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Response of Embedded Circular Flexible Foundations

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SYNOPSIS The stiffness matrix approach to the solution of vertical vibrations of a circular flexible foundation embedded in a viscoelastic layered half-space have been described. Results of a parametric study, represented by displacement and soil reaction distributions and impedances, indicate a significantly different responses of flexible and rigid foundations. Important parameters are identified and include: the stiffness ratio, depth of embedment, soil stratification and loading distribution. Influence of each of the parameters is discussed.

INTRODUCTION

Many authors have reported, e.g. Iguchi and Luco (1982), Riggs and Waas (1985), and Gaitanaros and Karabalis (1988), Gucunski and Peek (1993a), that the flexibility of a foundation may significantly influence its dynamic response. Effects of foundation flexibility are typically manifested by a decrease in the impedance of the foundation, and by stress and displacement distributions that differ significantly from those for a rigid foundation. Differences in response between rigid and flexible foundations become especially important in higher frequency ranges. Iguchi and Luco (1981 and 1982), Whittaker and Christiano (1982), and Gucunski and Peek (1993b) have also shown that both the distribution of the loading and the variation of the stiffness of the foundation affect its response.

The previously described studies deal with the response of flexible foundations on the surface of an elastic or viscoelastic half-space or a layered stratum, except for the study by Gaitanaros and Karabalis (1988) that deals with a foundation embedded in an elastic half-space. The purpose of this paper is to present effects of flexibility on the vertical response of a circular foundation for various embedment and soil layering conditions. The soil is modelled herein as a layered viscoelastic half-space by using the stiffness matrix approach [Kausel and Roesset (1981)]. The "ring method" approach [Lysmer (1965), Waas (1980)] was implemented in the analytical solution of the soil-foundation interaction problem. The second part of the paper presents results of a parametric study of flexible foundation embedded in a half-space and a layer over a half-space systems. The results include impedance functions as well as soil reaction and displacement distributions.

THEORETICAL BACKGROUND

The "ring method" approach assumes discretization of a circular foundation into a number of rings. The solution based on the evaluation of the foundation stiffness matrix and the matrix of influence coefficients for the layered system. The stiffness matrix of the plate is obtained by the finite difference energy method (FDEM). In this method the stiffness matrix of a ring is obtained from energy considerations, and the stiffness matrix of the foundation through an assembly process identical to the one for finite elements. The matrix of influence coefficients for the layered system is evaluated based on the stiffness matrix approach. In this approach the stiffness matrix of the layered system in the frequency-wave number domain is obtained from stiffness matrices of layers and of a half-space. The exact formulation of stiffness matrices by Kausel and Roesset (1981) was implemented. Once the stiffness matrix of the system is formed, influence coefficients representing the response of the system to unit harmonic loadings are evaluated. The advantage of the stiffness matrix approach with respect to some others is that by including the stiffness matrix of the half-space problems associated with energy radiation are completely eliminated.

The 'ring method' solution for vertical oscillations of a flexible circular foundation can be summarized by

$$[(\underline{S}_p-\omega^2\underline{M}_p)\underline{L}+\underline{I}]\underline{P}_s=\underline{P}_0 \tag{1}$$

where $\underline{S}_p$ and $\underline{M}_p$ represent the stiffness and the mass matrices of the foundation, respectively, $\underline{L}$ the matrix of influence coefficients for the soil system, and $\underline{I}$ the identity matrix. $\underline{P}_s$ and $\underline{P}_0$ are soil reaction and loading vectors, respectively. For an embedded foundation the assumption is that the bottom of foundation is on one of the layer interfaces. It is also assumed that the thickness of the foundation and a radius of a rigid core (Fig. 1) are small relative to the layer thickness. In that case the influence coefficient matrix $\underline{L}$ derived for an assumption of the continuity of the layers can be considered as sufficiently accurate.

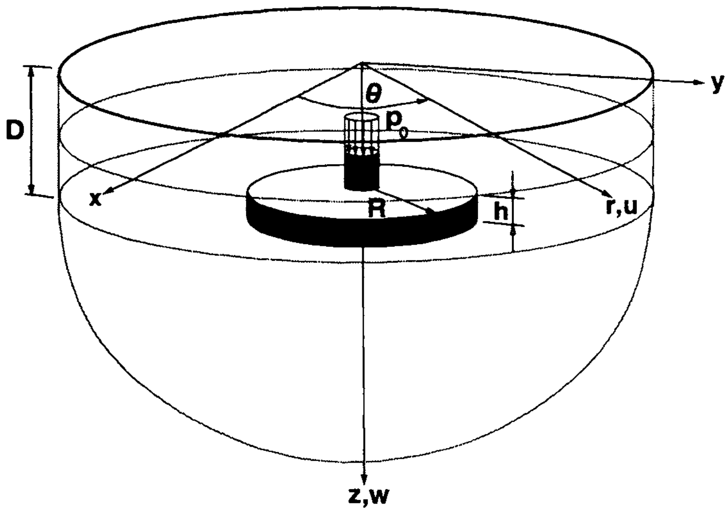

Fig. 1. A Schematic of an Embedded Foundation.

The influence coefficients in matrix $\underline{L}$ for interface l are given by

$$L_{ij}^{l}=\frac{1}{\Delta R_j}\int_{k=0}^{\infty}[J_1(k(R_j+\Delta R_j))(R_j+\Delta R_j)-J_1(kR_j)R_j]\,kJ_0(kr_i)\,w_l^1(k)\,dk \tag{2}$$

and represent a response of ring i due to unit harmonic loading on ring j. Vertical displacement $w_l^1(k)$ is the vertical displacement of interface l due to unit vertical loading on interface l in the frequency-wave number domain. It is obtained from the equilibrium equation for the in-plane motion of the soil system

$$\underline{S}(k)\underline{u}(k)=\underline{q}(k) \tag{3}$$

where $\underline{S}(k)$ is the stiffness matrix of the system, and $\underline{u}(k)$ and $\underline{q}$ (k) are vectors of interface displacements and loadings in the radial and vertical directions respectively. Coefficients L_{ij}^1 are obtained by numerical integration. Once the soil reaction distribution $\underline{P}_s$ is calculated the vertical displacement distribution can be obtained from

$$(\underline{S}_p-\omega^2\underline{M}_p)\underline{w}=\underline{P}_0-\underline{P}_s \tag{4}$$

Details of the derivation of the above equations were described by Gucunski and Peek (1993a).

