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SYNOPSIS An improved analytical method to calculate the likelihood of soil liquefaction is de-
scribed. The method is based on an expanded list of case histories of liquefaction and no-lique-
faction and employs earthquake magnitude and hypocentral distance to describe the intensity of shak-
ing at a site. The new list of case histories is compiled from a complete re-evaluation of previ-
ously published case histories and field data observed in more recent earthquakes. Specific appli-
cations of the proposed procedure are described and an example analysis is presented.

INTRODUCTION

Throughout the past decade, liquefaction due to
earthquake shaking has been the subject of nu-
merous studies. Considerable gains toward an
understanding of liquefaction have evolved from
extensive laboratory studies. In addition,
various laboratory testing procedures have been
developed to supply soil parameters needed for
analytical techniques that predict liquefaction
potential at a site. Data from field observa-
tions of liquefaction have been utilized to
study the phenomenon as it occurs in the field
and to develop empirical procedures for prelim-
inary investigations of liquefaction.

Yegian and Whitman (1978) proposed an empirical
procedure for liquefaction which is based on
interpretation of field data using earthquake
parameters such as magnitude, M, and hypo-
central distance, R, to describe the earthquake
intensity. The paper discusses the merits of
using M and R as opposed to acceleration, which
has more commonly been used by other investi-
gators to describe earthquake intensity. The
list of case histories used by Yegian and Whit-
man to develop the liquefaction criteria they
proposed was that which was published earlier
by Seed and Idriss (1971), with some additional
data points. In the past few years the need
for re-evaluation and expansion of the currently
used list to include data obtained from more
recent earthquakes has been apparent. The au-
thors of this paper completed such an overall
review of all the case histories discussed and
presented by the original investigators. Due
to space limitations this updated list, which
includes a total of about 322 data points corre-
sponding to 80 locations and 19 different earth-
quakes, is not presented in this paper, but will
be published separately.

A simple analytical technique for liquefaction
based on field data compiled in the new list
of case histories is presented in this paper.
A new criterion for liquefaction is proposed
which employs earthquake magnitude and distance

to describe the seismic event, and standard
penetration test results to represent the soil
condition. Deterministic and probabilistic
applications of the method are discussed and an
example analysis is included.

ANALYSIS FOR LIQUEFACTION

Yegian and Whitman (1978) proposed the use of
the parameter Liquefaction Potential Index,
LPI, for the evaluation of the liquefaction
potential at a site. LPI, which is inversely
related to factor of safety against liquefac-
tion, can be expressed as:

$$LPI = \frac{\text{stress parameter, } S_c}{\text{strength parameter } \bar{S}_c} \qquad (1)$$

If

$LPI > 1$; liquefaction is likely to occur

$LPI < 1$; liquefaction is not likely to occur

In the investigations reported herein, the
parameter LPI was employed and an expression
for it was developed as follows: the stress
parameter was assumed to have the form

$$S_c = e^{c_1 M}(R + 25)^{c_2} \cdot \frac{\sigma_v}{\bar{\sigma}_v} \qquad (2)$$

where M is the earthquake magnitude, R is the
hypocentral distance in KM, σ_v is the total
vertical stress, $\bar{\sigma}_v$ is the effective vertical
stress, and c_1 and c_2 are constants. The form

for the strength parameter was assumed to be:

$$\bar{S}_c = c_3 N_c^{c_4} \qquad (3)$$

where N_c is the standard penetration test value, (SPT), corrected for the overburden pressure as suggested by Seed (1976):

$$N_c = N (1 - 1.25\log \bar{\sigma}_v (tsf)) \qquad (4)$$

where N is the SPT recorded in the field. Combining Eqs. 1, 2 and 3, LPI can be expressed as:

$$LPI = \frac{e^{c_1 M}(R + 25)^{c_2}}{c_3 N_c^{c_4}} \cdot \frac{\sigma_v}{\bar{\sigma}_v} \qquad (5)$$

The values of the constants c_1, c_2, c_3 and c_4 were evaluated using simulation techniques and the 322 data points corresponding to liquefaction and no-liquefaction case histories. A nonlinear multi-regression analysis following an iterative approach was employed in which values of the constants c were assumed and the difference between $\ln S_c$ and $\ln \bar{S}_c$ was computed for each case history. If the difference for a case was positive and the case was a 'no' liquefaction or negative and the case was a 'yes' liquefaction, then this difference was squared and saved; otherwise it was discarded. The best estimate of the constants c were evaluated by minimizing the sum of these squared differences for the entire case history list. The minimum of the sum of the squares is a measure of the uncertainty in both the interpretation technique and the data. The result of these investigations shows that the best estimates of the values of the constants are

$$c_1 = 0.2$$

$$c_2 = -0.4$$

$$c_3 = 0.464$$

$$c_4 = 0.4$$

Hence, the equation for the mean value of LPI can be written as:

$$LPI = \frac{e^{0.2M}(R + 25)^{-0.4}}{0.464 \, N_c^{0.4}} \cdot \frac{\sigma_v}{\bar{\sigma}_v} \qquad (6)$$

