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Attenuation Characteristics of Ground Strains Induced Juring Earthquake

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YNOPSIS: The appropriate estimation of ground strains induced during earthquake is indispensable 'or the seismic design of buried lifeline facilities such as pipeline systems. The ground strains nduced during earthquakes are calculated with use of the dense instrument array data observed luring past 78 earthquakes for the surface ground at the Public Works Research Institute (PWRI) in 'sukuba Science City in Japan. Based on the multiple regression analysis for the calculated ground strains, the empirical formulae of attenuation of maximum ground strains for such the ground condition as the PWRI campus are proposed in terms of earthquake magnitude and epicentral distance, and the attenuation characteristics of ground strains are investigated.

INTRODUCTION

It is well recognized that dynamic behavior of lifeline facilities such as tubular piping systems embedded in ground essentially depends on the dynamic response of subsurface grounds. The seismic deformation method, which considers ground strains induced during earthquakes as seismic effects instead of inertia forces, was developed and is now in practical use for seismic design of extended structures embedded Although investigations in ground[PWRI (1977)]. on actual ground strains induced during earthquake are essential to assess appropriate seismic effects to be considered in the seismic deformation method, few studies have been conducted, mostly due to the lack of measured data [Arakawa et.al.(1985)].

In this paper, the ground strains are tried to estimate with use of the acceleration data observed by dense instrument array for the surface ground at the Public Works Research Institute during past 78 earthquakes. The ground strains are calculated by a standard three dimensional finite element analysis. Further, the empirical formulae of attenuation of maximum ground strains in terms of earthquake magnitude and epicentral distance, induced by multiple regression analysis are proposed for such the ground condition as the PWRI campus.

ARRAY INSTRUMENTATION AT PWRI

There are tow local laboratory arrays called Field-A and Field-B at the PWRI campus, as shown in Photo. 1. The Field-B locates about 600 m far away from the Field-A. The subsurface geological condition around the PWRI is almost uniform, i.e., diluvial sandy and silty deposits with approximate thickness of 50 m rest on gravel formations as shown in Fig. 1. The shear wave velocity of the upper diluvial and lower gravel deposits is approximately 250 m/s and 400 m/s, respectively. Thus the objective ground at these Fields belongs to the ground condition with the average stiffness according to the engineering view-point.



Photo. 1 Laboratoy Array at PWRI Campus



Fig. 1 Soil Profile around Fields-A and B

Fig. 2 shows the instrumentation at Field-A and Field-B. The 13 three-components accelerometers are installed at Field-A, that is, 3 on the ground surface, 5 at the depth of 2 m and 5 at the depth of about 50 m, along a cross shaped configuration with each length of 100 m. The 6 three-components accelerometers are installed at Field-B, that is, 1 at the depth of 2 m, 4 at the depth of about 50 m and 1 at the depth of 96 m, along a L shaped configuration with the length of 100 m and 50 m. The direction of the cross configuration at Field-A and Field-B as well as the direction of the installed at so the direction of the installed at field-B.

Signals from 19 accelerometers at both Field-A and Field-B are simultaneously transmitted by cable to the central processing room shown in Photo. 1, where the signals are digitized with a time interval of 1/100 second by 12 bits AD converters. The observation was started partially in July, 1979 and totally in December, 1980.

CALCULATION METHOD FOR GROUND STRAINS

Calculation Method

For estimating the ground strains. the acceleration records are converted to the displacement records by the double integration considering thefrequency domain of accelerometer. Based on the three-components displacement calculated at each observation point, the ground strains are induced as follows.

