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Three Dimensional Wave Propagation due to Pile Driving

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Three Dimensional Wave Propagation due to Pile Driving

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SYNOPSIS: A very economical and efficient method to construct pile foundations or sheet walls is
given by the driving of the pile itself in the case of prefabricated piles or of the steel-sheet pi-
pe for concrete piles in situ and also by the driving of the sheet piles in the case of sheet walls.
In spite of its efficiency this method underlies, because of environment protection reasons to cer-
tain restrictions, that concern the influence of the produced shock waves during the driving proce-
dure to neighboring buildings and constructions.
For the theoretical calculation of this influence at first the free-field response of the ground due
to the propagated shock waves will be requiered. The source wave is generally of transient nature.
The authors deal in this contribution with the theoretical calculation of free-field magnitudes for
an elastic homogeneous half-space as an adequate model for an idealized ground.

INTRODUCTION

The problem of the three dimensional wave pro-
pagation in elastic media and particularly in
those media restricted by a natural boundary, as
the elastic half-space by its surface, has been
intensivelly investigated in the last years by
many authors. The most of the known solutions
appears in the frequency domain. Johnson, 1974,
yields the first analytical and complete solu-
tion of the problem in time domain. Allthough
the solution is mathematically komplex the ex-
pressions obtained there are very simple.Because
of some slight but essential mistakes in the so-
lution of Johnson, Mitakidis, 1989, resolved the
three dimensional boundary value problem follow-
ing the work and the solution technics of John-
son. The solution represents exact and in mathe-
matically closed form evaluated Green's func-
tions for the elastic half-space which are
pointed out by an absolutely stable behavior in
their numerical treatment.

SOLUTION OF THE PROBLEM IN TIME DOMAIN

The problem of the three dimensional wave propa-
gation in the elastic homogeneous half-space
consists in the solution of the Lamé-Navier
equation of motion.

$$\rho \frac{\partial^2}{\partial t^2} u(x,t) = f(x,t) + (\lambda+\mu) \nabla(\nabla u(x,t)) + \mu \Delta u(x,t) \quad (1)$$

In eq. (1) is u the displacement vector, x the
location vector to the considered point A in the
interior of the half-space,f displays the force
as source for the propagated elastic disturbance
at the source point Q. The infinite elastic me-
dium is given by its Lamé constants λ and μ,
Fig. 1. In order to proceed in the solution of
eq. (1), the source function f, a single point
load in the interior of the half-space will be
loclized in time and space with Dirac-delta
functions:

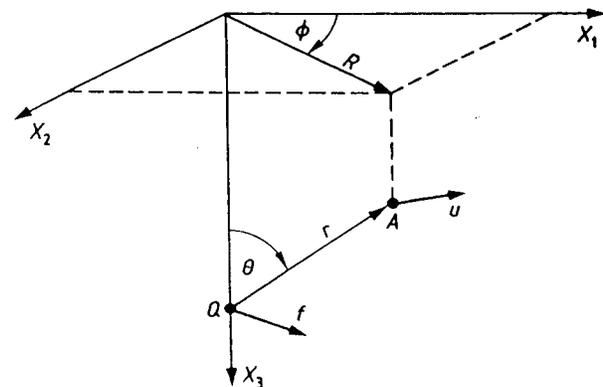$$f(x,t) = (F_1 e_1 + F_2 e_2 + F_3 e_3)\delta(x_1 - X_1)\delta(x_2 - X_2)\delta(x_3 - X_3)\delta(t-T) \quad (2)$$

Fig. 1: Geometry of the problem: displacement u
at point A due to force f at a point Q

In eq. (2) x_1, x_2, x_3 and t corresponds to the
location and the arrival time of the disturbance
at the controll point A, X1, X2, X3 and T cor-
responds to the location and the moment of ap-
plying of the force f at the source point Q. For
a force vector, as given by eq. (2) the solution
of eq. (1) corresponds to a Green's function so
that we can write

$$u(x,t) = g(x,t; X,T)$$
$$= q_1(x,t; X,T)e_1 + q_2(x,t; X,T)e_2 + q_3(x,t; X,T)e_3 \quad (3)$$

Physically g (x,t; X,T) means the displacement
vector at moment t at the controll point A with
location vector x = (x1, x2, x3.) due to an im-
pulvive single point load F applied at time T
at the source point Q with location vector
X = (X_1, X_2, X_3).
The eq. (1) is solved analytically by the aid of
the Cagniard-De Hoop transform technic,
Mitakidis 1989, accordingly to the work of

Johnson, 1974. The complete solution will be represented here for the case $X_3 = 0$, Point A at the surface, because the most interesting phenomena take place at the surface of the ground. For this case the solution has the form:

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \mathbf{F} \qquad (4)$$

where

$$\mathbf{G} = \mathbf{G}^P + \mathbf{G}^S \qquad (5)$$

The upper indizes P and S in (5) characterize the P - and S - wave parts respectivelly of which consist the solution vector G. For the problem considered here only the two components G_{13} and G_{33} will be used. After Mitakidis, 1989, they have the following form:

$$G_{13}^P = \frac{1}{\pi^2 \mu r} \frac{\partial}{\partial t} \int_0^{P_p} U\left(t - \frac{r}{V_p}\right) Re \left\{ \frac{2q\,\eta_p \eta_s \cos\phi}{\sigma \left(\frac{t^2}{r^2} - \frac{1}{V_p^2} - \rho^2\right)^{1/2}} \right\} d\rho \qquad (6)$$

$$G_{33}^P = \frac{1}{\pi^2 \mu r} \frac{\partial}{\partial t} \int_0^{P_p} U\left(t - \frac{r}{V_p}\right) Re \left\{ \frac{\eta_p^2 \gamma}{\sigma \left(\frac{t^2}{r^2} - \frac{1}{V_p^2} - \rho^2\right)^{1/2}} \right\} d\rho \qquad (7)$$