PARAMETRIC STUDY

A parametric study of the vertical response of a flexible foundation embedded in viscoelastic half-space and layered half-space systems was done. A study of layered systems was limited to a single layer over a half-space to simplify description of effects of parameters of the study. The parameters used in the study include:

(1) the stiffness ratio $s_r=E_ph^3/(\rho_{HS}V_{sHS}R^3)$
(2) dimensionless frequency $a_0=\omega R/V_{sHS}$
(3) shear wave velocity ratio V_{s1}/V_{sHS}
(4) layer thickness to foundation radius ratio R/d_1

where E_p and h are the modulus of elasticity and the thickness of the plate respectively, and ρ_{HS} and V_{sHS} are the mass density and the shear wave velocity of the half-space respectively. V_{s1} and d_1 are the shear wave velocity and the thickness of the surface layer respectively, and ω is the circular frequency. The Poisson's ratio of the plate ν_p was kept constant at 0.25, and the plate was for all cases divided into twenty rings. The plate is massless, of uniform thickness, and has a rigid core of radius 0.18R. The Poisson's ratio of soil ν was kept constant at 0.35 and the damping ratio ξ at 0.02. The damping is included in the model in terms of the complex shear modulus

$$G^*=G(1+i2\xi) \tag{5}$$

Flexible Foundation Embedded in a Half-Space

The response of the foundation embedded in a half-space was examined for variation of the stiffness ratio s_r and the depth of embedment to radius ratio D/R. Influence of the stiffness ratio s_r and the dimensionless frequency a_0 on the displacement distribution for D/R=1 is illustrated in Fig. 2. For each frequency the displacements are normalized by the absolute value of the maximum displacement for that frequency. They are presented by their real (a) and imaginary (b) components as a function of the normalized radius r/R. The position of the rigid core can be clearly recognized in the displacement distributions for the most flexible foundations with s_r=0.01, and s_r=0.1 (not shown here). It should be noted that for higher frequencies essentially only the core moves while the remaining portion merely follows the developed elastic waves. Even though the stiffness ratio s_r=100 should represent a rigid foundation (Gucunski and Peek, 1993b) there are significant displacement variations even for low a_0. The above observations are in correlation with the soil reaction distributions presented in Fig. 3. The soil reactions are normalized by the intensity of applied loading. For s_r=0.01 there is soil reaction concentration in the vicinity of the edge of the rigid core, while the reaction is essentially zero outside the core. For the stiffest foundation (s_r=100) a soil reaction concentration is noticeable at the edge of the foundation.

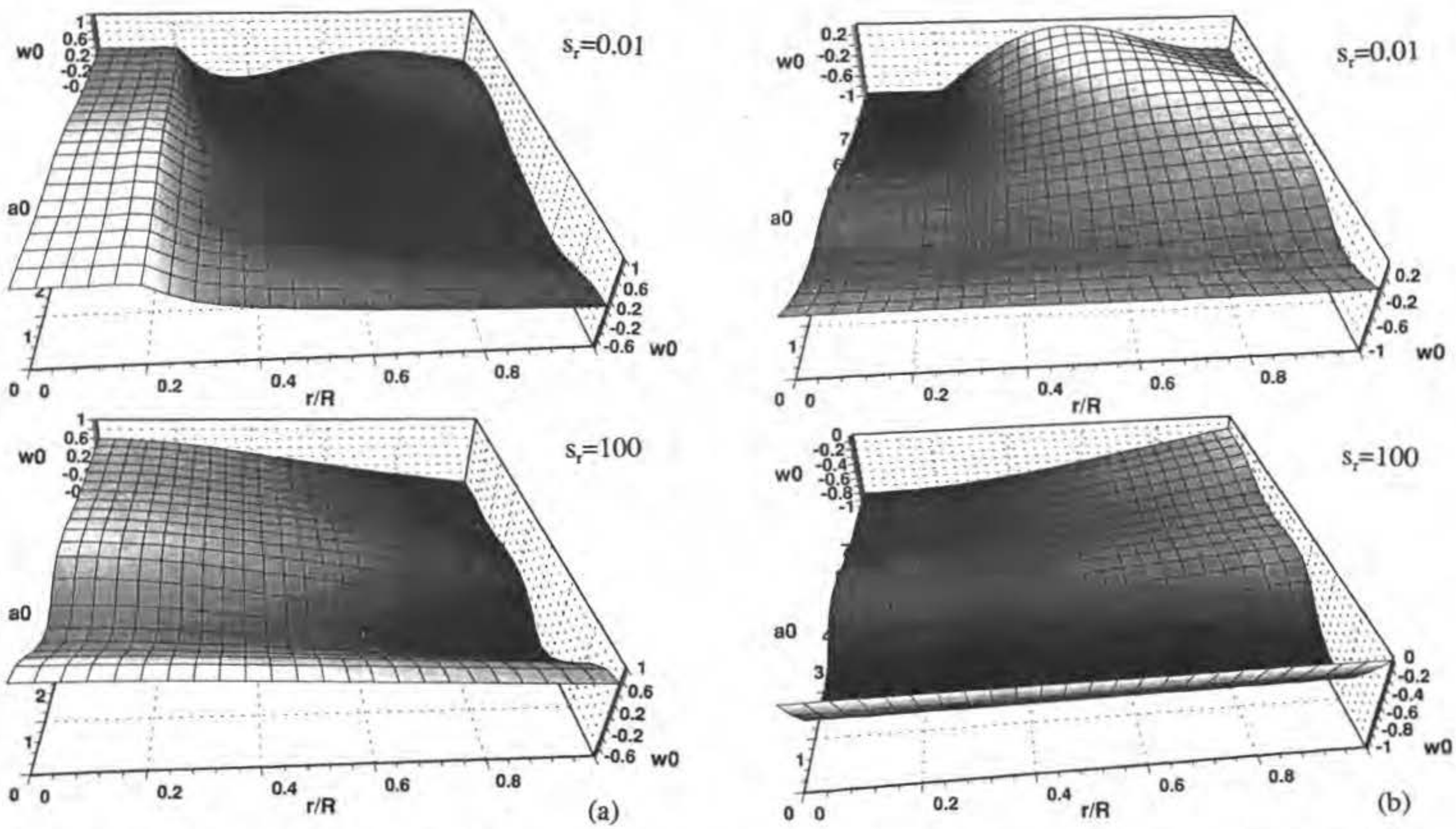

Fig. 2. Displacement Distribution for a Foundation Embedded in a Half-Space for D/R=1; (a) Real, (b) Imaginary Part.