A tetrahedron consisting of 4 points (i, j, m and p) as shown in Fig. 3 is supposed to calculate the strains of the tetrahedron as the

ground strains. According to a standard thre dimensional finite element analysis procedur [Zienkiewicz (1971)], the ground displacemer u(t), v(t) and w(t) in x(East-West),



Fig. 2 Array Configuration at PWRI Campus

(North-South) and z(Up-Down) directions, espectively, at the free position with oordinates x, y and z in the tetrahedron are ssumed to be linear as:

 $u(t) = \alpha_{1} + \alpha_{2} X + \alpha_{3} Y + \alpha_{4} Z$ $v(t) = \alpha_{5} + \alpha_{6} X + \alpha_{7} Y + \alpha_{8} Z$ (1) $w(t) = \alpha_{9} + \alpha_{10} X + \alpha_{11} Y + \alpha_{12} Z$

here: α_{-i} (i=1 ~ 12) represent constants

etermining α , by prescribing coordinates of he four observation points i, j, m and p, Eq. 1) can be written in the form as:

$$\begin{cases} u(t) \\ v(t) \\ w(t) \end{cases} = \frac{1}{6V} \begin{bmatrix} u_{1}(t) & u_{J}(t) & u_{m}(t) & u_{P}(t) \\ b_{1}(t) & v_{J}(t) & v_{m}(t) & v_{P}(t) \\ w_{1}(t) & w_{J}(t) & w_{m}(t) & w_{P}(t) \end{bmatrix} \cdot H$$

$$(2)$$

here

$$H = \begin{cases} a_{1} + b_{1} x + c_{1} y + d_{1} z \\ a_{1} + b_{3} x + c_{3} y + d_{3} z \\ a_{m} + b_{m} x + c_{m} y + d_{m} z \\ a_{p} + b_{p} x + c_{p} y + d_{p} z \end{cases}$$

 x_{κ} , y_{κ} , z_{κ} (k=i, j, m, p): coordinates of k-th observation point

 $\begin{array}{c} u_{\kappa}(t), \ v_{\kappa}(t), \ w_{\kappa}(t) \ (k=i, \ j, \ m, \ p): \ ground \\ \ displacement \ calculated \ at \ k-th \\ \ observation \ point \end{array}$

$$a_{i} = \begin{vmatrix} x_{j} & y_{j} & z_{j} \\ x_{m} & y_{m} & z_{m} \\ x_{p} & y_{p} & z_{p} \end{vmatrix} \qquad b_{i} = -\begin{vmatrix} 1 & y_{j} & z_{j} \\ 1 & y_{m} & z_{m} \\ 1 & y_{p} & z_{p} \end{vmatrix}$$
(3)
$$c_{i} = -\begin{vmatrix} x_{j} & 1 & z_{j} \\ x_{m} & 1 & z_{m} \\ x_{p} & 1 & z_{p} \end{vmatrix} \qquad d_{i} = -\begin{vmatrix} x_{j} & y_{j} & 1 \\ x_{m} & y_{m} & 1 \\ x_{p} & y_{p} & 1 \end{vmatrix}$$

Other constants a_{κ} , b_{κ} , c_{κ} , and d_{κ} (k=j, m, p) can be obtained by changing the subscript in the order of j, m and p.

$$6V = \begin{bmatrix} 1 & x_{1} & y_{1} & z_{1} \\ 1 & x_{j} & y_{j} & z_{j} \\ 1 & x_{m} & y_{m} & z_{m} \\ 1 & x_{p} & y_{p} & z_{p} \end{bmatrix}$$
(4)

On the other hand, the strains at the free position in the tetrahedron are represented as:

$$\begin{array}{c} \partial \mathbf{v} / \partial \mathbf{z} + \partial \mathbf{w} / \partial \mathbf{y} \\ \partial \mathbf{w} / \partial \mathbf{x} + \partial \mathbf{u} / \partial \mathbf{z} \end{array}$$

Substitution of Eq.(2) into Eq.(5) gives the ground strains as:

$$\{ \varepsilon \} = \frac{1}{6V} [B] \{ \delta \}$$

$$= \frac{1}{6V} [B_{\downarrow}, B_{\downarrow}, B_{m}, B_{P}] \{ \delta \}$$
(6)

where

$$\begin{bmatrix} \mathbf{B}_{\kappa} \end{bmatrix} = \begin{bmatrix} \mathbf{b}_{\kappa} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{c}_{\kappa} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{d}_{\kappa} \\ \mathbf{c}_{\kappa} & \mathbf{b}_{\kappa} & \mathbf{0} \\ \mathbf{0} & \mathbf{d}_{\kappa} & \mathbf{c}_{\kappa} \\ \mathbf{d}_{\kappa} & \mathbf{0} & \mathbf{b}_{\kappa} \end{bmatrix} \qquad (\mathbf{k}=\mathbf{i}, \mathbf{j}, \mathbf{m}, \mathbf{p}) \qquad (7)$$

