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Seismic Response of Pile Supported Structures

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SYNOPSIS Earthquake loads are applied to the foundation mainly through shear waves in the underlying soil. A method is presented for analyzing the alteration in the response of a structure by adding piling to the foundation. The method of computation consists of a series of transfer matrices which are used to form a stiffness matrix of the foundation system. Central to the computation is the modelling mode for the soil-pile interaction; several alternatives are presented. Observations regarding the effect of several design parameters, based on a numerical example, are discussed.

INTRODUCTION

Unlike the dynamic loads generated by machinery, earthquake loads are applied to the foundation, mainly through shear waves in the underlying rock and/or soil. The displacement of a pile foundation during an earthquake arises from the shaking of the soil-pile system and the inertia of the superstructure. Nair (1968) and Tajimi (1977) discussed the problem of seismic effects on piles in comprehensive state-of-the-art reports. The response of a structure to seismic disturbance is a very complex phenomenon. The most refined continuum or finite element methods available are based on simplified models and the use of greatly simplified material properties. Yet considerable progress in analytic procedure and evaluation of soils for dynamic response have been made since Nair's (1968) report; see Margason (1977), Zeevaert (1977) and Oweis (1980).

Often, some form of shallow foundation, such as a mat, is a viable alternative to a deep foundation. It is a hypothesis of this presentation that the fundamental frequency of the system may be altered by the use of piles. Energy released by seismic activity in the form of waves when transferred through a soil media to a structure may be expected to be modified by reflection, refraction and damping, so that the energy spectrum at the structure is substantially different than that at bedrock. Thus the soil acts as a filter. It appears (Tajimi 1977) that a long pile acts substantially together with the soil when subjected to seismic activity. This seems to be a highly reasonable compatibility assumption. Moreover, it is postulated that the stiffness of the pile-soil system is increased, thus changing the filter characteristics of the soil. Prediction of the change in response due to the presence of piling could be influential in design.

The purpose herein is (1) to introduce a procedure for analyzing the response of structures on piling to earthquake excitation which can utilize any linear or piecewise linear soil-pile interaction model, including measured influence coefficients; (2) to suggest the suitability of the beam on spring foundation model, Winkler (1867), in any of 5 different forms including an all purpose layered model for most design and many analysis purposes; and (3) to explore the soil filter modification by piling hypothesis.

METHOD OF ANALYSIS

Saul (1968, 1979, 1980) derived the stiffness matrix [S] for a rigid foundation on piles (thus 6 kinematic degrees of freedom - 3 in transition and 3 in rotation) with pile variables including; number, material, spacing, position or elevation, or inclination such that the pile loads {Q} and displacement {Δ} are related by

$$\{Q\} = [S]\{\Delta\} \tag{1}$$

The stiffness matrix is given by

$$[S] = \sum_{i=1}^{n} [cap]_i [b']_i [cap]_i^T \tag{2}$$

where i is one of n piles; $[c]_i[a]_i$ and $[p]_i$ are transformation matrices for coordinates, batter, and principal axes rotation of pile i respectively; and $[b']_i$ is the stiffness matrix for pile i in member coordinates. The statics problem may be solved for unknown loads or displacements given the corresponding displacements or loads using Eq.1. Pile forces and displacements can then be found in pile head coordinates parallel to the foundation coordinates by

$$\{x\}_i = [c]_i^T\{\Delta\} \text{ and } \{F'\} = [ap]_i[b']_i[ap]_i^T[c]_i^T\{\Delta\} \tag{3}$$

or, in member principal axes by

$$\{x\}_i = [cap]_i^T\{\Delta\} \text{ and } \{F\}_i = [b']_i[cap]_i^T\{\Delta\} = [b']_i\{x\} \tag{4}$$

The dynamic problem may be stated

$$[M]\{\ddot{\Delta}\} + [C]\{\dot{\Delta}\} + [S]\{\Delta\} = \{P(t)\} \tag{5}$$

where [M] is mass and mass moment of inertia, [C] viscous damping, {P(t)} a forcing function and $\dot{\Delta}_i$ and $\ddot{\Delta}_i$ are velocity and acceleration. The undamped free vibration form

$$[M]\{\ddot{\Delta}\} + [S]\{\Delta\} = \{0\} \tag{6}$$

leads to the eigenvalue problem

$$([S] - \omega^2[M])\{\phi\} = 0 \tag{7}$$

which will yield 6 frequencies ω_i and 6 mode shapes $\{\phi\}_i$. Definition of mass and damping are subject to a wide variation in modeling and interpretation; however, it is proposed that 30% of the soil within the pile group above the pile inflection point be included with the mass of the structure and that damping, when needed, be assigned by ratio of critical in modes ξ_i from soil test data. Damping is then given by

$$[C] = [M][\Phi][2\xi\omega][\Phi]^T[M] \tag{8}$$

where [Φ] is a matrix of mode shapes by columns which are orthonormal with respect to mass.