Influence of the stiffness ratio on the impedance coefficients k and c for D/R=1 is shown in Fig. 4. The impedance function in this case is described as a ratio of the total loading to the displacement of the core. The following relationship between the impedance and impedance coefficients is used

$$K = K_s(k + ia_0c)(1 + i2\xi) \tag{6}$$

For lower stiffness ratio, small variations with frequency of both coefficients ratios can be observed. The other results not presented herein indicate the same for other D/R ratios. The impedance coefficients take larger values for larger stiffness ratios. Such behavior is quite expected considering the fact that a larger area of a foundation is participating in the load transfer for stiffer foundations.

Influence of the depth of embedment is presented in Fig. 5 for stiffness ratio s_r=0.01. After the initial increase, the k and c curves, for all D/R ratios, approach the corresponding curves for the half-space. Brief calculations indicate that these approaches occur at frequencies close to the first natural frequency of vertical oscillations of the soil layer above the foundation. An assumption applied here is that the foundation represents a rigid base. Similar behavior is observed for other stiffness ratios. Variations of displacement with frequency become more pronounced for higher D/R ratios. For higher D/R ratios it may be observed that the displacement distribution closely resembles the displacement distribution of a foundation with a lower stiffness ratio for a lower D/R ratio. On the other hand, the soil reaction distribution is very little affected by the depth of embedment. Finally, the static stiffness as a function of the stiffness ratio and the depth of embedment is shown in Fig. 6. The stiffness is normalized with respect to the static stiffness for D/R=0 and s_r=100. As expected, the stiffer the foundation, the higher the static stiffness. It can be observed that even a small depth of embedment causes a significant increase of the stiffness of a flexible foundation. On the other hand, an increase of the depth of embedment is more important for a stiff foundation.

Flexible Foundation Embedded in a Layer over a Half-Space

In this part of the study an embedded flexible foundation was placed at the bottom of a layer overlying a half-space. The response of the foundation was examined with respect to the stiffness ratio s_r, depth of embedment to radius ratio D/R, where D is equal to the thickness of the surface layer d_1, and the shear wave velocity ratio V_{s1}/V_{sHS}. The stiffness ratio influences the soil reaction and displacement distributions in a way similar to that for a half-space. The soil reaction distribution, for a constant stiffness ratio,

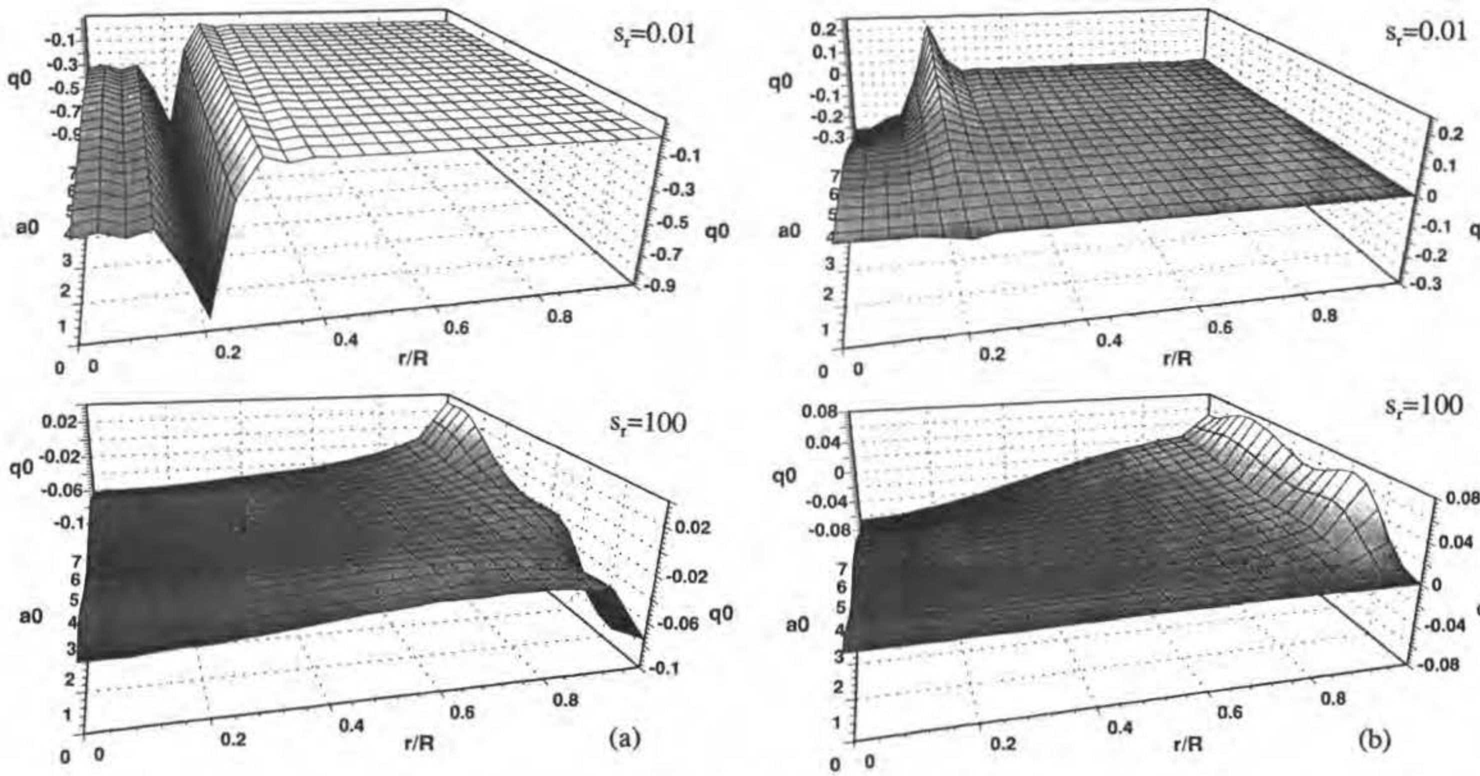

Fig. 3. Soil Reaction Distribution for a Foundation Embedded in a Half-Space for D/R=1; (a) Real, (b) Imaginary Part.

