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Application of Fuzzy Set Theory in Option of Response Spectra

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SYNOPSIS: In order to consider the fuzziness of seismic intensity indecated by zoning map and seismic code in seismic design of structure, an innovative method in option of design spectrum in consideration of fuzziness of regional seismic hazard and site condition is presented herein. The solution of the problem can be divided into two steps: 1. to define and establish a fuzzy set for predicting and judging regional seismic hazard and site condition by applying fuzzy mathematics; 2.further to find out fuzzy vector of design spectrum depending on the fuzzy set of regional seismic hazard and site condition by applying fuzzy comprehensive judgement theory.

FOREWORD

At present, the equivalent base shear method and the vibration mode superposition resolution method are widely used for the aseismic checking stipulated by the aseismic design codes in all countyies. As the methods mentioned above are all based on the response spectrum theory, it is, then, very important to calculate rationally the design response spectrum of specified construction site. Practically, there are about two ways to calculate the design response spectrum: one is to select the appropriate design response spectrum according to the requirement of the codes and other is to calculate the design response spectrum in the light of seismic hazard in combination with seismic response analysis of ground strata. In China, the latter is mainly used for urban seismic microzonation and major projects, while the former for large quantity of ordinary projects to be widely used. However, it should be recognized that the design response spectrum set up according to the aseismic design is determined by seismic zoning and site conditions, which are both of randomness and fuzziness. The randomness may be solved by the use of probability analysis approach, however,the fuzziness,different from randomness in essense,can only be treated by using fuzzy mathematical method. In view of this, in the paper, based on some new stipulations for design response spectrum in China's aseismic design code of building a method for calculating design response spectrum in consideration of the effect of fuzzy factors is presented and the concrete steps of the proposed method are described with algorithms. As the method suggested is rather simple, it can be used in aseismic design for ordinary and major projects .

STIPULATIONS OF CHINA'S NEW ASEISMIC DESIGN CODE OF BUILDING FOR DESIGN RESPONSE SPECTRUM

In the light of China's specific conditions, in Referense[1],based on the China's conventional method for calculating the design response spectrum considering intensity of earthquake and site condition, a new method is presented for comprehensively selecting design response spectrum in consideration of the quake intensity and magnitude,epicentral distance and site conditions. The major improvement of the method lies in that not noly the effect of site condition on the characteristic of quake dynamic spectrum but also the effect of quake magnitude and epicentral distance on the shape of the spectrum are considered. The design response spectrum stipulated by the China's new aseismic design code of building on the basis of the proposal in Reference [1] has the from as shown in Fig. 1.

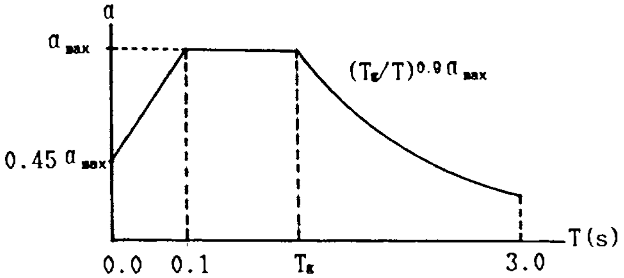


Fig.1 Design response spectra stipulated by China's new Aseismic design code of building

α_{max} shown in Fig. 1 is the peak value of response spectrum which is relevant with the expected quake magnitude and may be determined according to Table 1 for various earthquake defence level (or the ground motion severity of various return periods).

The characteristic period T_g is relevant to site classification and epicentral distance and is determined according to Table 2.

The appraisal standards of site conditions are given in Table 3, where H represents the thickness of overburden covering on the bedrock and Vs is the average shear wave velocity within 15m under the ground surface.

Tab.1 Maximum spectral coefficient α_{max} (g)

Earthquake defence level \ Intensity at site	6	7	8	9
I (minor earthq.)	0.040	0.080	0.160	0.320
II (moderate earthq.)	0.113	0.225	0.450	0.900
III (major earthq.)	0.3	0.5	0.9	1.4

Tab.2 Characteristic periods T_g (s)

Near or Far shock	Site Condition			
	I	II	III	IV
Near shock	0.20	0.30	0.40	0.65
Far shock	0.25	0.40	0.55	0.85