PILE-SOIL INTERACTION MODEL

The relationship between a force F_{ji} applied at the head of pile i and the 6 corresponding displacements $\{x\}_i$ of the pile head is given by

$$\{F\}_i = [b']_i\{x\}_i \quad (9)$$

where $[b']_i$ is as previously defined. If the load-displacement relationship is nonlinear it may be approximated as being piecewise linear in which case both the forces $\{F\}_i$ and displacements $\{x\}_i$ are increments within an interval. Since Eq. 9 is defined in member coordinates, which are the principal axes and the centroid, the stiffness matrix $[b']_i$ is necessarily sparse; that is, the stress resultants are uncoupled. Thus, only the b'_{ii} are non-zero excepting the coupling terms for shear and flexure in the vertical planes. These are given, see inset Appendix I for coordinate system, as b'_{15} and b'_{24}.

The stiffness coefficients b'_{ij} may be determined in any of several ways including by field or model tests or by the use of analytical models. Models suggested as being best include those advanced by Novak (1974) (a generalized Winkler model) and the beam on a spring foundation (Winkler model) as summarized by Saul (1968, 1977, 1980) for the lateral (flexural) modes. Models best suited for the axial component include those by Poulos (1972, 1977), Novak (1974), Vesić (1977), and Randolph and Wroth (1978); for the torsional those by O'Neill (1969), Poulos (1975) and Novak and Howell (1977).

Values of b'_{ij} for the flexural mode for a beam on spring foundation are summarized in Appendix I for 4 different models for long piles and in Appendix II for a segment of a short or layered system pile. When using the latter the stiffness matrix of each segment is computed, the parts summed into a global pile matrix, and the values desired extracted by matrix condensation. Alternative values given by Novak (1974) are

b'_{11} and b'_{22} $\quad (EI/L^3)F_{11}(\lambda)_1 \quad (10)$

b'_{44} and b'_{55} $\quad (EI/L)F_7(\lambda)_1 \quad (11)$

b'_{15} and b'_{24} $\quad (EI/L^2)F_9(\lambda)_1 \quad (12)$

where the $F_i(\lambda)_1$ are frequency dependent functions and it is assumed that the pile has a pinned end at length L, an unfortunate constraint.

The axial stiffness influence coefficient b'_{33} is given variously as follows:

Saul (1968) $\quad k_L(AE/L) \quad (13)$

Poulos (1972) $\quad E_sD/I \quad (14)$

where E_s is the modulus of elasticity of the soil, D the pile diameter, and I a pile settlement influence coefficient values of which are determined by integration of Mindlin's Problem, Part I (1936), an elasticity formulation in a half space. Later Poulos changed the coefficient to

Poulos (1977) $\quad E_sL/I_p \quad (15)$

where I_p is another form of pile settlement influence coefficient.

Novak (1974) $\quad (EA/L)F_{18}(\lambda)_1 \quad (16)$

which is similar to Eqs. 10 through 12 and 13.

Vesić (1977)

$$[(\beta+\alpha_s(1-\beta))(L/AE)+C_p\beta/(Dq_o)+C_s(1-\beta)/(Lq_o)]^{-1} \quad (17)$$

in which β is the ratio of load taken by the point, C_p and C_s are empirical coefficients, α_s is a skin friction distribution factor and q_o is the ultimate point resistance.

Randolph and Wroth (1978)

$$G_Lr_o\left[\frac{4}{\eta(1-\nu)} + \frac{2\pi\rho L \tanh(\mu L)}{\xi\, r_o\, (\mu L)}\right]\left[1 + \frac{4L\tanh(\mu L)}{\eta(1-\nu)\pi\lambda r_o(\mu L)}\right]^{-1} \quad (18)$$

where G_L is the soil shear modulus of elasticity at the tip, r_o the pile radius, L the pile length; ξ, η, ρ, μ and λ various parameters, and ν Poisson's ratio for the soil.