The above equation for LPI can be used for a particular site to deterministically evaluate the liquefaction potential: when the computed LPI is greater than 1, liquefaction is 'expected' to occur. However, an analysis of liquefaction potential involves many uncertainties. Quantification and incorporation of these uncertainties in the analysis are essential for a realistic assessment of the likelihood of liquefaction. Thus, Eq. 6 yields the mean value of LPI using mean or 'expected' values of the parameters which define LPI. In, addition, it is necessary to compute the coefficient of variation of LPI in order to make predictions of the probability of liquefaction. Assuming that the earthquake parameters are 'given', the coefficient of variation of LPI, V_{LPI}, is given by:

$$V_{LPI}^2 = 0.035 + (0.16)\frac{Var.N}{N^2} + \left[1 + (\frac{\sigma_v}{\bar{\sigma}_v})^2\right]\frac{Var.\gamma}{\gamma^2}$$
$$+ (\frac{\gamma_w}{\bar{\sigma}_v})^2 Var.dw \qquad (7)$$

where Var.N, Var.γ and Var.dw are the variances of the field blow counts, total unit weight, and the depth of the water table respectively; and γ_w is the unit weight of water. The constant term in Eq. 7 is due to the uncertainty in the c parameters which define the equation for LPI (Eq. 6). The rest of the terms in Eq. 7 describe the uncertainties in the soil parameters used in the liquefaction analysis for any site. Typical values of V_{LPI} range from 0.2 to 0.50; the lower value corresponds to the uncertainty only in the liquefaction analysis procedure which is proposed herein (Eq. 6).

The use of V_{LPI}, together with the mean value of LPI computed from Eq. 6, can provide an estimate of the conditional probability of liquefaction, defined as

$$P\left[LIQ.\right] = P\left[LPI > 1 \big| M \text{ and } R\right] \qquad (8)$$

Evaluation of $P\left[LIQ.\right]$ requires an assumption regarding the probability density function. Yegian and Whitman (1978) suggested the lognormal distribution for LPI. The form of LPI as given in Eq. 6 is very similar to that of peak ground acceleration. Donovan (1973) has shown that measured ground acceleration is lognormally distributed. Thus, LPI is also assumed to be lognormally distributed. The conditional probability of liquefaction is then determined by computing first the standardized variable U,

$$U = \frac{m_{\ln LPI}}{\sigma_{\ln LPI}} \qquad (9)$$

where,

$$m_{\ln LPI} = \ln LPI - 0.5\sigma_{\ln LPI}^2$$

and

$$\sigma_{\ln LPI}^2 = \ln(V_{LPI}^2 + 1)$$

Using the computed U and the normal tables, the conditional probability of liquefaction can be determined.

The procedure proposed above differs from the previous model developed by Yegian and Whitman (1978) in that the liquefaction criterion used herein is based on the new, expanded data bank. Furthermore, the parameters and the constants defining LPI in this investigation were determined in a more refined and rigorous manner, thus increasing the reliability of the model. In comparison to the previous criterion for liquefaction, this procedure predicts larger LPIs, but yields smaller coefficients of variation due to the larger number of data points used.

APPLICATIONS

Liquefaction Analysis

As described above, Eq. 6 together with the coefficient of variation of LPI (Eq. 7), can be used to predict the conditional probability of liquefaction. Fig. 1 shows results of a probability calculation made assuming the typical range 0.2-0.50 for V_{LPI}. This plot can be used in engineering practice as a preliminary study of liquefaction for selected design seismic events. Note that the curve corresponding to V_{LPI} = 0.0 describes a deterministic analysis of liquefaction.
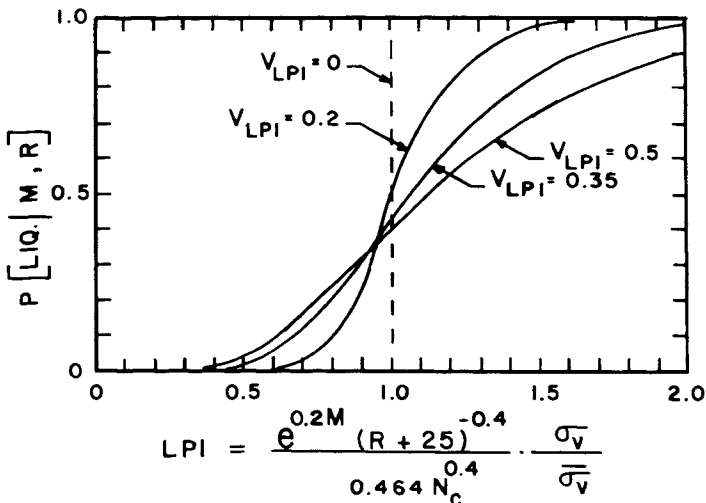

$$LPI = \frac{e^{0.2M}(R+25)^{-0.4}}{0.464 \, N_c^{0.4}} \cdot \frac{\sigma_v}{\overline{\sigma_v}}$$