Tab.3 Classification for site conditions

<table>
<tr><th>Shear Wave Speed of Subsoil
$\bar{V}s$ (m/s)</th><th colspan="5">Thickness of Overburden Layer H (m)</th></tr>
<tr><th></th><th>0</th><th>0<H<3</th><th>3<H<9</th><th>9<H<80</th><th>H>80</th></tr>
<tr><td>Vs>500</td><td>I</td><td></td><td></td><td></td><td></td></tr>
<tr><td>500≥$\bar{V}$s>250</td><td></td><td colspan="2">I</td><td colspan="2">II</td></tr>
<tr><td>250≥$\bar{V}$s>140</td><td></td><td>I</td><td colspan="2">II</td><td>III</td></tr>
<tr><td>$\bar{V}$s≤140</td><td></td><td>I</td><td>II</td><td>III</td><td>IV</td></tr>
</table>

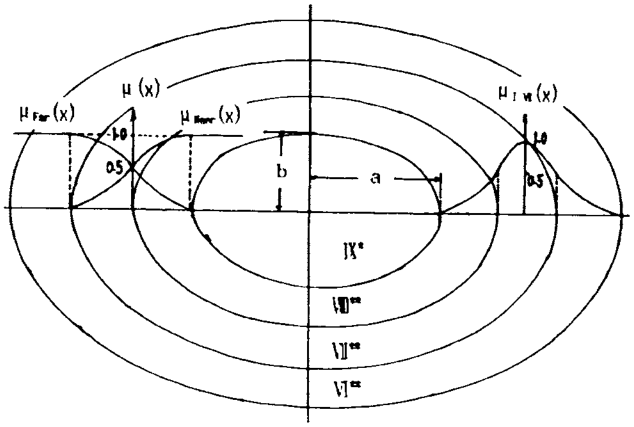
In Table 2, near source earthquake and distant source earthquake are the design earthquakes classified in the light of the magnitude and distance from focal area under the same felt intensity. What is called focal area refers to the focal area of higher intensity which can bring about to the site the quake effect coming up to the designed intensity. When the earthquake intensity in the focal area is two degrees or more greater than that at the site considered as the distant shock, otherwise, the near shock.

THE FUZZY PREDICATION OF EARTHQUAKE INTENSITY

Though the randomness of the occurance of earthquake is to a varying degree considered in the existing China's earthquake intensity zoning map, yet not the fuzziness of the intensity concept implicated in zoning. Thus, the predicted intensity based on the fuzzy idea is certain to be infiltrated with fuzziness. Beside, it would be rational that the everchanging intensity in the intensity zoning map is separated up artificially by distinct margins. In order to study the fuzziness of intensity zoning, we have, based on the data of intensity attenuation in the report of China's earthquake intensity zoning map, set up, given the epicentral intensity condition the membership function in east and west China, which features that the intensity will attenuate with the distance:

$$\mu(x)=e^{-\left(\frac{x-m}{c}\right)^2} \qquad (1)$$

where the parameters are shown in Table 4. The change of membership function of intensity to distance is displayed in Fig. 2.



* -- Epicentral Intensity
**-- Felt Intensity

Fig.2 Membership Functions of Fuzzy Intensity and Far/Near Distance Shock

In table 4, a,b and r represent respectively the distances of given intensity along the directions of earthquake structure and vertical earthquake structure (e.g. major and minor axes of elliptical isoseisms) and the equivalent epicentral distance. Given the epicentral intensity, the fuzzy predictions of earthquake intensity is virtually to find the membership degree of given distance to the grade of earthquake intensity, and hence may be expressed by the following fuzzy vector:

$$\underset{\sim}{I}=\sum_{i=1}^{n} b_i/I_i \qquad (2)$$

where b_i is the membership degree of given distance to the intensity and may be calculated by the parameters in Table 1 by use of Equation (1).