In the case of steep radiation: $\sin\vartheta < V_s/V_p$

$$G_{13}^S = \frac{1}{\pi^2 \mu r} \frac{\partial}{\partial t} \int_0^{P_s} U\left(t - \frac{r}{V_s}\right) Re \left\{ \frac{-\eta_s q\,\gamma \cos\phi}{\sigma \left(\frac{t^2}{r^2} - \frac{1}{V_s^2} - \rho^2\right)^{1/2}} \right\} d\rho \qquad (8)$$

$$G_{33}^S = \frac{1}{\pi^2 \mu r} \frac{\partial}{\partial t} \int_0^{P_s} U\left(t - \frac{r}{V_s}\right) Re \left\{ \frac{2\eta_s \eta_p (q^2 - \rho^2)}{\sigma \left(\frac{t^2}{r^2} - \frac{1}{V_s^2} - \rho^2\right)^{1/2}} \right\} d\rho \qquad (9)$$

In the case of flat radiation $\sin\vartheta > V_s/V_p$ and $t > r\sqrt{\rho^2 + \frac{1}{V_s^2}}$ are

$$G_{13}^S = \frac{1}{\pi^2 \mu r} \frac{\partial}{\partial t} \int_0^{P_1} U(t - t_1) Re \left\{ \frac{-\eta_s q\,\gamma \cos\phi}{\sigma \left(\frac{t^2}{r^2} - \frac{1}{V_s^2} - \rho^2\right)^{1/2}} \right\} d\rho \qquad (10)$$

$$G_{33}^S = \frac{1}{\pi^2 \mu r} \frac{\partial}{\partial t} \int_0^{P_1} U(t - t_1) Re \left\{ \frac{2\eta_s \eta_p (q^2 - \rho^2)}{\sigma \left(\frac{t^2}{r^2} - \frac{1}{V_s^2} - \rho^2\right)^{1/2}} \right\} d\rho \qquad (11)$$

For $t < r\sqrt{\rho^2 + \frac{1}{V_s^2}}$ eqs. (11) and 12) become the form:

$$G_{13}^S = \frac{1}{\pi^2 \mu r} \frac{\partial}{\partial t} \int_0^{P_1} U(t - t_1) Im \left\{ \frac{\eta_s q\,\gamma \cos\phi}{\sigma \left(\frac{1}{V_s^2} + \rho^2 - \frac{t^2}{r^2}\right)^{1/2}} \right\} d\rho \qquad (12)$$

$$G_{33}^S = \frac{1}{\pi^2 \mu r} \frac{\partial}{\partial t} \int_0^{P_1} U(t - t_1) Im \left\{ \frac{-2\eta_p \eta_s (q^2 - \rho^2)}{\sigma \left(\frac{1}{V_s^2} + \rho^2 - \frac{t^2}{r^2}\right)^{1/2}} \right\} d\rho \qquad (13)$$

In the above eqs. (6) ./. (13) the following symbols have been used:

$u(t)$: is the Heaviside's unit step function

$$P_p = \left(\frac{t^2}{r^2} - \frac{1}{V_p^2}\right)^{1/2}$$

$$P_s = \left(\frac{t^2}{r^2} - \frac{1}{V_s^2}\right)^{1/2}$$

$$V_p = \left(\frac{2(1-\nu)\mu}{(1-2\nu)\rho}\right)^{1/2} \quad \text{Longitudinal wave velocity}$$

$$V_s = \left(\frac{\mu}{\rho}\right)^{1/2} \quad \text{shear wave velocity}$$

$$\eta_s = \left(\frac{1}{V_s^2} + \rho^2 - q^2\right)^{1/2} \quad ; \quad \eta_p = \left(\frac{1}{V_p^2} + \rho^2 - q^2\right)^{1/2}$$

$$\gamma = \eta_s^2 + \rho^2 - q^2 \quad ; \quad \sigma = \gamma^2 + 4\eta_p \eta_s (q^2 - \rho^2)$$

$$P_1 = \left[\left(\frac{\frac{t}{r} - \left(\frac{1}{V_s^2} - \frac{1}{V_p^2}\right)^{1/2} \cos\vartheta}{\sin\vartheta} \right) - \frac{1}{V_p^2} \right]^{1/2}$$

$$t_1 = \frac{r}{V_p} \sin\vartheta + r\left(\frac{1}{V_s^2} - \frac{1}{V_p^2}\right)^{1/2} \cos\vartheta$$

For the components with upper index p the separation variable q is given by

$$q = -\frac{t}{r} \sin\vartheta + i\left(\frac{t^2}{r^2} - \frac{1}{V_p^2} - \rho^2\right)^{1/2} \cos\vartheta \qquad (14)$$