The methods of Saul (1968) and Novak (1974) must use a coefficient k_L or $F_{18}(\lambda)_1$ between 0.5 and 1.0 in most cases. If the pile tip moves, it is particularly necessary that $k_L < 1.0$.

The torsion constant b'_{66} has variously been given as follows:

O'Neill (1969) $\quad (4\pi r^2 G_s G_p J)^{1/2} \quad (19)$

where J is the torsional constant of the pile.

Saul (1968) $\quad k_T(GJ/L) \quad (20)$

Poulos (1975) $\quad GD^3(F_\phi/I_\phi) \quad (21)$

where F_ϕ is a soil slip factor and I_ϕ a charted influence factor.

Novak and Howell (1977) $\quad (G_pJ/L)(\omega L)\coth(\omega L) \quad (22)$

The coefficient k_T in Eq. 20 would usually be 1 or greater and may be as high as 6.

For computation the flexural coefficients of Appendices I or II are preferred, the axial coefficient given by Eqs. 15, 17 or 18 and the torsional value by Eqs. 19, 21 or 22. If the pile is long, that is βL or $\psi L \geq \pi$ where

$$\beta^4 = k_sD/(4EI) \quad \text{and} \quad \psi^5 = \eta D/(EI) \quad (23)$$

and k_s and ηz are moduli of subgrade reaction constant with depth or increasing linearly with depth, respectively, the coefficients b'_{ij} determined from the relationships of Appendix I and the aforementioned equations are readily calculated. In the case of short piles, a nonlinear soil, nonprismatic piles, or piles in a layered media the flexural coefficients are determined from the stiffness values for a segment as given in Appendix II where the modulus of subgrade reaction k_s is assumed constant over the segment. The axial and torsional components may be determined separately and superimposed or included in each segment by increasing the size of the member stiffness matrix.

A computer program, called PILE, has been written to compute the pile stiffness matrices $[b']_i$ and the foundation stiffness matrix [S]. Options are available with which to compute displacements, forces or eigenvalues. If the latter, a diagonal mass matrix is supplied.

EARTHQUAKE ANALYSIS

The ground surface acceleration $\ddot{U}$ is obtained from earthquake input at bedrock for a given soil profile. Any method may be used for the computation such as the numerical integration techniques outlined by Schnabel, et al.

(1972). The mass matrix is estimated; it includes the structure, foundation and added mass of soil. Since the inflection point for a fixed head pile is about $3/\beta$ (see Eq. 23) about $1/\beta$ to $1.5/\beta$ depth of soil enclosed by the pile pattern may be included. The response for a given structure on a shallow foundation to the input $\ddot{U}$ may be obtained following Richart, et al. (1970) to obtain the response and/or response spectrum. The natural frequencies are also readily determined.

Piles may be added to the above shallow foundation. A preliminary design can be done by rough approximation by obtaining a set of pseudo earthquake loads

$$\{Q\} = [M]\{\ddot{U}\} \quad (24)$$

where $\{\ddot{U}\}$ is a vector of the free-field surface accelerations. These loads are used for an initial pile design. The pile foundation may then be analyzed using Eq. 7 to obtain the natural frequencies of the new system. Correction should be made for group action such as given by Wolf and von Arx (1978). The system may then be analyzed using compatibility of strains such as by Oweis (1980) or as suggested by Matlock, et al. (1978). Thus, the two designs, alike in all respects with the exception of added piling are expected to perform quite differently when subjected to earthquake excitations.

A numerical example was carried out using the procedure described above but is not reproduced here since the method of, and the data from, a single example is insufficient to establish general conclusions. Discussion of the effects of several parameters is, however, warranted. The soil profile used in the example consisted of alternating layers of sand and clay with known densities and strength properties as given by Schnabel et al. (1972). The Kern County earthquake of 1952 scaled to earthquake magnitude 7.4 was used. The design parameters included piles of different materials, pile batter, number of piles in the group, and pile spacing. The foundation was also analyzed without any piles.

CONCLUSIONS

The addition of piles appears to significantly increase the natural frequencies of the system compared to the same foundation without piles. The pile system may be arranged to increase the foundation stiffness by increasing the number of piles, battering piles, using piles of higher individual stiffness and/or increasing the spacing between piles. The effect of length of piling has not yet been evaluated.

The use of piling to alter the natural frequency of a foundation system and thus the response of a structure to seismic disturbance seems reasonable since it appears to change the stiffness of the underlying soil. The natural frequencies of the foundation system are readily obtained using the method of computation and models given. Considerable work remains to be done to evaluate relative effects of the various parameters of soil and pile on overall response for a wide variety of cases.