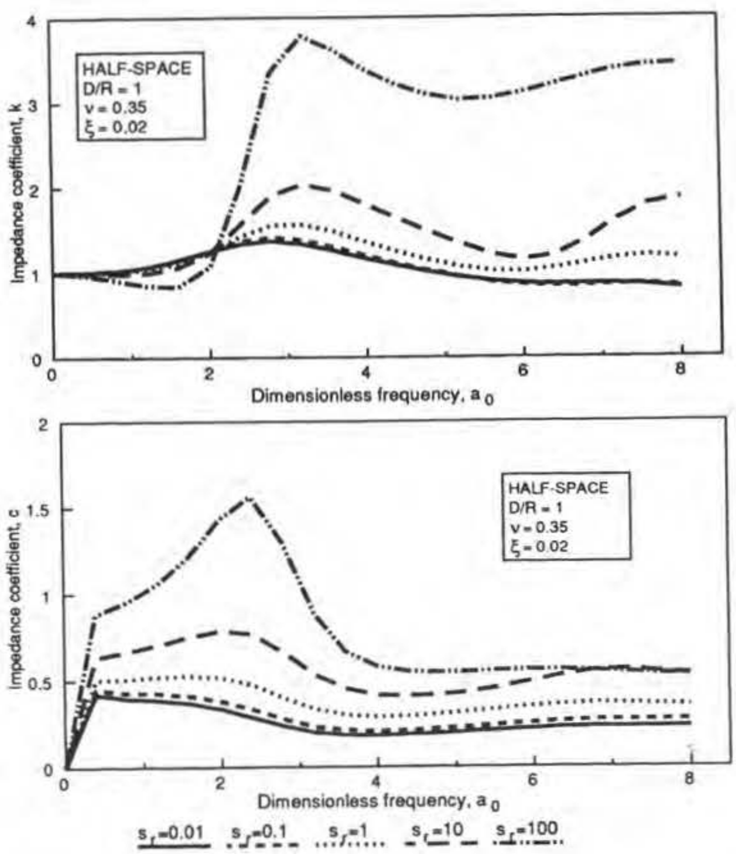

Fig. 4. Impedance Coefficients for a Half-Space as a Function of the Stiffness Ratio for D/R=1.

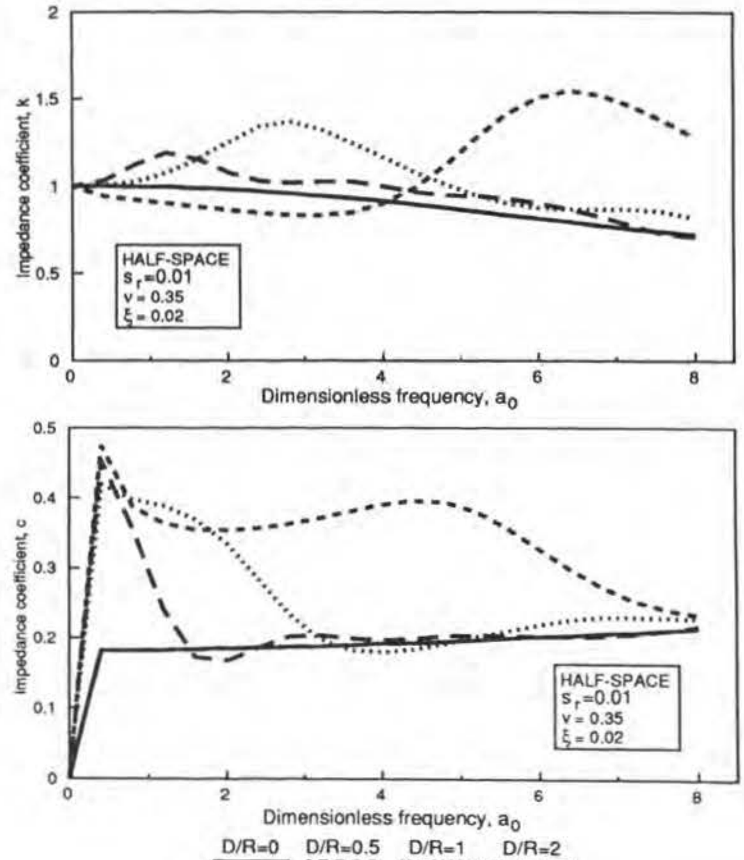

Fig. 5. Impedance Coefficients for a Half-Space as a Function of the D/R Ratio for s_r=0.01.

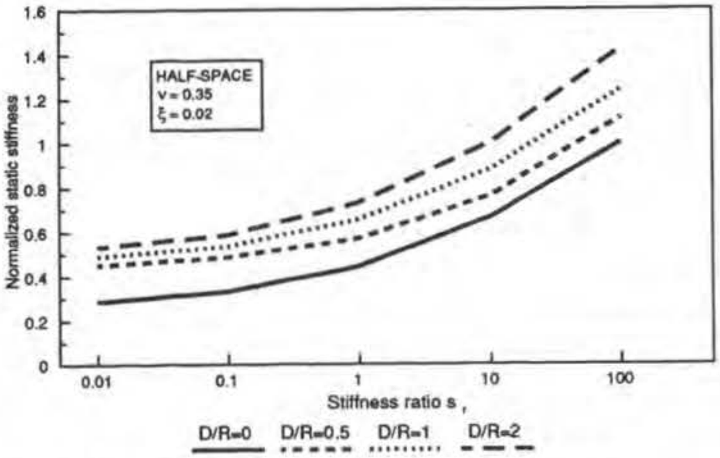

Fig. 6. Normalized Static Stiffness for a Half-Space.

seems to be little sensitive to variations of both D/R and V_{s1}/V_{sHS} ratios. The displacement distribution is affected by the both ratios. Figure 7 illustrates the effect of embedment given by D/R=0.5, and for the V_{s1}/V_{sHS} ratio of 0.25. From the other results it was observed that increasing the V_{s1}/V_{sHS} ratio results in lower variations of displacement with frequency. A stiffer overlying layer forces the foundation to "follow" the motion of the layer. Also, for higher V_{s1}/V_{sHS} and D/R ratios differences in displacement distributions are less stiffness ratio dependent. This is illustrated in Fig. 8 for D/R=1 and V_{s1}/V_{sHS}=2, and should be compared to the displacement distributions in Fig. 2.