When the construction site is not located on

Tab.4 Coefficients m and c in membership function in East and West China

Io	Region	Parameter	Felt Intensity															
			≥10			9			8			7			6		
			a	b	r	a	b	r	a	b	r	a	b	r	a	b	r
≥10	East	m	6	4	5	18.5	13.5	16.0	40	32.5	36	87.5	78	82.5	192	187	189.5
		c	7.2	4.8	6	7.8	6.6	7.2	18	16.2	16.8	39	38.4	39	86.4	92.4	89.4
	West	m	18	3	7.5	41.5	8	18.5	53.5	13.5	27	69	23	40	89	39.5	59.5
		c	21.6	3.6	9	6.6	2.4	4.2	7.8	4.2	6	10.8	7.2	9.6	13.2	12.6	13.4
	East	e	0.746			0.649			0.548			0.399			0.000		
	West	e	0.986			0.977			0.959			0.928			0.866		
9	East	m				6	3.5	4.5	19	11	14.5	40.5	24	31.5	86.5	53	68
		c				7.2	4.2	5.4	8.4	4.8	6.6	17.4	10.8	13.8	37.8	24	30
	West	m				6	3	4	18.5	9.5	13	37	20.5	27.5	73.5	45.5	57.5
		c				7.2	3.6	4.8	7.8	4.2	6	14.4	9	11.4	29.4	21	24.6
	East	e				0.812			0.817			0.800			0.786		
	West	e				0.866			0.854			0.821			0.766		
8	East	m							3	2	2.5	11.5	7.5	9.5	31.5	20	24.5
		c							3.6	2.4	3	6.6	4.2	5.4	17.4	10.8	12.6
	West	m							5	2	3	16	8	11	36	22.5	28.5
		c							6	2.4	3.6	7.2	4.8	6.6	16.8	12.6	15
	East	e							0.745			0.762			0.776		
	West	e							0.917			0.838			0.751		
7	East	m										4	2.5	3	13	11	11.5
		c										4.8	3	3.6	6.6	7.2	6.6
	West	m										5	2.5	3.5	17	9.5	12.5
		c										6.6	3	4.2	8.4	5.4	6.6
	East	e										0.781			0.329		
	West	e										0.866			0.812		
6	East	m													3.5	2	2.5
		c													4.2	2.4	3
	West	m													5	2.5	3.5
		c													6.6	3	4.2
	East	e													0.821		
	West	e													0.866		

Note: Io means epicentral intensity.

the direction of major or minor axis of isoseismic lines and the effect of tectonics is necessary to be taken into consideration, the radii of major and minor axes should be calculated according to the eccentricity e and the site coodinates (x,y) by using the following equations:

$$b = \sqrt{x^2 (1-e)^2 + y^2}$$

$$a = \sqrt{\frac{x^2 (1-e)^2 + y^2}{1-e^2}} \qquad (3)$$

and the membership degree may be calculated by substituting them into the membership functions of major and minor axes of corresponding conditions, selecting the larger one as the fuzzy prediction results of the site intensity.

THE FUZZY APPRAISAL OF NEAR AND DISTANT SHOCKS

Studies show that apart from the site condition, the earthquake magnitude and the distance also have a bearing on acceleration response spectrum. The field investigation of earthquake disaster indicates the earthquake ground motion characteristics and destruction property at two sites with same ground motion severity, in which one site is located in epicentral area of a low mediate to mediate earthquake and the other is far away from epicentral area of a major earthquake. Generally speaking, the frequency spectrum of the former will consist of more short frequency components, while that of the latter less. As to the former, the corresponding earthquake destruction will be more serious to the lowstorey buildings with high stiffness, while for the latter, it will be more serious to the long period high-flexible structures. That the different frequency contents of near and distant shock on the design response spectrum are considered in new aseismic design code of building virtually reflects the different feature of the damadge consequence in near and far source area. However, from the stipulations of codes it is not difficult to find ou that, despite the clear definition of near shock and distant shock, there is fuzziness in concept of intensity and seismic zoning, hence there is also fuzziness in distinguishing between near and distant shocks For this reason, we have set up membership function s of near and distance shocks for East China and West China by using the fuzzy prediction of earthquake intensity, and suppose that the membership function of near and distant shock is also regarded as semi-normal distribution as shown in Equation (1) (Fig. 2), and the coefficients concerned are meaning of the symbols is the same as above-stated. The fuzzy vactor of near and distant shock can thus be calculated:

$$\underset{\sim}{L} = \sum L_i/R_i \qquad (4)$$

where L_i is the membership degree of near shock or distant shock R_i, which can be obtained by using the parameters in Table 5 and Equation (1).