In eqs. (8), (9), (12) and (13) q is given by

$$q = -\frac{t}{r} \sin\vartheta + i\left(\frac{t^2}{r^2} - \frac{1}{V_s^2} - \rho^2\right)^{1/2} \cos\vartheta \qquad (15)$$

Finally in eqs. (10) and (11) q has the form

$$q = -\frac{t}{r} \sin\vartheta + \left(\frac{1}{V_s^2} - \frac{t^2}{r^2} + \rho^2\right)^{1/2} \cos\vartheta \qquad (16)$$

By ommiting the differentiation after the time the expression in eq. (6) till (13) yield the response of the half-space surface to a Heaviside unit-step function, Fig. 2 ./. 4.
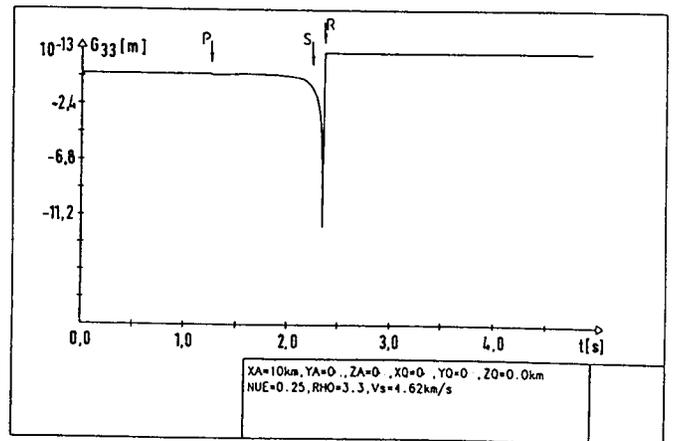

Fig.2a: Green's function G_{33}

Excitation of the half-space surface Heavidide's unit step-function
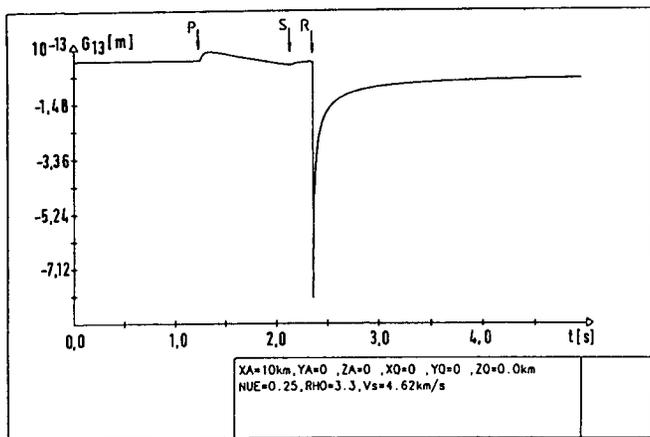
Fig. 2b: Greens's function G_{13}

Excitation of the half-space surface
by Heavidide's unit step function

Figure 2 shows the surface behavior of the half
space due to the Heavidide's unit step function
acting at the half-space surface. The epicentral
distance X_A of the controll point A has been
choosen to be 10 km, the Poisson ratio V of the
medium ammounts 0,25 , the density $\rho = 3.3$ t/m³
(rock material) and the shear wave velocity is
$v_s = 4.62$ km/s.

All the three wave types of the half-space oc-
cure very clearly and they are pointed out by
the symbols P for the Longitudinal wave, S for
the shear wave and R for the well known Rayl-
leigh-wave.
In Figure 3 the source of the elastic disturban-
ce moves from the surface to a depth of Z_Q
= 2.0 km. All the other parameters remain the
same. This is the case of flat-radiation.

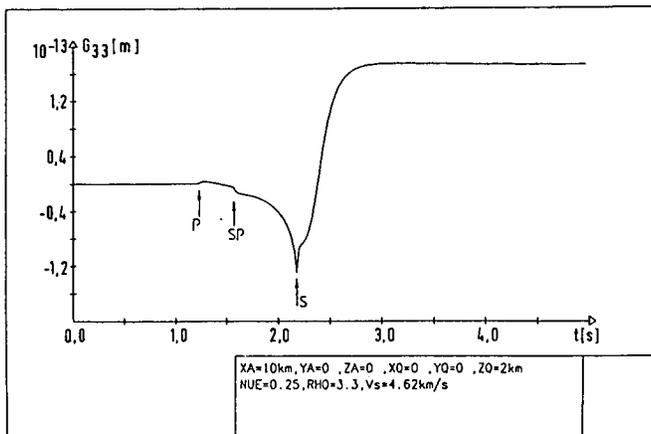Fig. 3a: Green's function G33
            Hypocentral distance Z_Q = 2.0 km
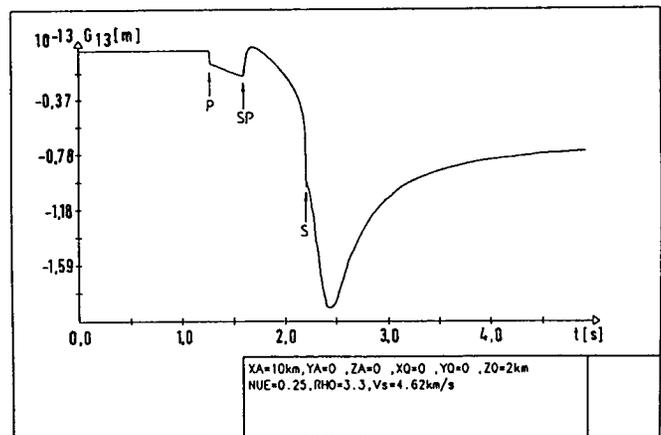   Epicentral distance X_A = 10.0 km