The impedances are strongly influenced by the all three factors. Figure 9 is an illustration of the effect of the stiffness ratio for D/R=0.5 and V_{s1}/V_{sHS}=0.5. As in Fig. 4, the stiffness coefficient k rapidly increases with an increase in the stiffness ratio. Similar trend can be observed for the damping coefficient c for frequencies below the previously described natural frequency of vertical oscillations of a layer above the foundation. The effect of the D/R ratio on impedance coefficients is small for all stiffness ratios for low V_{s1}/V_{sHS} ratios (e.g. V_{s1}/V_{sHS}=0.25). Effect of D/R ratio increases as V_{s1}/V_{sHS} ratio increases, as shown in Fig. 10 for s_r=1 and V_{s1}/V_{sHS}=2. On the other hand, the effect of V_{s1}/V_{sHS} ratio on the impedance coefficients is more important for lower D/R ratios. The static stiffness increases with an increase of the stiffness, D/R and V_{s1}/V_{sHS} ratios. A stiffer overlying layer is of exceptional importance for a more flexible foundation, as shown in Fig. 11. Effects of the stiffness and D/R ratios are similar to those for a half-space.

Finally, soil reaction and displacement distributions and impedances for a foundation with loading applied on a rigid core were compared with those for a hypothetical foundation with a uniformly distributed loading. Displacement distributions in Fig. 12 for a foundation with

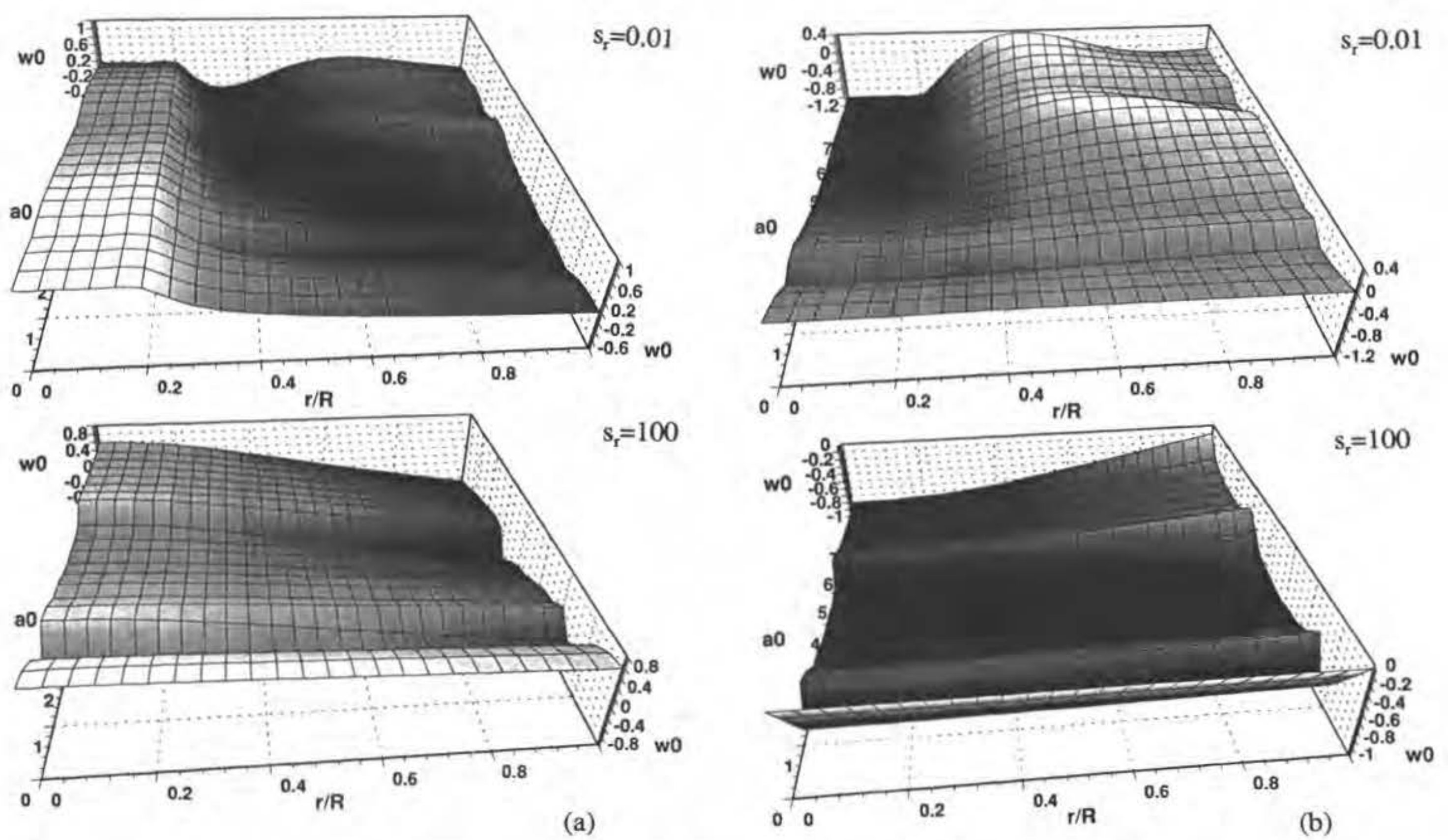

Fig. 7. Displacement Distribution for a Layer over a Half-Space for D/R=0.5 and V_{s1}/V_{sHS}=0.25; (a) Real, (b) Imag. Part.