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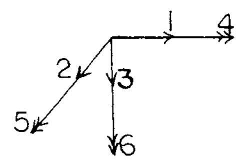
Appendix I

Nonzero Coefficients for Semi-Infinite Piles in Flexure

Model	b'_{11} & b'_{22}	b'_{44} & b'_{55}	b'_{15} & $-b'_{24}$ **
A. Single Layer			
1. Beam on a* Constant Spring Foundation	$(1+\delta)\kappa_i\beta_i^2$ where $\kappa_i = 2\beta_i EI_i$	$\delta\kappa_i$	$\delta\kappa_i\beta_i$
2. Beam on a Linearly Increasing Spring Foundation	$(0.427+0.672\delta)\lambda_i\psi_i^2$ where $\lambda_i=EI_i\psi_i$	$1.503\delta\lambda_i$	$\delta\lambda_i\psi_i$
B. Two Layer (ℓ is the unsupported length of cantilever)			
1. Cantilever Beam Adjoining Model A1	$3[1+\delta(1+2\beta_i\ell)]\beta_k^2\kappa_i t_i$	$\delta(3+6\beta_i\ell+6\beta_i^2\ell^2 +2\beta_i^3\ell^3)\kappa_i t_i$	$3\delta(1+2\beta_i\ell+\beta_i^2\ell^2)\beta_i\kappa_i t_i$
	where $t_i = 1/[3+6\beta_i\ell+6\beta_i^2\ell^2+(1+\delta)2\beta_i^3\ell^3+\delta\beta_i^4\ell^4]$		
2. Cantilever Beam Adjoining Model A2	$3(1+3\delta)[\psi_i+\delta(1.686+\psi_i\ell -\psi_i)]\lambda_i\psi_i^2 s_i$	$4\delta(6.918+9.204\psi_i\ell +5.058\psi_i^2\ell^2+\psi_i^3\ell^3)\lambda_i s_i$	$6\delta(3.068+3.732\psi_i\ell+\psi_i^2\ell^2)\lambda_i\psi_i s_i$
	where $s_i=1/(18.417\delta+[1+\delta(\ell-1)][6.918(1+3\delta)\psi_i+9.204(1+\delta)\psi_i^2\ell+1.686(3+\delta)\psi_i^3\ell^2+\psi_i^4\ell^3])$		



* Subscripts i must be adjusted for the direction or axis of bending. For the fixed condition $\delta=1$ and for the pinned end $\delta=0$.

** The form of b'_{24} is always the same as the form for b'_{15} but negative. The values may not be the same however since $\psi_1 \neq \psi_2$ or $\beta_1 \neq \beta_2$ unless piles have $I_x=I_y$ and $D_x=D_y$.

Appendix II

Stiffness Matrix for a Pile Segment

$$[k]_i = \begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7\\8 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} T3_y & 0 & 0 & T5_y & T4_y & 0 & 0 & -T6_y \\ 0 & T3_x & -T5_x & 0 & 0 & T4_x & T6_x & 0 \\ 0 & -T5_x & T1_x & 0 & 0 & -T6_x & T2_x & 0 \\ T5_y & 0 & 0 & T1_y & T6_y & 0 & 0 & -T2_y \\ T4_y & 0 & 0 & T6_y & T3_y & 0 & 0 & -T5_y \\ 0 & T4_x & -T6_x & 0 & 0 & T3_x & T5_x & 0 \\ 0 & T6_x & T2_x & 0 & 0 & T5_x & T1_x & 0 \\ -T6_y & 0 & 0 & -T2_y & -T5_y & 0 & 0 & T1_y \end{bmatrix}_k \end{array}$$

where,
$T1_i = (C'S' - CS)\kappa q$
$T2_i = (C'S - CS')\kappa q$
$T3_i = 2(CS + C'S')\kappa\beta^2 q$
$T4_i = 2(C'S + CS')\kappa\beta^2 q$
$T5_i = (S'^2 + S^2)\kappa\beta q$
$T6_i = 2SS'\kappa\beta q$

$q = 1/(S'^2-S^2)$
$C = \cos\beta L$
$S = \sin\beta L$
$C' = \cosh\beta L$
$S' = \sinh\beta L$
$\kappa = 2\beta EI$
$\beta^4 = k_s D/(4EI)$

Segment i of Pile with Member Degrees of Freedom