Fig. 3b: Green's function G_{13}

In Figure 3 we can see, that a very interesting
phenomenon coccurs as the wave source leaves
the surface of the half-space: The main signal
of the arriving R-wave dissapears and another
signal between the P- and S-Wave appears. This
is the refracted at the surface of the half-
space SP-Wave. The SP-Wave, or also known as
head wave, occurs only in the case of flat radi-
ation, it means in the case, that the radiation
angle   satisfy the condition: $\sin \vartheta > v_s / v_p$,
Fig. 1. Finally in Fig. 4 the source lies in a
depth of Z_Q = 10 km and the epicentral distance
of the considered point A ammounts 2 km. Here we
have the case of steep radiation, $\sin \vartheta < v_s / v_p$,
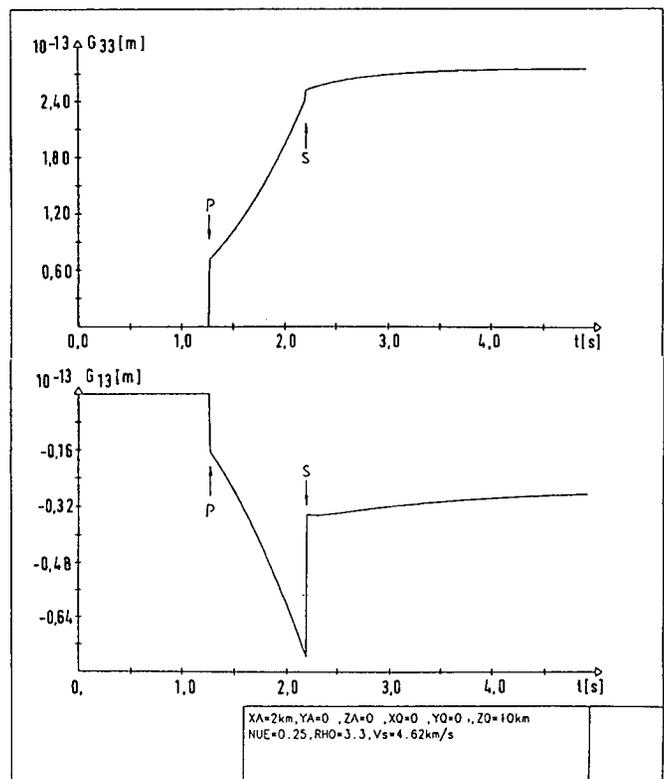for which only the two main wave types P+S exist.

Fig. 4a: Green's function G33

    4b: Green's function G_{13}

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SYSTEM RESPONSE TO ARBITRARY TRANSIENT LOADS

Because of the fact that the Green's function represents also the transfer function of the system half-space, they can be used to calculate by the aid of the Duhamel's convolution integral the system response to any arbitrary transient load, eq. (17).

$$X(t) = \int_0^t \frac{df(t-\tau)}{d\tau} \, g(\tau) \, d\tau \qquad (17)$$

X(t) : System response at time t

f(t) : Load function

g(t) : Transfer function of the considered system, must be known.

For the considered system of the elastic homogeneous half-space we obtain

$$W_{ij}(t) = \int_0^t \frac{df_i(t-\lambda)}{d\lambda} \, G_{ij}(\lambda) \, d\lambda \qquad (18)$$

The half-sinus impuls load:

We consider a half-sinus impuls with a duration Timp and a peak magnitude P_0.

$$f(t) = \begin{cases} P_0 \sin\Omega t & \text{for } 0 \leqslant t < \pi/\Omega \\ \\ 0 & \text{for } t \geqslant \pi/\Omega \end{cases}$$

Setting the load function f(t) and the evaluated Green's function G_{33} and G_{13} in eq. (18) we obtain for the vertical and radial displacement components W_{33} and W_{13} of the half-space surface due to a half-sinus impuls:

$$W_{33}(t) = \int_0^t \frac{df(t-\lambda)}{d\lambda} \, G_{33}(\lambda) \, d\lambda \qquad (19)$$

$$W_{13}(t) = \int_0^t \frac{df(t-\lambda)}{d\lambda} \, G_{13}(\lambda) \, d\lambda \qquad (20)$$