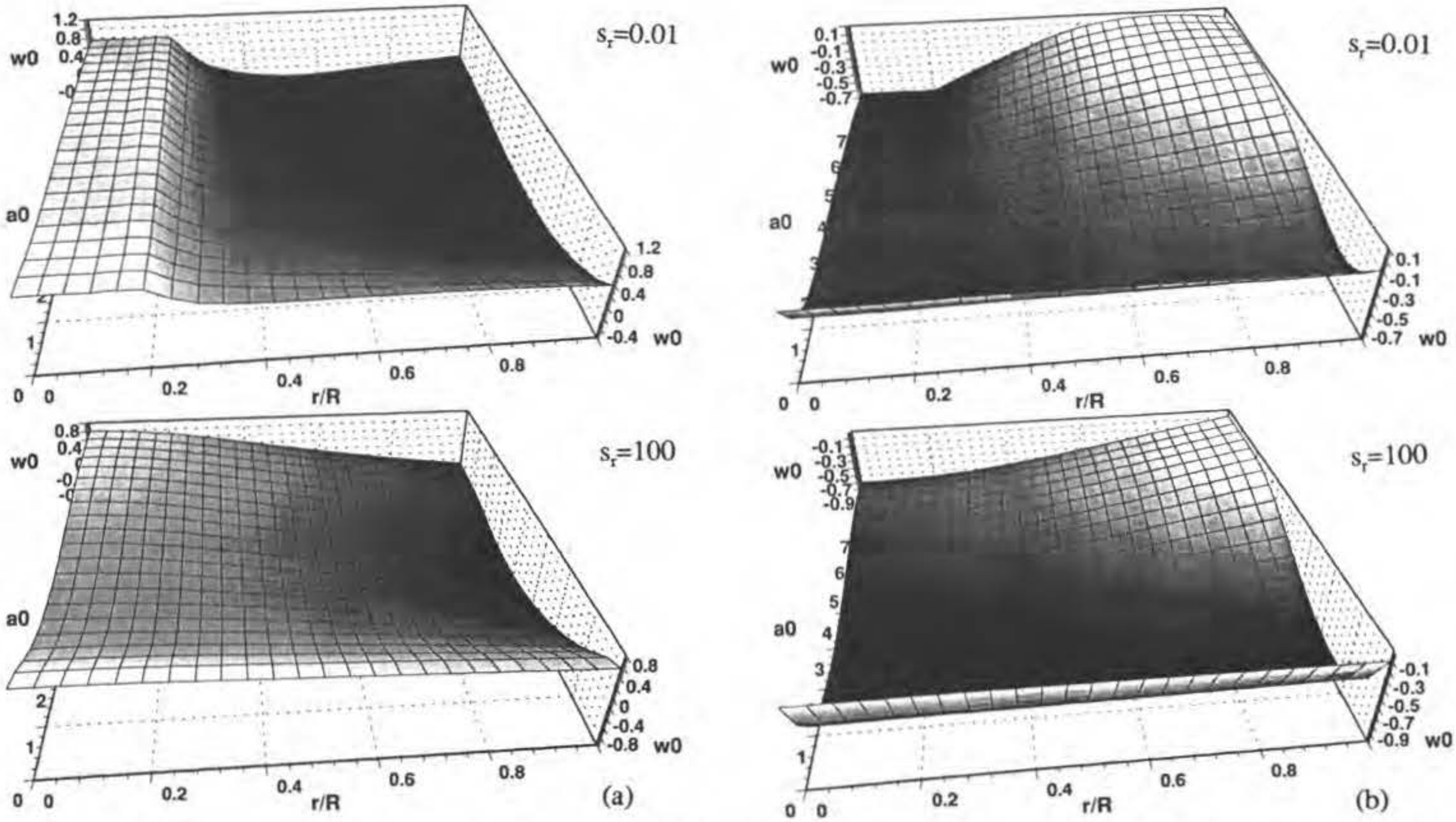

Fig. 8. Displacement Distribution for a Layer over a Half-Space for D/R=1 and V_{s1}/V_{sHS}=2; (a) Real, (b) Imaginary Part.

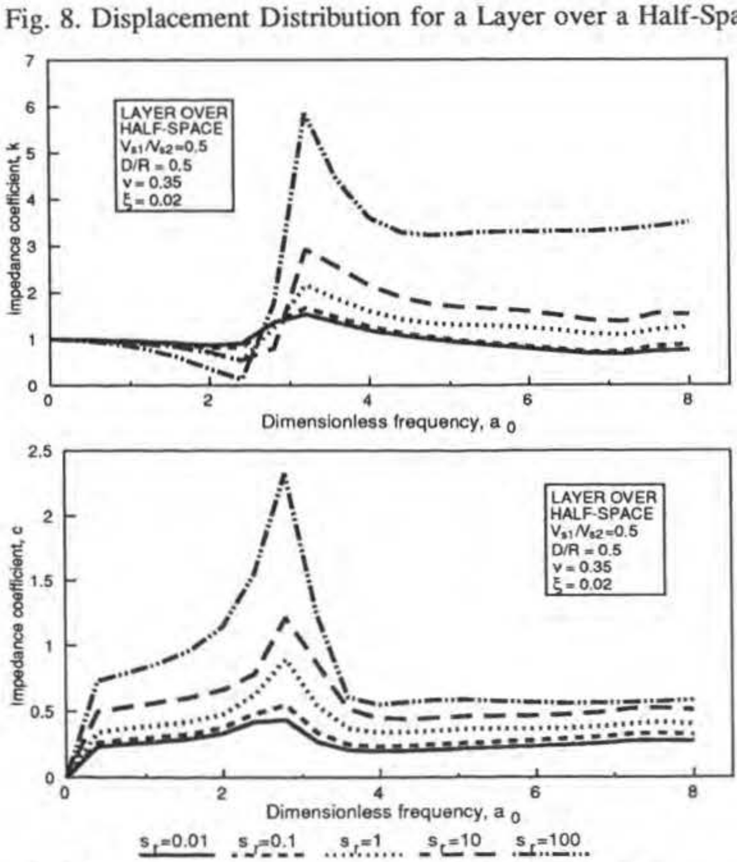

Fig. 9. Impedance Coefficients for a Layer over a Half-Space as a Function of s_r for D/R=0.5 and V_{s1}/V_{sHS}=0.5.

