soil foundation used in the development of the  $p$ - $y$  curves are reported in Table 2.

The ultimate horizontal soil resistance was determined according to Broms' theory (1964<sup>a</sup>; 1964<sup>b</sup>), while the initial modulus of horizontal was determined assuming for the ratio  $E_{sv}/G_o$  a value equal to 0.5 (Castelli & Maugeri, 2009).

Table 2. Soil properties used in the development of  $p$ - $y$  curve.

Depth (m)	Soil	Undrained shear strength (kPa)	Friction angle	Initial shear modulus $G_o$ (MPa)
0-6	Peat	15	-	46.8
6-20	Clay	25	-	121.5
20-30	Sandy Silt	-	33°	649.8

The agreement between the present numerical analysis and the results provided by the other formulations is found to be satisfactory, including those provided by the proposed pseudo-static approach.

### CONCLUDING REMARKS

A simple engineering model and a simplified analysis procedure were outlined for the kinematic bending of a single pile under the passage of seismic waves which impose curvatures and, thereby, lateral strains on the piles along their whole length.

To evaluate the internal response of piles subjected to earthquake loading, a simplified pseudo-static method based on the “ $p$ - $y$ ” subgrade reaction approach has been developed. The method involves the evaluation of the *free-field* soil movements caused by earthquake and the analysis of the response of the pile to the maximum *free-field* soil static movements plus a static loading at the pile head.

This approach could be attractive for practicing engineers because it is simpler than dynamic analyses, and permits to evaluate a bending distribution which is independent of the time history.

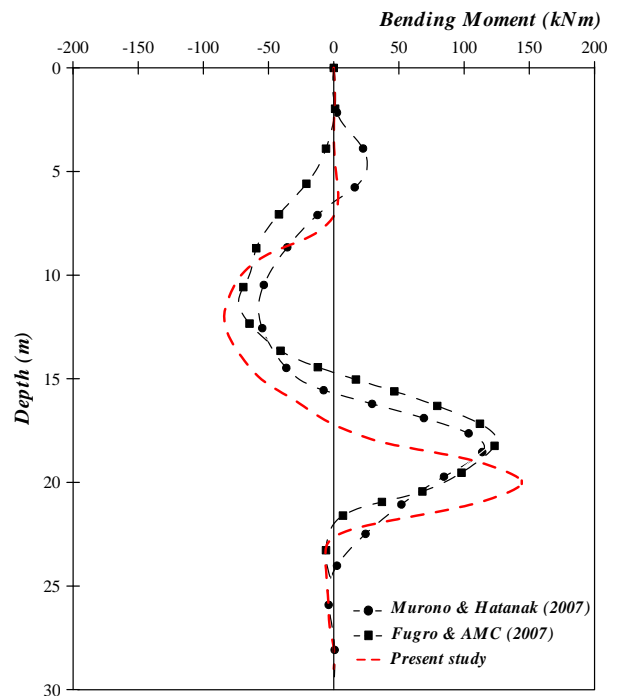


Fig. 7. Numerical results: bending moment profiles.

### ACKNOWLEDGEMENTS

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