where

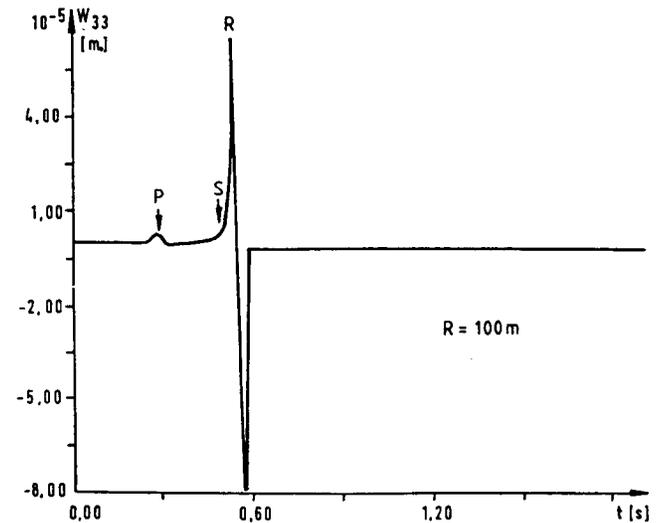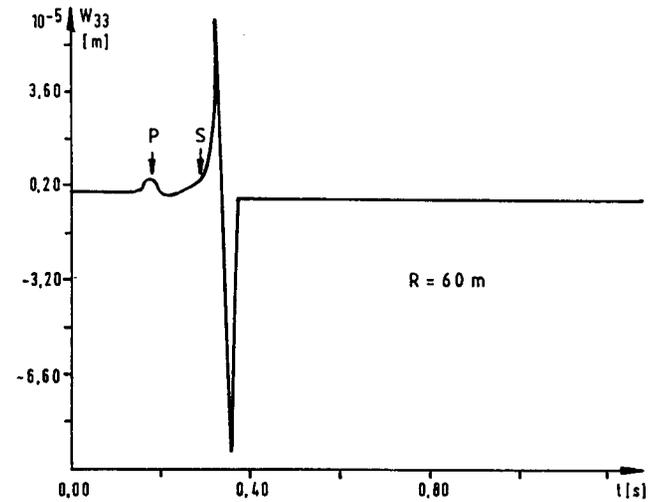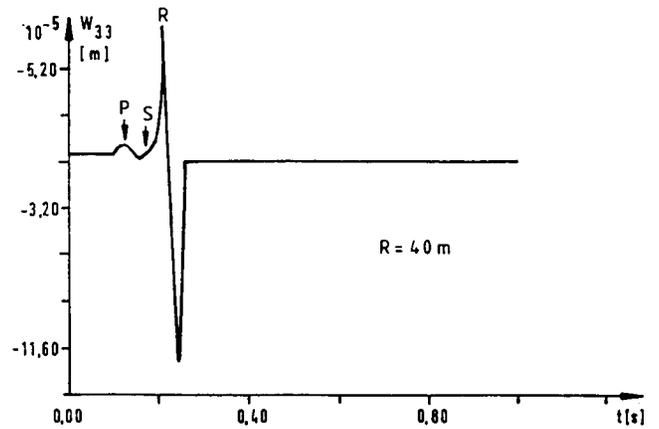$$\frac{df(t-\lambda)}{d\lambda} = -\Omega P_0 \cos\Omega(t-\lambda)$$

Fig.5 Free field amplitude due to a half-sinus impuls at the surface of the half-space P_0 = 2000 kN; ρ = 1.8 t/m³, V = 0.33 Timp = 0.05 s
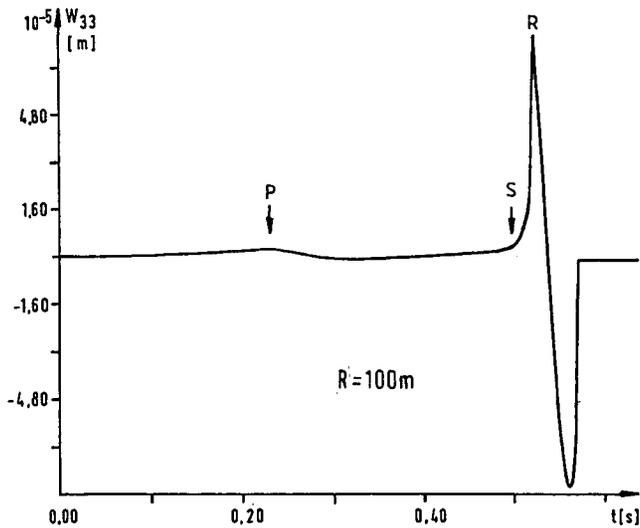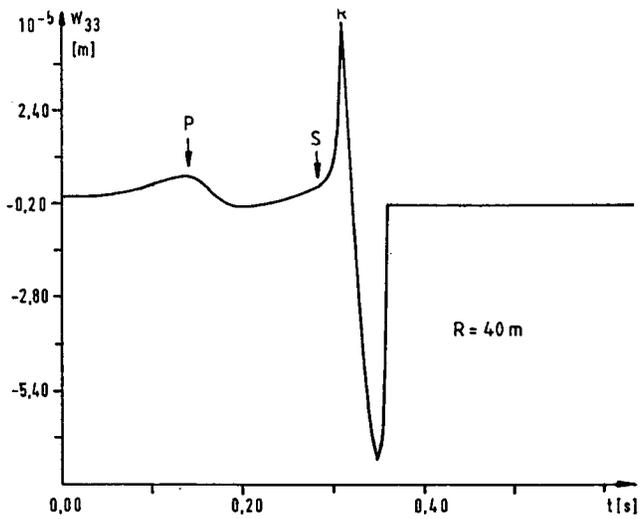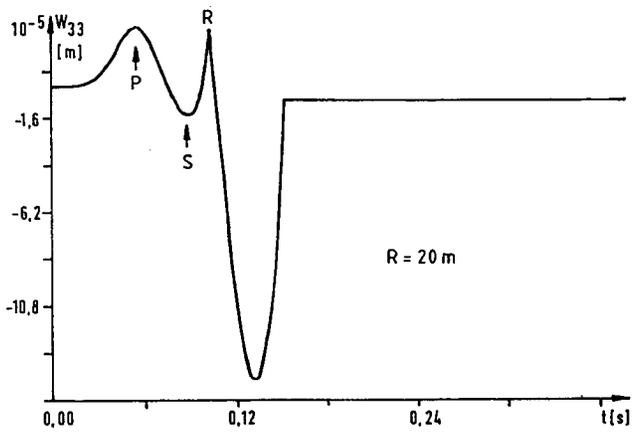
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Figures 5 and 6 display the graph of the displacement component W33 as given by eq. (19) evaluated for various distances R from the source. In both figures we see clearly all the three wave types of the half-space excited at its surface. They are denoted by the symbols P, S and R.
The Rayleigh wave is dominant and occurs just after the S-Wave signal.
For materials with great Poisson ratios , Figure 6 ($\nu = 0.49$), as for example water saturated soils, the P-Wave compared to the S-Wave arrives with a considerable strong signal. This phenomenon has been very often neglected in the past years.