FUZZY COMPREHENSIVE APPRAISAL OF SITE CLASSIFICATION

According to the stipulations for site classification in aseismic design code of building, the thickness of overburden layer H and the average shear wave velocity Vs

Tab.5 Membership degree coefficient of near or distant shock

Region I		Parameters	a		b		r		Eccentricity	
			m	c	m	c	m	c	Near Shock e	Far Shock e
⩾11	E	N / F	37/109	32.4/54	14/57	16.8/34.8	23/79	22.8/44.4	0.899	0.852
	W	N / F								
10	E	N / F	12/55	15.6/36	8/46	13.2/32.4	10/50	14.4/33.6	0.649	0.548
	W	N / F	36/60	13.2/15.6	6/17	4.8/8.4	15/32	8.4/12	0.977	0.959
9	E	N / F	12/55	16.8/34.8	7/33	9.6/21.6	9/43	13.2/27.6	0.817	0.800
	W	N / F	12/49	15.6/28.8	6/28	8.4/18	8/37	12/22.8	0.854	0.821
8	E	N / F	6/46	13.2/34.8	4/29	8.4/21.6	5/35	10.8/25.2	0.762	0.776
	W	N / F	10/50	14.4/33.6	4/33	9.6/25.2	6/41	12/30	0.838	0.751
7	E	N / F	8/47	12/34.8	5/31	14.4/16.8	6/38	13.2/25.2	0.329	0.329
	W	N / F	10/54	16.8/36	5/29	10.8/18	7/39	13.2/25.2	0.812	0.812

Note: E--East, W--West, N--Near Shock, F--Far Shock, I--Intensity.

hould be selected as the appraising factors. or the convenience of description and alculation, according to the classification rincipal of the standards (Table 3), the ite classification category is first defined s 10 cases: I(1), I(2), I(3), I(4), II(1), II(2), II(3), III(1), III(2), IV(1), the orrespondings to Table 3 are shown in Table . The membership function of appraising actors subordinate also to the normal dis- ribution in Equation (1), and the corres- onding coefficients are given in Table 7. iven the membership function of appraising actors, the fuzzy vectors can be obtained:

$$\underset{\sim}{S} = \underset{\sim}{A}.\underset{\sim}{RR} \qquad (5)$$

where $\underset{\sim}{RR}$ is the fuzzy relation between the ten cases and appraising factors as shown in Table 6,and can be determined by Equation (1) and the parameters in Table 7;$\underset{\sim}{A}$ is the fuzzy set of appraising factors which represents the affect of the objects appraised. When operating Equation (5), we prefer to use the "weighted aggregative method", thus,we have:

$$\underset{\sim}{A} = 0.6/H + 0.4/Vs \qquad (6)$$

Tab.6 Code names of universe for appraisal category of site condition

Shear Wave Speed of Subsoil $\bar{V}s$ (m/s)	Thickness of Overburden Layer H(m)				
	0	0<H<3	3<H<9	9<H<80	H>80
Vs>500	I(1)				
500≥$\bar{V}$s>250		I(2)		II(1)	
250≥$\bar{V}$s>140		I(3)	II(2)		III(1)
$\bar{V}$s≤140		I(4)	II(3)	III(2)	IV(1)

Tab.7 Parameters of subordinate degree of site classification

Parameter / Type	Thickness of Overburden Layer H		Shear Velocity $\bar{V}s$	
	m	c	m	c
I(1)			550	60
I(2)	4.5	5.4	375	150
I(3)	1.5	1.8	195	66
I(4)	1.5	1.8	100	48
II(1)	100	109.2	375	150
II(2)	41.5	46.2	195	66
II(3)	6.0	3.6	100	48
III(1)	100	24.0	195	66
III(2)	44.5	42.6	100	48
IV(1)	100	24.0	100	48

When we have the fuzzy vectors of the ten ceses in site classification category S, we can, according to the extension principle, convert the fuzzy vectors of the ten cases into fuzzy sets of 4 kinds of sites stipulated in design code by using the following equation:

$$\underset{\sim}{SS} = \frac{(S1,S2,S3,S4)_{max}}{SS1} + \frac{(S5,S6,S7)_{max}}{SS2} + \frac{(S8,S9)_{max}}{SS3} + \frac{(S10)}{SS4} \qquad (7)$$

this is the appraisal result of site classification when considerring the fuzziness. When other information sources (including other appraising standards and expertise) are used for site appraising,see Reference 2.

SELECTION OF DESIGN RESPONSE SPECTRUM

We know from the above discussion that the design response spectrum of China's aseismic design standars is determined by intensity, near or distant shock and site class. If the above three factors marking the design response spectrum are all be treated with the fuzzy set theory, they should be design response spectrum fuzzy comprehensive appraisal factor category and the fuzzy set on it may be written as:

$$\underset{\sim}{E} = \frac{E1}{I} + \frac{E2}{S} + \frac{E3}{R} = (0.4,0.35,0.25) \qquad (8)$$

where E1,E2 and E3 are weighting coefficients respectively for intensity I, site class S and near or distant shock R.