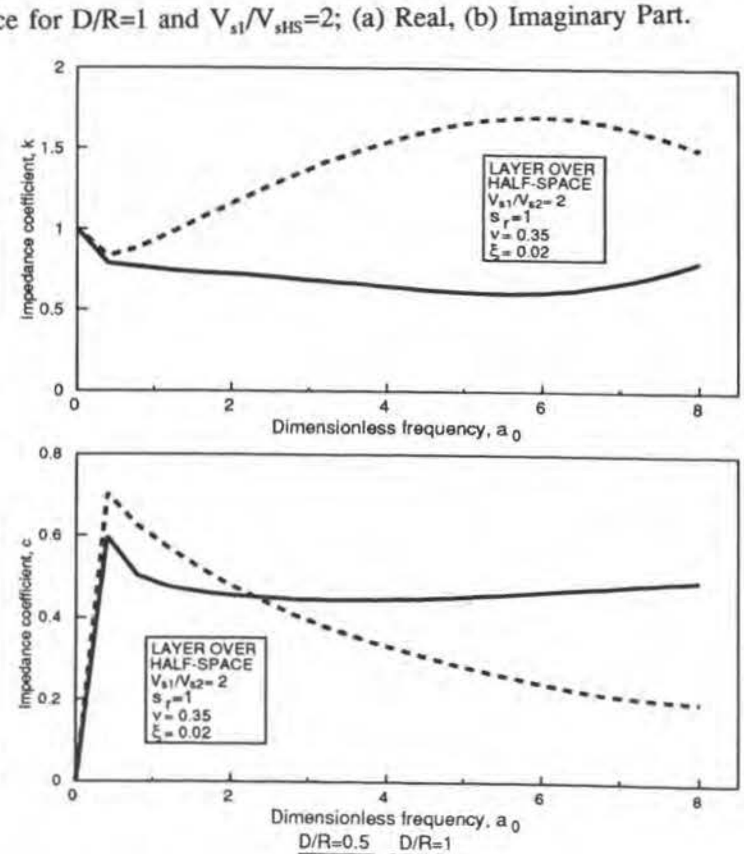

Fig. 10. Impedance Coefficients for a Layer over a Half-Space as a Function of D/R Ratio for s_r=1 and V_{s1}/V_{sHS}=2.

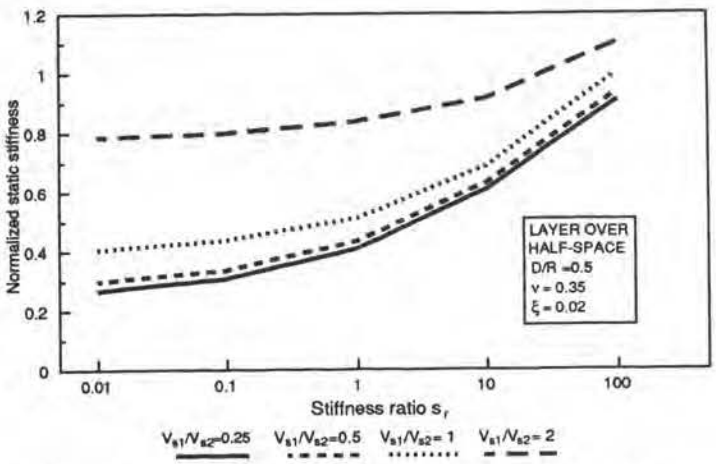

Fig. 11. Normalized Static Stiffness for a Layer over a Half-Space as a Function of V_{s1}/V_{sHS} Ratio for D/R=0.5.

a uniformly distributed loading differ significantly from those in Fig. 2. Similar differences can be observed for soil reaction distributions from a comparison of Figs. 13 and 3. The soil reaction concentration in the vicinity of the edge of the rigid core of a foundation with a lower stiffness ratio is much less pronounced for the case with uniform loading. At the same time, the soil reaction concentration at the edge of a foundation with a higher stiffness ratio (not shown here) is more pronounced. It may be described that foundations with a uniform loading distribution fit well into a previously suggested classification of surface foundations [Gucunski and Peek (1993b)]. In this classification, foundations with the stiffness ratio higher than 10 can be considered as rigid. Impedances for the two loading conditions differ significantly too, especially for lower stiffness ratios. Figure 14 illustrates their comparison for s_r=1. Generally, the uniform loading gives significantly higher impedance coefficients.

CONCLUSIONS

The results of the study indicate that the vertical response of flexible circular foundations, represented by the displacement and soil reaction distribution and impedances, differs significantly from the response of the rigid ones. All of the variables examined-the stiffness ratio s_r, depth of embedment to radius ratio D/R, and the shear wave velocity ratio V_{s1}/V_{sHS}-affect the response. Distribution of loading represents another important variable. Loading applied at the central portion of the foundation tends to modify the behavior of what would otherwise be considered a rigid foundation to that characteristic of flexible foundations.

