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K. Nakagawa
Osaka City University, Japan

K. Soga
University of Cambridge, Cambridge, UK

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Nonlinear Cyclic Stress-Strain Relations of Soils

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K. Nakagawa
Professor of Geosciences, Osaka City University, Osaka, Japan

K. Soga
Lecturer of Engineering, University of Cambridge, Cambridge, UK

SYNOPSIS A cyclic torsional shear testing system was developed to measure the dynamic properties of soils at a wide range of strain levels (10^{-4}~1%). Use of proximity transducer and pneumatic actuator in a closed loop system enabled us to measure the deformation at very small strains. A new simple nonlinear model of $G/G_{max}=1/(\alpha+\gamma^{\beta})$ agreed well with the test results of various geologic materials. In this model, parameter α represents the strain at which the stiffness starts to decrease, and parameter β controls the rate of the stiffness degradation. Loose sands had larger α and β, whereas clays and mudstones had smaller α and β. A unique relationship of β=0.2logα+0.3 was also found from the compiled data.

INTRODUCTION

In order to predict more accurate site response during earthquakes, there is a need to investigate the deformation mechanisms of geologic materials under dynamic loading. The two purposes of this study were to measure nonlinear cyclic stress-strain relations at a wide range of strain levels and to model them for better physical interpretation.

In the past, the stress-strain relations at very small strains, say 10^{-2} to 10^{-4}%, had been measured by a resonant column device. However, it did not easily measure the stress-strain relations at large strain levels because of the fundamental principle of the device. Thus, a combination of the resonant column and cyclic torsional shear tests is now commonly used to obtain the stress-strain relations at a wide range of strain levels.

In this study, an improved cyclic torsional shear device was developed to investigate the deformation of strain levels between 10^{-4} and 1%. Use of a proximity transducer enabled us to measure the cyclic stress-strain relations at the very small strain levels common in the range of the resonant column tests.

A simple mathematical expression was proposed in this study to model the backbone stress-strain curves measured by the new device. This model also agreed well with other published data of various soils including sands, silts, clays, and mudstones.

TESTING APPARATUS AND MATERIALS

Cyclic Torsional Shear Device

A schematic diagram of the cyclic torsional shear device developed for the investigation is shown in Fig. 1. This testing device has the following unique characteristics compared to similar devices used in the past investigations.

- It can measure strains ranging from 1% to 10^{-4}% or less with the use of a proximity transducer. The sensitivity of the transducer is on the order of 0.1μm.

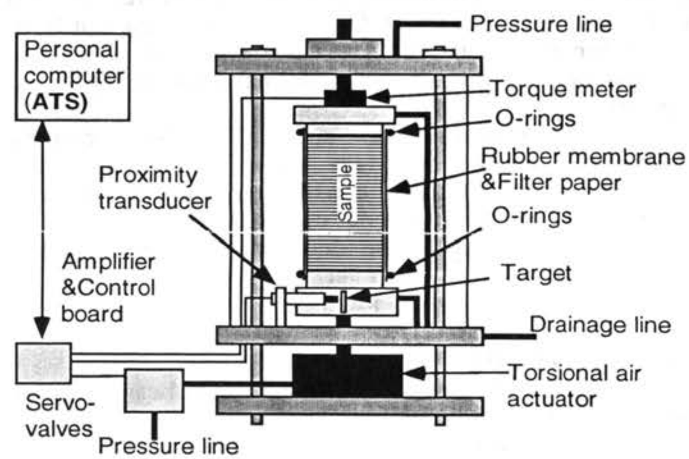

Fig. 1 Schematic diagram of the torsional shear testing system

- A pneumatic actuator is used for torsional driving force. This actuator provides small vibrations during the operation. The supporting system of this actuator is more compact than that of a hydraulic actuator.
- The device is controlled by a closed loop system using two servo-valves and ATS software (Sousa (1993)).

We installed a proximity transducer on the pedestal of the device as shown in Fig. 1. Since the displacement of a specimen could be measured without any contact with other parts of the device, the boundary was totally free at the bottom, whereas it was fixed at the top. A number of spikes were driven into the porous metals of the top cap and pedestal in order to avoid sliding between the specimen and the top cap or pedestal.

In torsional shear of a solid cylindrical specimen, the applied shear strain in the specimen increases with increasing distance from its center. Thus, the shear strain reported in this paper was that at the peripheral part of the specimen.