Fig. 1 Conditional Probability of Liquefaction versus LPI

Pore Pressure Prediction

Procedures currently used for estimation of pore pressures in sands are based on the earthquake-induced shear stresses obtained from the application of the one-dimensional shear wave propagation theory, and on the laboratory pore pressure data on cyclically-loaded specimens of sands. Such procedures for pore pressure prediction involve many uncertainties, and are complicated and expensive to apply. Yegian (1980) proposed an empirically developed model for the prediction of pore pressures in loose, saturated sands. The model employs LPI to define a threshold event causing 100% pore pressure response with normalized laboratory cyclic behavior curves, in order to predict the excess pore pressure generated during events smaller than the event causing liquefaction. The pore pressure response, r_u, is defined as:

$$r_u = \frac{\Delta u}{\overline{\sigma}_v} \qquad (12)$$

in which Δu is the excess pore water pressure for level ground conditions. Thus, a pore pressure response of 100% (r_u = 1) indicates that the sand under study has liquefied. In engineering practice, common analysis of liquefaction involves the determination of whether or not the pore pressure response is greater than 1 for a given seismic event. Thus, a computed LPI less than 1 may indicate that the sand is not likely to liquefy, but does not indicate the level of excess pore pressure that might still be generated during that particular event. Such increases in excess pore pressure below levels causing liquefaction may be of such magnitude as to reduce the effective stresses in the soil to levels consequential to the dynamic response of the deposit, and to the settlement of a structure founded on the deposit.

The liquefaction analysis procedure described in this paper can be used to develop such a model for excess pore pressure prediction as a function of earthquake magnitude and distance. Yegian (1980) showed that the pore pressure response parameter, r_u, can be related to LPI as:

$$r_u = \frac{2}{\pi} \arcsin (LPI)^{\frac{1}{2\alpha\beta}} \; ; \; LPI < 1.0 \qquad (13)$$

where, α is a curve-fitting parameter used to relate laboratory pore pressure data for a particular soil to the ratio of the number of equivalent cycles of stress application to the number of cycles causing the parameter, r_u, to be equal to 1. The parameter β is the slope of the laboratory strength data when plotted on log-log paper. Definition of possible ranges for these parameters was attempted on the basis of published laboratory strength and pore pressure data. Values of β were estimated from laboratory strength curves suggested by various investigators for different types of tests and sands. Based on this review, values of β ranged between 0.10 and 0.25, with an average value of 0.19. A similar study of published data on excess pore pressure plotted against the normalized number of cycles yielded a range of values for α between 0.5 and 1.0. Seed and Booker (1977) recommended a typical value of 0.7.

Using the ranges for β and α given above, together with the equation for LPI (Eq. 6), plots of pore pressure response versus earthquake magnitude, distance and soil strength are generated as shown in Fig. 2. This plot can be used in preliminary studies to determine expected buildup of excess pore pressure in a particular sand deposit during a given seismic event. Fig. 2 demonstrates that while an LPI of 0.8 (factor of safety of 1.25) may imply safety against liquefaction, there may be a pore pressure response of up to 50%.

Thus the model described enables quick evaluation of pore pressures for preliminary studies and provides an opportunity to combine future field data and laboratory data on pore pressures in a simple, logical and consistent manner.
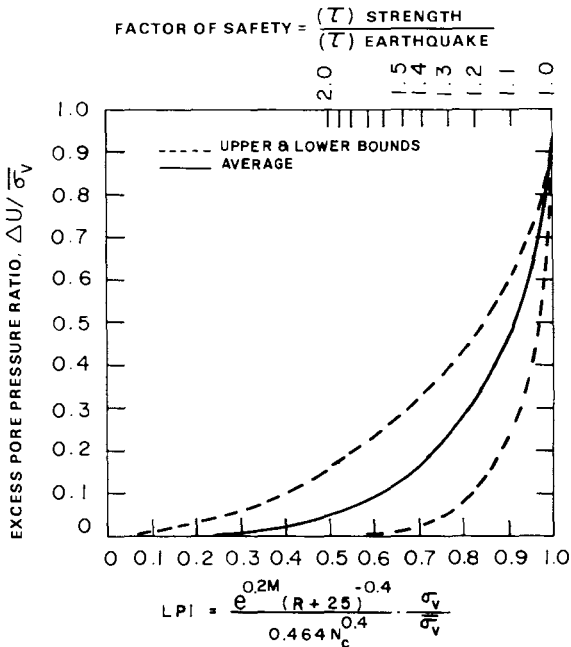
of the form:

$$A = k_1 e^{k_2 M}(R + 25)^{k_3} \qquad (14)$$

where, k_1, k_2 and k_3 are constants.