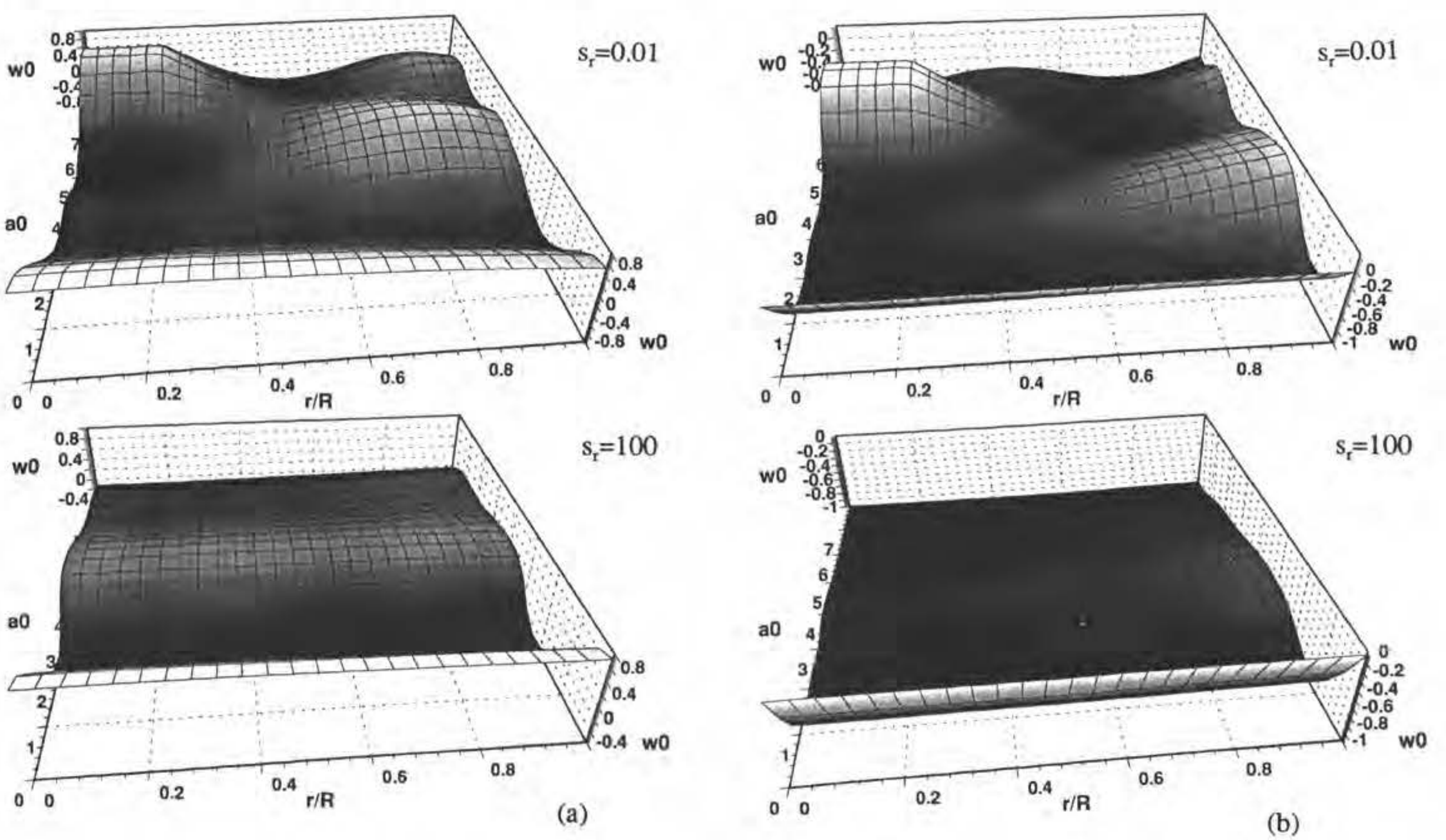

Fig. 12. Displacement Distribution for a Foundation with Uniformly Distributed Loading; (a) Real, (b) Imaginary Part.

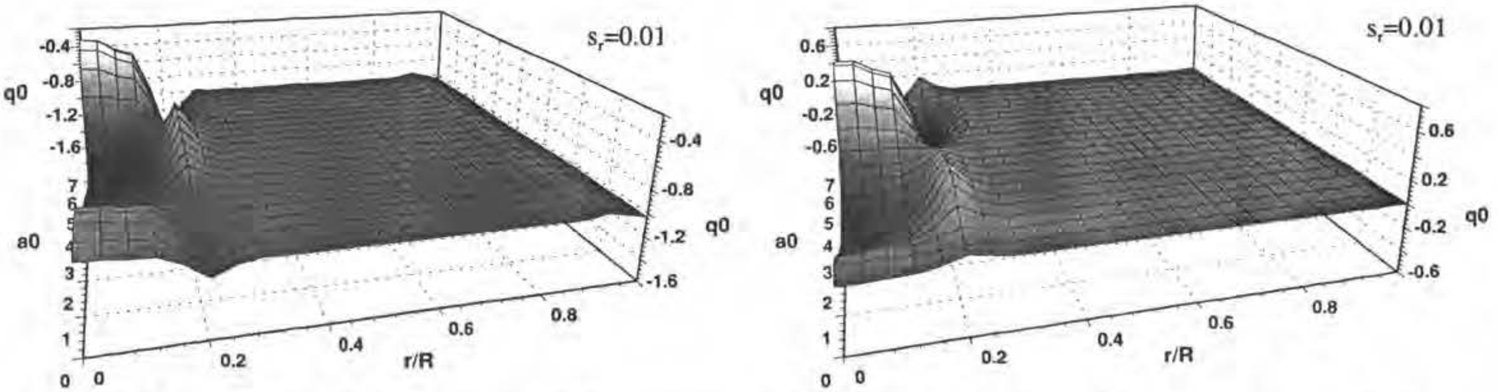

Fig. 13. Soil Reaction Distribution for a Foundation with Uniformly Distributed Loading; (a) Real, (b) Imaginary Part.

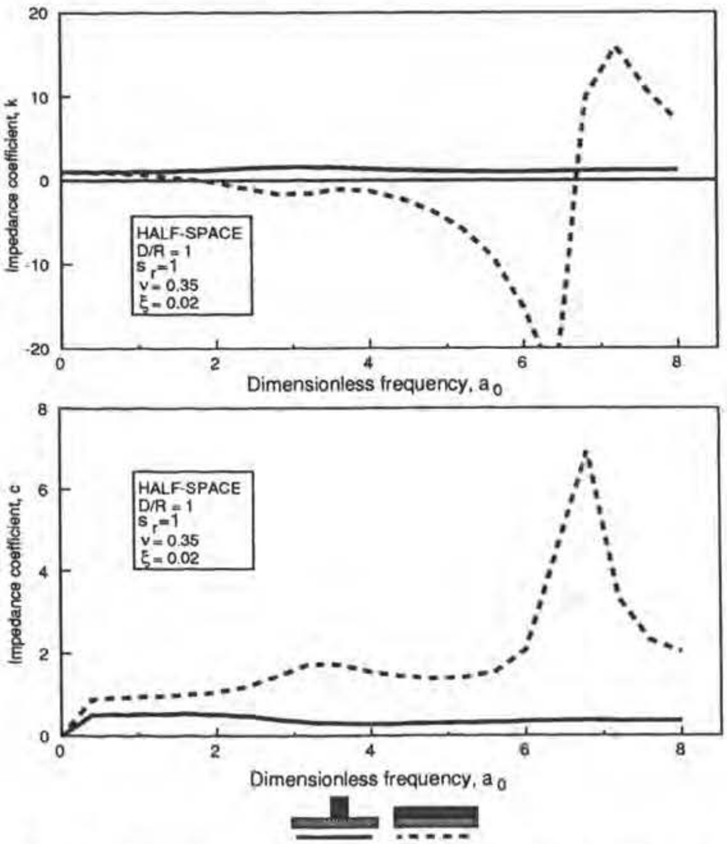

Fig. 14. Impedance Coefficients for a Half-Space as a Function of Loading Distribution for D/R=1 and s_r=1.

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