Harmonic point load:

We consider a harmonically oscillating vertical load

$$f^H(t) = P\exp(i\Omega t)$$

Fig.6 Free field amplitude due to a half-sinus impuls at the surface of the half-space P_0 = 2000 kN; ρ = 1.8 t/m³; ν = 0.49 Timp = 0.05 s
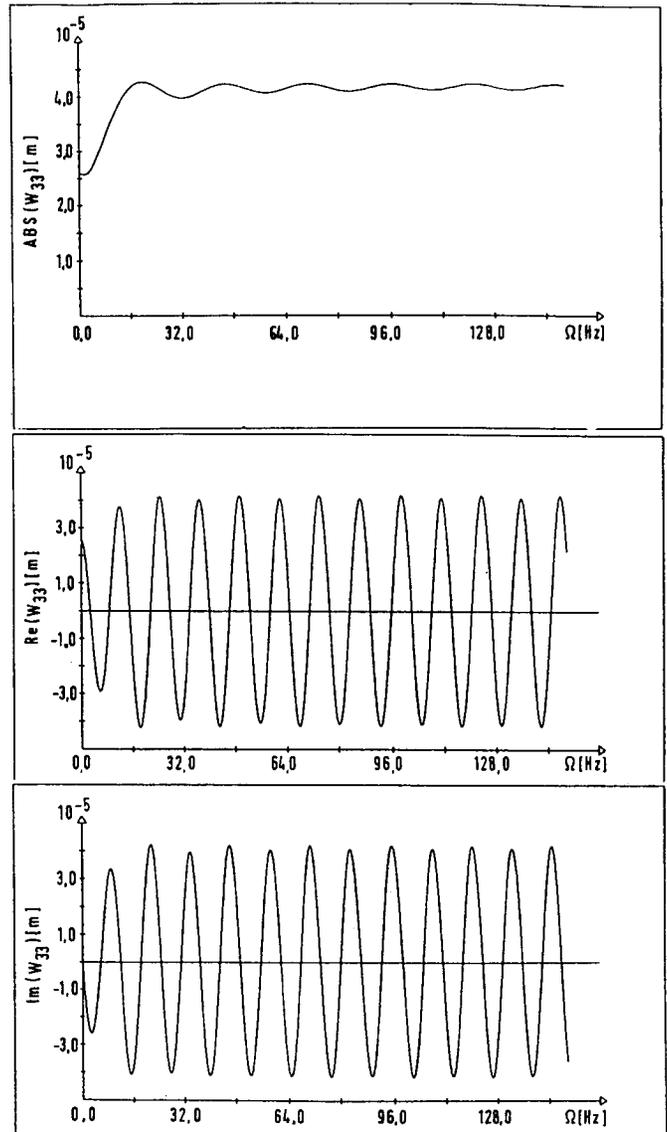
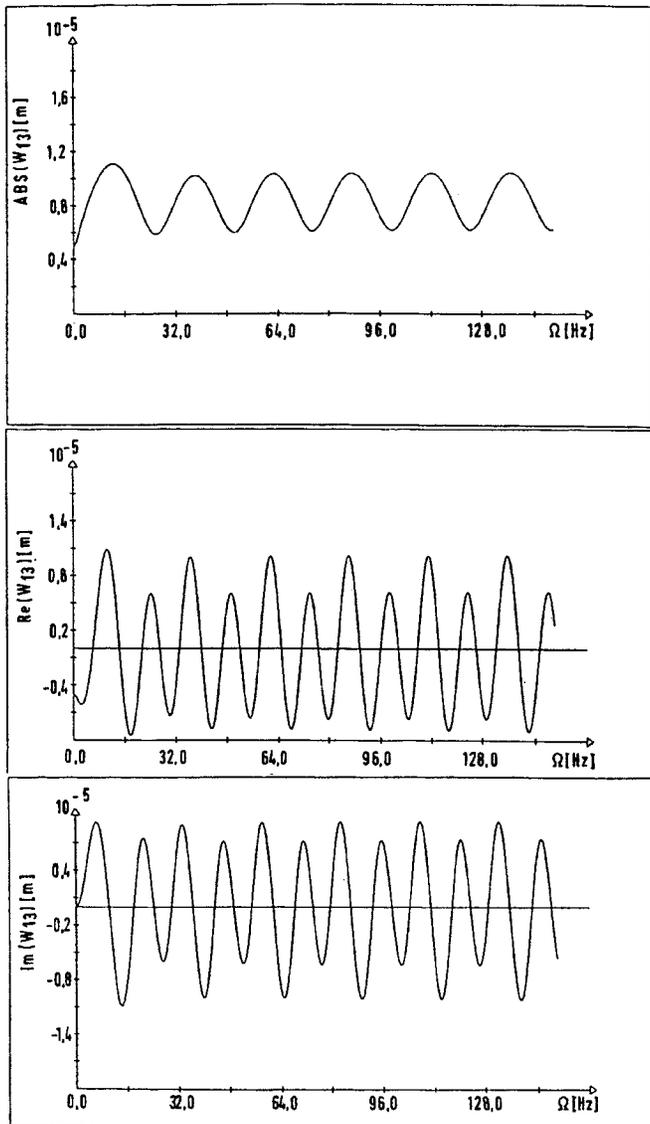Fig. 7: Free field amplitude due to harmonic load, displacement W_{33}