$$\left\{ \boldsymbol{\delta} \right\} = \begin{bmatrix} \boldsymbol{\delta} & \mathbf{i} \\ \boldsymbol{\delta} & \mathbf{j} \\ \boldsymbol{\delta} & \mathbf{m} \\ \boldsymbol{\delta} & \boldsymbol{\rho} \end{bmatrix} \qquad (8)$$

$$\{\delta_{\kappa}\} = \begin{cases} u_{\kappa} \\ v_{\kappa} \\ w_{\kappa} \end{cases} \qquad (k=i, j, m, p) \qquad (9)$$

As mentioned above, the ground strains can be estimated with use of the acceleration records observed at 4 points. However, it should be noted that the ground strains { ε } estimated by Eq. (6) represent the average ground strains in a tetrahedron because the displacements are assumed to be linear as shown in Eq. (1), and further , the strains $\varepsilon \times$, $\varepsilon \times$ and $\gamma \times$ for the tetrahedron shown in Fig. 3 are not affected by the Point-p but only three Points-i, j and m when the i-j-m plane is parallel to the x-y plane.



Fig. 3 Tetrahedron for Calculation of Ground Strains

Formation of Tetrahedrons for Observation Points

The 8 tetrahedrons are formed based on the 10 observation Points to calculate the ground strains in the upper level ground and lower level ground with the depth of GL-2m to GL-46m at Field-A as shown in Figs. 4(a) and 4(b), respectively. The 4 upper-side tetrahedrons (Zone I ~ IV) are formed by the 6 observation Points-A2CO, A2E2, A2N2, A2W2, A2S2 and A46C0 to calculate the strains at the upper level ground, and the 4 lower-side ones(Zone V ~ VM) are by the 6 Points-A46C0, A47E2, A47N2, A46W2, A45S2

(5)

and A2C0 at the lower level ground. With use of these 4 upper-side and 4 lower-side tetrahedrons, the ground strains ε_{\times} , ε_{\vee} and $\gamma_{\times \nu}$ for each tetrahedron, that is, at the depth with GL-2m and GL-46m, can be estimated, respectively.

Further, the 2 tetrahedrons are considered to calculate the ground strains in the lower level ground with the depth of GL-2m to GL-53m and GL-53m to GL-96m at Field-B as shown in Fig. 4(c). The tetrahedron as Zone X is formed by the 4 observation Points-B53C0, B52W1, B53S1 and B2CO, and the one as Zone X is by B53C0, B52W1, B53S1 and B96C0. Based on these tetrahedrons, the ground strains ε_{X} , ε_{Y} and γ_{XY} for each tetrahedron, that is, at the depth with GL-53m, can be estimated.



(a) Upper Level Ground at Field-A



(b) Lower Level Ground at Field-B



(c) Lower Level Ground at Field-B

Fig. 4 Tetrahedrons Formulated to Calculate Ground Strains

CALCULATION FOR GROUND STRAINS DURING EARTHQUAE

The array data have been obtained at the PWF campus during past 100 earthquakes between 19° and 1989. Among those data, the data obtained during 78 earthquakes with the Japa Meteorological Agency (JMA) magnitude of 4.0 c greater are used for this analysis. Fig. shows the relation between a magnitude (M) ar an epicentral distance (\bigtriangleup) for 78 earthquake that is, the magnitude distributes within 4 to 7.9 and the epicentral distance distribute within 3 km to 758 km. It can be noted that the earthquake with large magnitude and shor epicentral distance has not occurred so muc around the observation site.