Shear stress applied to the specimen was measured by a torque meter. The torque meter was installed on the fixed shaft above the top cap. Four electrical resistance strain gages were mounted on the circular shaft by two perpendicular 45 degrees helices that were diametrically opposite one another.

The data noise was reduced by a stacking operation so that a smooth cyclic stress- strain curve could be obtained. In this operation, sets of hysteresis stress-strain loops were first recorded individually. Then, the loops were added together to reduce the noise by counterbalancing. A reasonable number of stacking operations was 20 times for strain levels of 10^{-4}%, 5 times for 10^{-3}%, and 1 time for 10^{-2}% and larger. At small strain levels of less than 10^{-3}%, the reduction of stiffness by several number of cycles was considered minimal.

Testing Procedure and Tested Soils

The tested soils included sands, undisturbed clays and reconstituted clays. The physical properties of these tested soils are shown in Table 1.

The approximate size of the specimens was 5cm in diameter and 10 cm in height. Sand specimens were created by air pluviation in a mold. Clay specimens were cut from a block sample. A strip filter paper was placed around the clay specimen, allowing radial drainage for faster initial consolidation.

Confining pressure was applied to the specimen isotropically in the cell (see Table 1). All the clay specimens were initially consolidated to the desired pressure for a duration of one day, ensuring fully consolidated specimens in the secondary compression stage. Dispersed kaolinite, however, required two days of consolidation because of its low permeability.

Test Interpretation

For interpreting the test results, the secant shear modulus of a soil was defined as follows:

$$G = \tau_c / \gamma_c \tag{1}$$

where τ_c is the cyclic stress amplitude corresponding to the strain amplitude γ_c.

The area ΔW, which encloses the hysteresis loop, was used to define the material damping ratio λ by the following equation:

$$\lambda = \frac{1}{2\pi}\frac{\Delta W}{G\gamma^2} \tag{2}$$

Both G and λ depend on the cyclic strain amplitude. Although many mathematical equations were proposed in the past to approximate the measured backbone stress-strain curves, a new mathematical expression was used in this study because it gave a better curve fitting to the measured data. The expression can be written as

$$\frac{G}{G_{\max}} = \frac{1}{1+\alpha|\gamma|^{\beta}}, \tag{3}$$

where G_{max}, α and β are material constants. G_{max} is the initial shear modulus at γ=0, and α and β define the nonlinearity of a backbone stress-strain curve. The material damping ratio λ could be obtained from Eq. (3) using the Masing criteria.

Table 1 Physical and dynamic properties of tested soils

Soils	Dr or PI	σ_c'(kPa)	G_{max}(kPa)	α	β
SANDS					
Toyoura	60.4	98	85000	1042	1.03
Kyobashi	31.8	98	81300	244	0.73
Sanrihama	18.5	98	62300	578	0.93
	~35.2		~82200	~1727	~1.09
CLAYS					
Flocculated	29	30~400	32000	700	0.85
Kaolinite			~126000	~2700	~1.05
Dispersed	6	50~100	45000	500	0.74
Kaolinite			~80000	~900	~0.80
Pisa Clay	10~55	50~135	29000	170	0.70
			~55000	~350	~0.89
Bay Mud	40	30~50	13100	30	0.53
			~13700	~50	~0.65

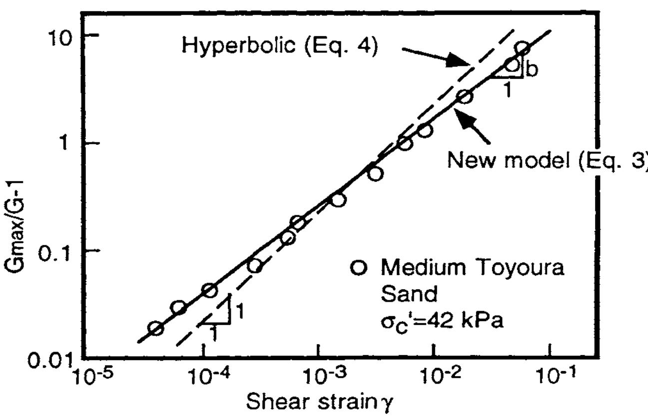

Fig.2 Model performance of Eq. (3)

Material parameters α and β can be obtained graphically. When the measured data is plotted in logγ vs. log((G_{max}/G-1), a straight line can be approximated as shown in Fig. 2. The α parameter is the value of G_{max}/G-1 at γ=1, and parameter β is the slope of the regression line. The data plotted in Fig. 2 is of medium Toyoura sand obtained by a combination of resonant column and cyclic torsional shear tests (Nakagawa (1986)).