For the appraised category, i.e. the design response spectrum category, according to the specified defense level,the intensity should be considered ranging from 6 to 9, site be classified into 4 classes and near or distant shock will have two cases (the distant shock may be neglected when intensity is in the area of intensity 9, thus, there are likely 28 combinations altogather, i.e. α_{611} ...... α_{941}), where α_{ijk} represents the code name of the design response spectrum according to different intensities, site conditions and near or distant shocks,where i (6 ... 9) represents intensity; j (1 ... 4), site conditions; k (1,2), near or distant shock. The classified combination of design response is in Fig. 3. In general, the fuzzy subset of design response spectrum appraisal category V may be written as:

$$\underset{\sim}{F} = \frac{F_{611}}{\alpha_{611}} + \frac{F_{621}}{\alpha_{621}} + \ldots + \frac{F_{ijk}}{\alpha_{ijk}} + \ldots + \frac{F_{941}}{\alpha_{941}} \qquad (9)$$

where F_{ijk} is the appraised membership degree of design response spectrum.

Obviously, there exists a fuzzy relationship between the appraised category V and the appraising factors category U.For the design response spectrum,the fuzzy relationship can be repressented by the following matrix:

$$\underset{\sim}{RR} = \begin{array}{cc} & \begin{array}{ccc} \alpha_{611} & \cdots\cdots & \alpha_{941} \end{array} \\ \begin{array}{c} I \\ S \\ R \end{array} & \begin{bmatrix} \gamma_{11} & \cdots\cdots & \gamma_{129} \\ \gamma_{21} & \cdots\cdots & \gamma_{229} \\ \gamma_{31} & \cdots\cdots & \gamma_{339} \end{bmatrix} \end{array} \qquad (10)$$

where the elements should be composed of fuzzy vectors or membership functions of the above-said appraising factors.

Given the fuzzy set $\underset{\sim}{E}$ and the appraising matrix $\underset{\sim}{RR}$,the design response spectrum fuzzy set can be calculated by the following equation:

$$\underset{\sim}{F} = \underset{\sim}{E} \cdot \underset{\sim}{RR} \qquad (11)$$

considering the effects of all factors,it is suggested in the paper that the fuzzy vectors of the design response spectrum is calculated by using the "weighted mean" method.

The following is an example showing the option method and calculation steps for selecting design response spectrum considering fuzziness.

Suppose a site is situated in the western part of China with an epicentral intensity of 9,felt intensity of 7,the x,y coordinates of the site to the epicenter of 30 and 155 km, the thickness of earth covering bedrock of 27m and an average shear wave velocity Vs = 220.5m/s, find the fuzzy vector of design response spectrum.

1. The fuzzy vector of intensity is calculated by using Equation (2):

$$\underset{\sim}{I} = \frac{0.00025}{8} + \frac{0.96}{7} + \frac{0.36}{6}$$

2. The fuzzy vector of site classification by Equation (7):

$$\underset{\sim}{S} = \frac{0.34}{S1} + \frac{0.88}{S2} + \frac{0.5}{S3} + \frac{0.0}{S4}$$

3. The fuzzy vector of near or distant shock by Equation (4):

$$\underset{\sim}{L} = \frac{0.032}{R_{near}} + \frac{0.919}{R_{dist.}}$$

4. The design response spectrum vector of algorithmic design response spectrum category by Equations (8), (9) and (10):

$$\begin{aligned} \underset{\sim}{F} = {} & \frac{0.271}{\alpha_{611}} + \frac{0.460}{\alpha_{621}} + \frac{0.328}{\alpha_{631}} + \frac{0.493}{\alpha_{612}} + \frac{0.682}{\alpha_{622}} + \\ & \frac{0.549}{\alpha_{632}} + \frac{0.511}{\alpha_{711}} + \frac{0.700}{\alpha_{721}} + \frac{0.567}{\alpha_{731}} + \frac{0.741}{\alpha_{712}} + \\ & \frac{0.922}{\alpha_{722}} + \frac{0.789}{\alpha_{732}} + \frac{0.127}{\alpha_{811}} + \frac{0.316}{\alpha_{821}} + \frac{0.183}{\alpha_{831}} + \\ & \frac{0.349}{\alpha_{812}} + \frac{0.538}{\alpha_{822}} + \frac{0.363}{\alpha_{832}} \end{aligned}$$

It can be seen from the appraisal results that, owing to the dispersion of the values of appraising factors, the membership degree of design response spectrum is rather decentralized and the response spectrum with membership level above 0.5 are not few. For this case,according to the principle of maxinum membership degree, the design response spectrum of serial number α_{722} may be selected.