The output of the analysis is the annual probability of A exceeding a certain value 'a'. In a similar way liquefaction risk analysis can be performed since the equation for LPI has the same form as the attenuation law used in seismic hazard analysis. For a given sand deposit, LPI will be given by

$$LPI = k_1 e^{k_2 M}(R + 25)^{k_3} \qquad (15)$$

where

$$k_1 = \frac{1}{0.464 \, N_c^{0.4}} \cdot \frac{\sigma_v}{\overline{\sigma}_v}$$

$$k_2 = 0.2$$

and $k_3 = -0.4$

Using these parameters to define the attenuation law in the computer program one can compute the annual probability of LPI exceeding a certain value. If the value to be exceeded is assigned as 1.0 in the analysis, the output is the annual probability of liquefaction.

Liquefaction risk analysis can provide information enabling comparisons between the various factors contributing to the overall seismic risk to a constructed facility. The analysis can also be used to study the degrees of influence of the various parameters involved to identify the major factors contributing to likelihood of liquefaction occurring at a site.

EXAMPLE

FACTOR OF SAFETY = $\dfrac{(T) \text{ STRENGTH}}{(T) \text{ EARTHQUAKE}}$

Fig. 2 Excess Pore Pressure Ratio versus Liquefaction Potential Index, LPI.

Liquefaction Risk Analysis

Probabilistic seismic hazard analysis has received increased attention in the past decade. Computer programs are now available to compute annual probability of a certain seismic parameter exceeding a specified value. The input information to such an analysis includes the seismic source data and an attenuation law relating the seismic parameter of interest to earthquake magnitude and distance. The details of such an analysis are beyond the scope of this paper and are reviewed by Yegian (1979). These computer programs can also be used to evaluate an overall annual probability of liquefaction at a site. The attenuation law specified in seismic hazard analysis is usually

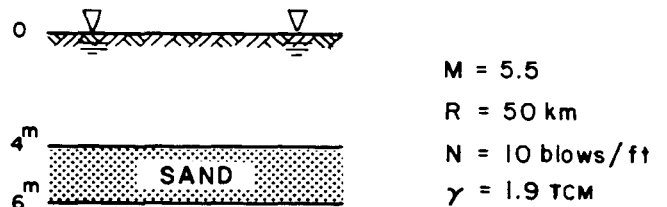
M = 5.5
R = 50 km
N = 10 blows/ft
γ = 1.9 TCM

Fig. 3 Example Site

Fig. 3 shows the pertinent data for the example site studied herein. The average SPT-value for the two meter sand layer shown in the figure is 10 blows/ft. The SPT-value corrected for the effect of the overburden stress, using Eq. 4 is about 14 blows/ft. The water table is at the ground surface and the unit weight of the overburden is 1.9 tcm. Considering a seismic event of magnitude 5.5 and distance 50 km, the average value of the LPI is computed from Eq. 6 to be 0.85. Assuming the following variance data for the site parameters:

$Var.N = 4$

$Var.\gamma = 0.04$

$Var.d_w = 1$

The coefficient of variation of LPI is computed from Eq. 7

$$V_{LPI}^2 = 0.035 + (0.16)\frac{4}{10^2} + (1 + (\frac{9.5}{4.5})^2)\frac{0.04}{1.9^2}$$

$$+ (\frac{1}{4.5})^2(1)$$

$$V_{LPI} = 0.39$$

Thus, employing Fig. 1 with LPI = 0.85 and $V_{LPI} = 0.32$, the probability of liquefaction of the sand layer given the postulated seismic event, is about 0.28. During this event, even though liquefaction is "not expected" to occur, the pore pressure response, r_u, for level ground condition is estimated to be 0.4 using average values of α and β parameters described earlier in this paper. The exact estimation of r_u for this site will require the evaluation of these parameters from laboratory tests on samples of the sand under study.

CONCLUSIONS

A simple analytical method to evaluate the likelihood of liquefaction at a site is described. An empirically developed criterion for liquefaction is presented which is based on a new, expanded list of case histories. Application of the method to evaluate liquefaction potential and to estimate earthquake-generated excess pore pressures are discussed. An example study is included.

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