Fig. 8   Free field amplitude due to harmonic
         load, displacement W_{13}

In order to calculate the system response to the
harmonic load we make use again of eq. (18) and
we obtain for the two components W_{33} and W_{13}

$$W_{33}(\Omega)= -i\Omega P \int_0^\infty exp(-i\Omega\lambda)G_{33}(\lambda)d\lambda \cdot exp(i\Omega t) \quad (21)$$

and

$$W_{13}(\Omega)= -i\Omega P \int_0^\infty exp(-i\Omega\lambda)G_{13}(\lambda)d\lambda \cdot exp(i\Omega t) \quad (22)$$

The integrant in eqs. (21) and (22) respresents
the steady state oscillation of the half-space
surface.

Figures 7 and 8 show the response of the half-
space surface due to a harmonic load with ampli
tude   P = 2000 kN acting in a depth of Z_Q = 100 m
in the interior of the half-space. The empicen-
tral distance of the control point amounts
R = 20.0 m.
The considered half-space has a Poisson ratio of
V= 0.33, a density ρ = 1.8 t/m³ and the shear
wave velocity is v_S = 200 m/s.
Figure 7 displays the vertical component W_{33}
after eq. (21).

The displacement components are given by their
absolute value and the phase to the applied
harmonic load is defined by

$$\phi = arc\,tg \frac{Im}{Re} \quad (23)$$

In Figure 8 we see the radial (horizontal)
component W_{13}, eq. (22)

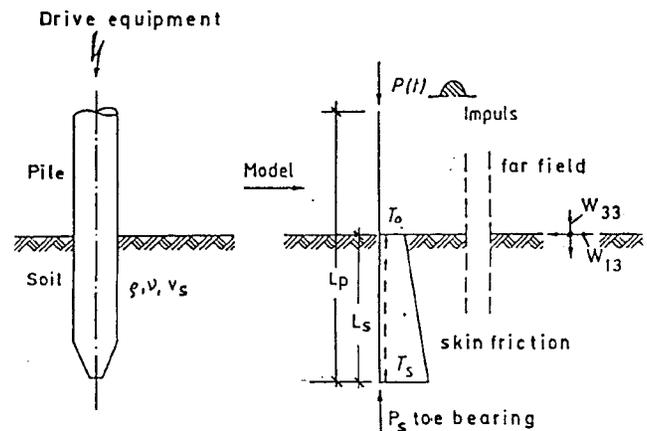MODEL FOR SIMULATION OF PILE DRIVING AND RESULTS

Fig. 9   Simulation model for pile driving –
         Free field response of the half-space
         surface

The created solutions for the free field respon-
se to a half-sinus impuls, eqs. (19,20), will be
used here to simulate the free field response on
the surface of a half-space, given by its densi-
ty ρ , Poisson ration v and shear wave vel city
v_S, due to a pile driving process. The calcula -
ted free field response is of practical interest
in the far-field. In the near field is the re-
sponse much more affected by the elasto-plastic
phenomena which take place during the driving
process and which phenomena cannot be consider-
ed by this theoretical solution. The pile consi-
dered here has a total length of Lp = 15.0 m.
The length of the pile shaft in the soil amounts
L_s = 9.0 m. The skin friction acts along the
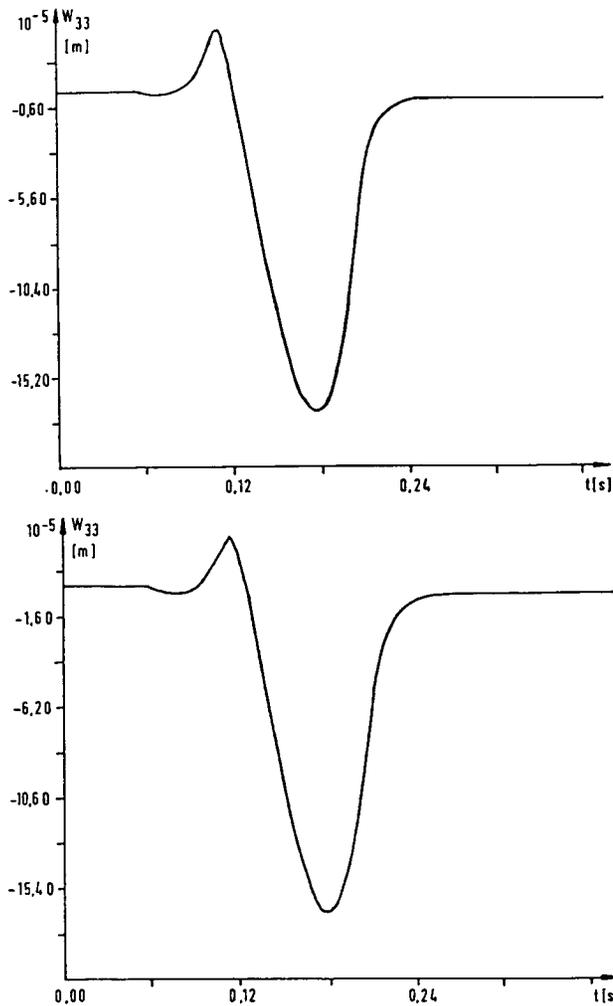length L_s with the ordinate T_0 at the surface of