METHOD FOR DETERMINING DESIGN RESPONSE SPECTRUM BASED ON MEMBERSHIP DEGREE

The results given in the previous section is the fuzzy subset of the design response spectrum, which not only indicates the design response spectra with certain degree of membership, but also show the membership degree to certain kinds of design response spectra, thus indicating objectively the uncertainty of design response spectrum. However,at present,the aseismic design is still based on one kind of crisp and definite design response spectrum, hence, how to determine the design response spectrum according to the above-said results is the key to use rationally the appraised results. It is sure the simplest way to select the design response spectrum with the highest membership degree in appraised fuzzy vector F as the design response sepectrum, however, it should be seen that although the relation of fuzzy subordination relation is to a certain degree taken into account by the design response sepectrum determined by the principle of maximum membership degree, yet the whole appraisal results are eventually not uesd. The more rational method is to use all the appraisal results comprehensively to determine the final design response spectrum. In consideration of this, it is proposed in the paper that to determine the final design response spectrum by using the membership degree square weighted mean method:

$$\alpha\ (T) = \sum_{i=1}^{n} F_i^2\, \alpha_i(T) \ / \ \sum_{i=1}^{n} F_i^2 \qquad (12)$$

where F_i is the membership degree of corresponding design response spectrum $\alpha_i(T)$,T is vibration period.By Equation (11),the design response spectrum which is determined by substituting the data of the algorithm in previous section into the equation is shown in Fig. 4. Various response spectra with the membership degree above 0.5 are also given in Fig. 4. It is also seen from the figure that the final design response spectrum differs from the spectrum determined by the principle of maximum membership degree and also from other some response spectra with next larger membership degree. Generally speaking, the final design response spectrum is close to some spectra with larger membership degree.

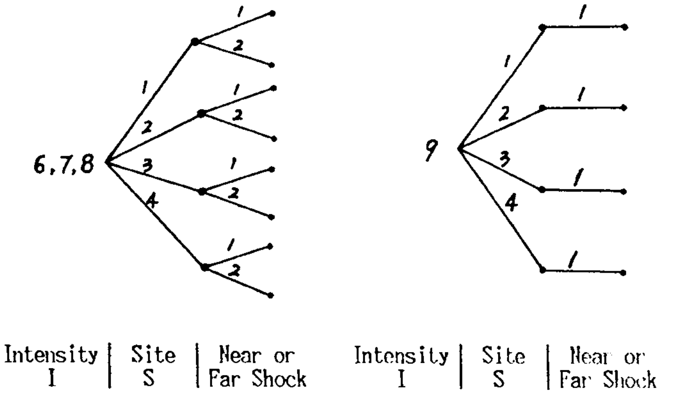


Fig. 3 A Schematic Chart of Seperating Universe

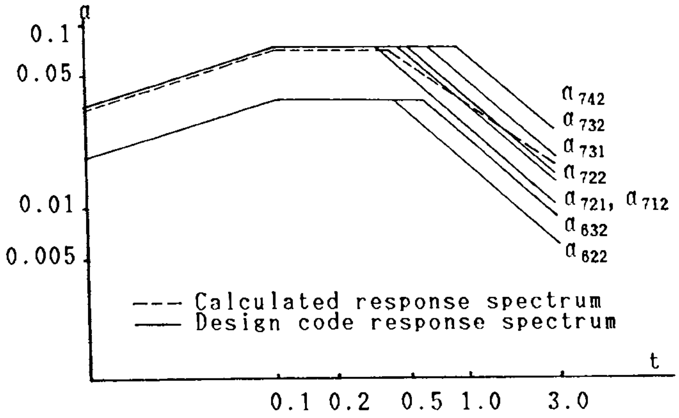


Fig. 4 Response spectrum based on membership degree

CONCLUSION

As the design response spectrum is subjected to the effects of fuzzy factors, such as earthquake intensity, near or distant shock, and site condition, a method f or selecting design response spectrum by using fuzzy comprehensive appraisal is proposed in the paper

This method can treat with the uncertainty of the response spectrum classification in a more rational way and provide a possible way for comprehensively using the expertise and other information sources. The method recommended is not only suitable for the specific construction sites,but also of certain practical meaning to urban microzonation.Though the method proposed in the paper takes as the objective the China's aseismic design code of building an theoretic basis and basic means are also suitable for other conditions.

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