TEST RESULTS

A typical example of the hysteresis loops measured by the new testing device is shown in Fig. 3. The loops were almost symmetric even at very small strain level of 10^{-3}%. At a smaller strain level of 10^{-4}%, some scatter in the loops was observed especially for the soft clay specimens, possibly as a result of the resolution of the torque meter.

An example of the curve fitting by Eq. (3) is shown in Fig. 4. Overall, Eq.(3) gave a reasonable approximation using only three material constants. The measured material parameters are listed in Table 1. More detailed description of the test results can be found in Soga (1994).

The material damping ratio λ of the tested soils increased with shear strain as shown in Fig. 4. Their values were plotted between the upper and lower bounds of the damping curves reported by Seed and Idriss (1970).

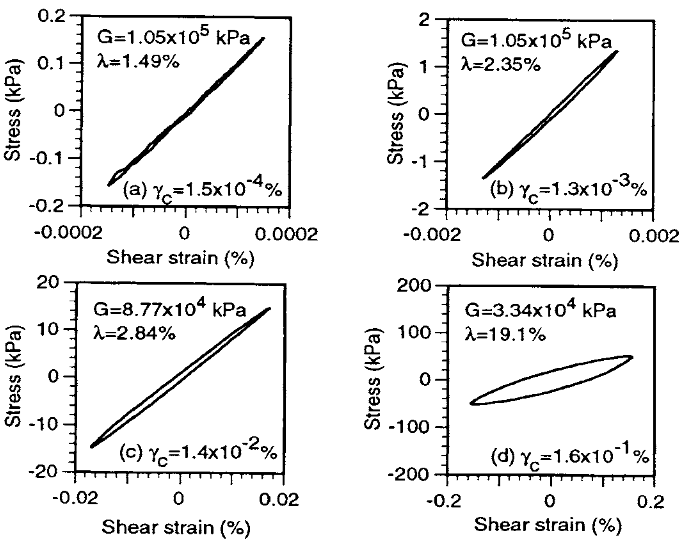

Fig. 3 Cyclic stress-strain curves of flocculated kaolinite
σc=400 kPa, Frequency=0.1Hz

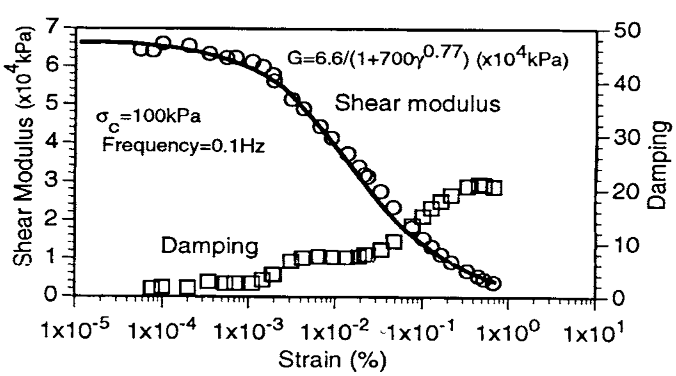

Fig. 4 Shear modulus degradation curve of dispersed kaolinite

DISCUSSION

Comparison to other nonlinear models

In the past, many constitutive models were developed to match the nonlinear stress-strain relations of soils (e.g., Hardin and Drnevich,1972;Tatsuoka and Shibuya,1991; Matasovic and Vucetic,1993). Some of the models used as many as five parameters to model nonlinearity. In general, as the number of model parameters increases, a better fit to the measured data can be obtained. However, too many parameters make it difficult to understand the effect of each parameter on the shape of the nonlinearity. Accordingly, the physical meaning of each parameter becomes obscure.

The most widely used nonlinear stress-strain models are the hyperbolic model and the Ramberg-Osgood model. Because of their relative simplicity, these models have been used widely to curve fit both static and cyclic nonlinear stress-strain relations of soils.

The hyperbolic model requires two material constants: G_{max} and γ_r. The model can be written as

$$\frac{G}{G_{\max}} = \frac{1}{1+\gamma/\gamma_r} \tag{4}$$

where $\gamma_r = \tau_{max}/G_{max}$, τ_{max} is the shear strength at $\gamma=\infty$, G_{max} is the shear modulus at $\gamma=0$, and G the secant shear modulus.

By requiring more parameters, the Ramberg-Osgood model may be able to curve fit the measured nonlinear stress-strain relations better than the hyperbolic model. The model can be written as follows (Tatsuoka (1979)):

$$\frac{G}{G_{\max}} = \frac{1}{1+a|\tau|^b} \tag{5}$$

where a, b, and G_{max} are material constants.