Fig. 10 Free field amplitude W_{33} (t)

  a) skin friction 0 %, toe bearing 100%
  b) skin friction 50 %, toe bearing 50%

Figure 10a shows the vertical component W_{33} (t) of the displacement on the half-space surface for the theoretical case of non existing skin friction. In Figure 10b the vertical displacement W_{33} (t) is displayed for the case that 50% of the impact magnitude will be emmitted from the pile to the soil by skin friction and the other 50 % by the toe bearing capacity.

By comparing both figures we see that skin friction affects the free field magnitude to become larger. This influence of the skin friction is pointed out more clearly in the figures 11a and 11b, which show the horizontal displacement component W_{13} (t). By considering of skin friction, 50 % of the impact magnitude, the free field amplitude is about two times larger as in the case of neglected skin friction.
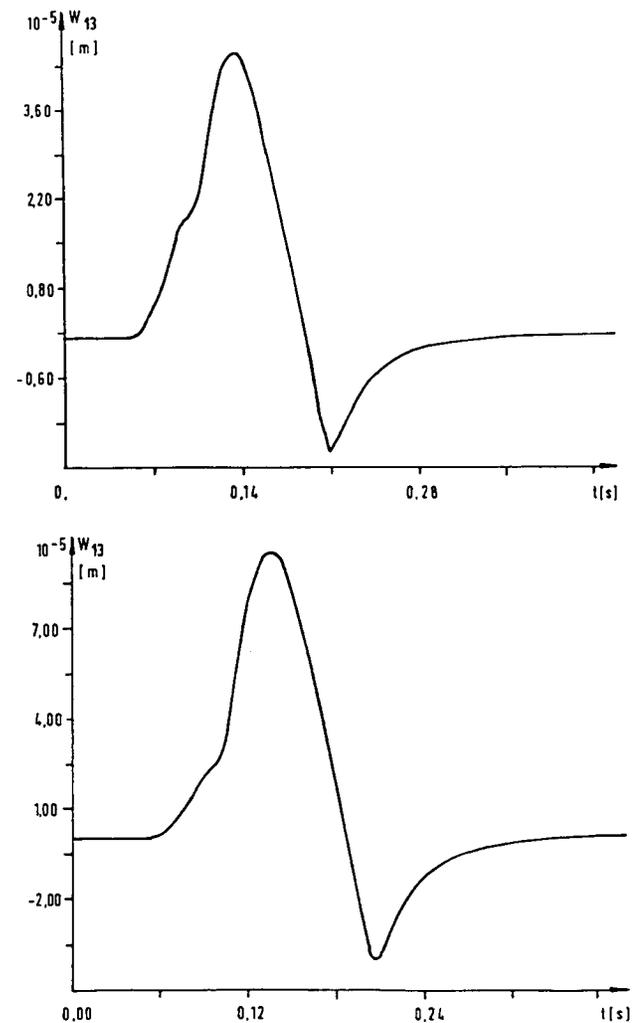

the half-space and T_S at the pile toe. The ratio T_0/T_S has been choosen to 0.5. At the pile toe acts also the toe bearing capacity P_S . This half-sinus impuls acting at the pile head has a peak magnitude of P_0 = 2000 kN and its duration amounts T_{imp} = 0.10 s. The wave propagation velocity in the pile is V_A = 4000 m/s (concrete pile). The sizes , and v_S for the considered soil are:

ρ = 1.8 t/m³; V = 0.33; v_S = 200 m/s

The influence of the skin friction is here considered by integrating the single-point load solution along L_S by the aid of Gaussian quadrature formula. The time stifting between the impact at the pile head and the Gaussian points along the length L_S has been also considered. The control point at the surface of the soil, where the signals are messured has a distance R = 20 m from the pile.

Fig. 11 Free field amplitude W_{13} (t)

  a) skin friction 0%, toe bearing 100%
  b) skin friction 50%, toe bearing 50 %

CONCLUSIONS

The phenomenon of wave propagation in the soil
is described by so called Green's function.
These solutions, gained by analytical methods,
are numerically very stable and thus extremely
suitable for investigations in various practical
application purposes. By use of the Green's func-
tions the authors calculated the free-field re-
sponse of the half space due to various loads
acting in the interior of the half-space or at
its surface, as half sinus impuls or harmonic
point load. Finally a pile driving process has
been simulated. The outhors investigated the in-
fluence of some substantial parameters as skin
friction and the toe bearing capacity of the pi-
le.

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