The model constants of the Ramberg-Osgood model can be determined by plotting the measured data in $\log(\tau/G_{max})$ versus $\log(G_{max}/G\text{-}1)$. As shown in Fig. 5, the Ramberg-Osgood model becomes a straight line in this plane.

Fig. 5 includes the measured nonlinear stress-strain relation of medium Toyoura sand, which was also presented in Fig. 2. The Ramberg-Osgood model matched poorly to the data at large stress/strain levels. The hyperbolic model demonstrated unsatisfactory curve fitting at small strains.

Using Equation (3) on other geologic materials

Compared to the hyperbolic model and the Ramberg-Osgood model, the model expressed by Eq. (3) matched the stiffness degradation of many geologic materials remarkably well as shown in Fig. 6. In addition to the data obtained from this study, the nonlinear stress-strain relations reported in the literature were used for examining the model performance.

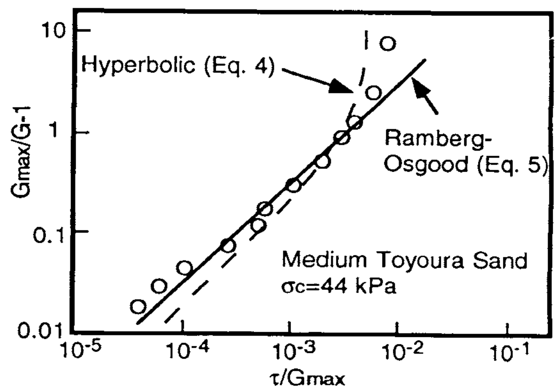

Fig. 5 Model performance of Eqs. (4) and (5)

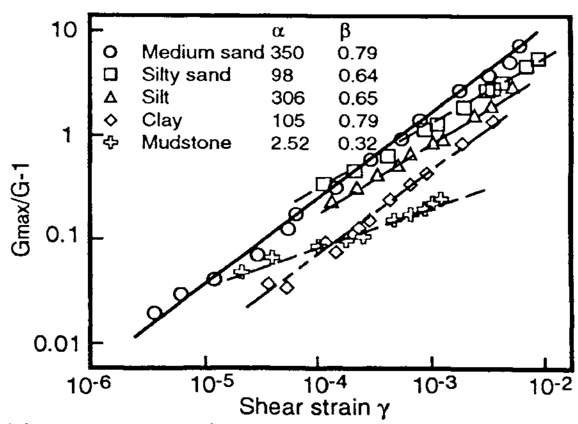

Fig. 6 Shear modulus degradation of various geologic materials

The effect of confining pressure on parameters α and β is shown in Fig. 7(data from Iwasaki, et al. (1978)). For sands, α increased with confining pressure. β was in the range of 0.84 and 0.91.

Material parameters α and β of clays decreased as the plasticity index *PI* increased (see Fig. 8 with data from Kokusho, et al. (1982)).

Relationship between α and β

The attractiveness of the model of Eq. (3) is that the effect of α and β on degradation curve is relatively easy to understand. The parameter α represents the location of the curve, whereas the parameter β represents the rate of the degradation.

The variations in the parameters α and β are considered to be material dependent. Loose sands tend to have larger α and β values. On the other hand, clays and mudstones have smaller α and β values. The parameters α and β are also expected to be sensitive to other factors such as disturbance, void ratio, confining pressure, and gradation.

The compiled sets of parameters α and β were plotted against each other in Fig. 9. It was found that the material parameters α and β were not independent of each other, but had a following relationship.

$$\beta = 0.2\log\alpha + 0.3 \qquad (6)$$

Thus, a two parameter model could be used for geologic materials using this relationship. A study of micromechanics may help us to explain this unique relationship for physical interpretation.

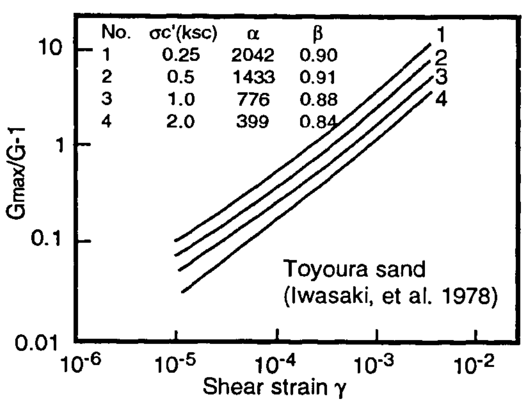

Fig. 7 Effect of confining pressure on α and β

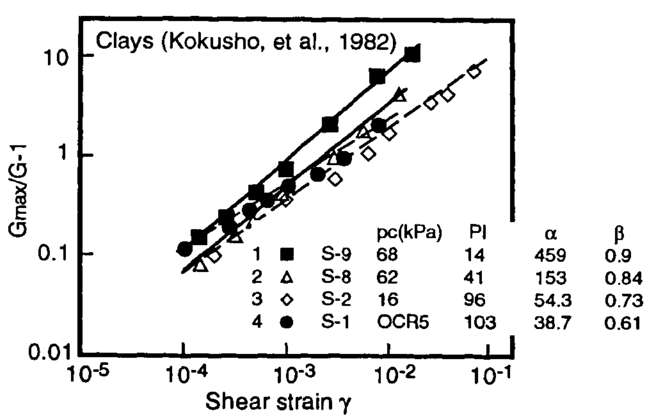

Fig. 8 Effect of plasticity on α and β

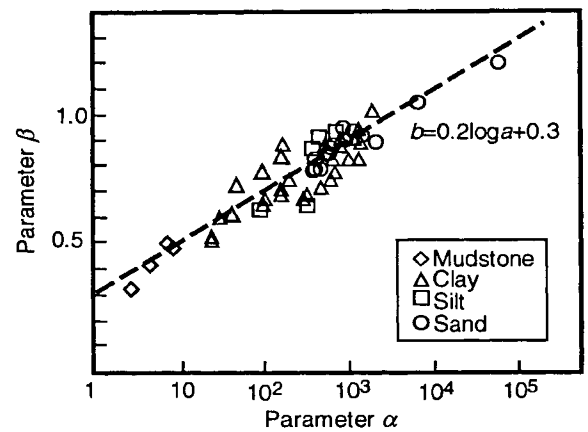

Fig. 9 Relationship between α and β

CONCLUSIONS

A simple nonlinear stress-strain model of Eq. (3) agreed well with the test results obtained by the new torsional shear device and with other published data of various geologic materials. The parameters α and β in Eq. (3) reflected the shape of the degradation curve. Loose sands had larger α and β values, whereas clays and mudstones had smaller α and β values. A unique relationship of β=0.2logα+0.3 was found from the compiled data of various soils.

If the material parameters, α and β, can be determined at small strains both in the laboratory and in the field, it is possible to predict the nonlinear stress-strain relations at large strains with the use of this new model. In turn, this wider prediction ability will allow more accurate site responses during earthquakes.

REFERENCES

Hardin, B.O. and Drnevich, V.P. (1972) : "Shear modulus and damping in soils : Measurement and parameter effects," Journal of SMFE Division, ASCE, Vol. 98, SM6, pp. 603-624

Iwasaki, T., Tatsuoka, F. and Takagi, Y. (1978) : "Shear moduli of sands under cyclic torsional shear loading, Soils and Foundations, Vol.18, pp.39-56

Kokusho, T., Yoshida, Y. and Esashi, Y. (1982) : "Dynamic properties of soft calys for wide strain range," Soils and Foundations, Vol. 20, pp.1-18

Matasovic, N. and Vucetic, M. (1993) : "Cyclic characterization of liquefiable sands," Journal of Geotechnical Engineering, Vol.119, No.11, pp.1805-1822

Nakagawa, K. (1986) : "A proposal to evaluate liquefaction resistance by developement of new in-situ testing technique," Chishitugakuronshu, Vol.27, pp.169-180 (in Japanese)

Seed, H.B. and Idriss, I.M. (1970) : "Soil moduli and damping factor for dynamic response analysis," EERC-70-10, Earthquake Engineering Research Center, UC Berkeley

Soga, K. (1994) : "Mechanical behavior and constitutive modelling of natural structured soils," Ph.D thesis, UC Berkeley

Sousa, J. (1993) : "Automated Testing System software", SHRP Equipment Corporation

Tatsuoka, F. (1979) : *Mechanics of Soils (III)*, Eds. Adachi, T. and Tatsuoka, F., Gihoudo, 338pp. (in Japanese)

Tatsuoka, F. and Shibuya, S. (1992) : "Deformation characteristics of soils and rocks from field and laboratory tests," Report of the Institute of Industrial Science, The University of Tokyo, Vol.37, No.1, 136pp.