-g. 6 shows the time histories of the threeomponents accelerations recorded at the Point-2CO(GL-2m) and Point-A46CO(GL=-46m) for the reld-A during the earthquake of February 27, 983 (EQ-28), with a JMA magnitude of 6.0 and an picentral distance to the site of 22km. The nplification ratio of maximum acceleration 2CO/A46CO is 2.04[101.3gals/49.7gals] and 2.61 92.7gals/35.5gals] for N-S component and E-W pmponent, respectively.

ig. 7 shows the time histories of the ground isplacement at the Points-A2C0 and A46C0 for Q-28, which are calculated by the double ntegration of acceleration records. In this nvestigation, acceleration records are ntegrated frequency domain with the lower and igher cut-off frequency of 0.2 Hz and 20 Hz. The amplification ratio of maximum displacement 2C0/A46C0 is 1.39 [1.29cm/0.93cm] and 2.05 1.80cm/0.88cm] for N-S and E-W component, espectively.



Fig. 8 shows the typical distribution of the maximum acceleration observed at the Point-A2C0 during 78 earthquakes. From this figure, it can be seen that the maximum acceleration is distributed within 5 gals to 130 gals, but more than 90 % of the data is less than 50 gals. Fig. 9(a) and (b) show the time histories of the upper and lower level ground strains, $\varepsilon \times$, $\varepsilon \times$ and $\tau \times \tau$ calculated in the Zones I and V at Field-A for EQ-28, respectively. As seen from Fig. 9, the maximum value of normal strains

 $(\varepsilon_{\times}, \varepsilon_{\times})$ at the upper level ground is $(100 \sim 200) \times 10^{-6}$ and larger than that at the lower level ground, which is about 50×10^{-6} . The maximum value of the shear strain (γ_{\times}) at the upper level ground is about 100×10^{-6} and larger than that at the lower level ground, which is about 50×10^{-6} .



Fig. 8 Maximum Acceleration Observed at Point-A2C0



ATTENUATION CHARACTERISTICS OF GROUND STRAINS

Practical Formula for Ground Motion

It is very important to estimate seismic effect properly in the practical design of structures. Up to the present, not a few attenuation equations of peak ground motions (acceleration, velocity and displacement) have been proposed and they are applied to the seismic design of structures.

the analyses Tn past on attenuation characteristics of maximum ground motions and absolute acceleration response spectra, based on acceleration records, the following empirical formula is often used as a practical one [Katayama et. al. (1978)].

 $X = a \times 10^{bM} \times (\varDelta + \varDelta \Box)^{C}$ (10)where

- ${\tt X}$: Maximum acceleration, velocity and displacement / absolute acceleration response spectral amplitude M : Magnitude of earthquake ∠ : Epicentral distance [km] \triangle \Box : Constant to adjust X for small epicental distance
 - a, b, c : Coefficients

Kawashima et. al. (1984) induced the following formulae for estimating the maximum acceleration ($\alpha_{\rm max}$) on the ground surface, with use of 394 components of acceleration records by SMAC accelerograph during past 88 earthquakes in Japan.

$$\alpha_{\max} = \begin{cases} 987.4 \times 10^{\circ.216M} \\ [for Group I] \\ 232.5 \times 10^{\circ.313M} \\ [for Group II] \\ 403.8 \times 10^{\circ.265M} \\ [for Group II] \\ (11) \end{cases}$$

where: Group I, ${\rm I\!I}$ and ${\rm I\!I\!I}$ indicate the classification of ground condition, i.e., rock or diluvium, alluvium and soft alluvium or reclaimed land, respectively.

According to the Eq. (10), the formula for the maximum acceleration on the ground at the PWRI campus is induced with use of 76 observation records obtained only for Point-A2C0, as follows:

$$\alpha_{\text{mex}} = 9.344 \times 10^{\circ.511M} \times (\ \ +30)^{-1.252}$$
(12)

This formula is also indicated in Fig. 8, and should be noted that the acceleration calculat by Eq. (12) is smaller than that by Eq. (11).

Application for Ground Strain

The attenuation characteristics of grou strains on the horizontal plane ε_{x} , ε_{y} a $\gamma_{\times \nu}$, which are important to be considered the seismic design of underground structure are discussed in this analysis. Since the pe values of ground strains are different among t tetrahedrons, the average of the peak grou strains over the 4 (Field-A) or 2 (Fieldtetrahedrons is defined as the maximum grou. strains. Further, the larger value of $\varepsilon \times a$ ε , is defined as the maximum normal strain ε The same expression with Eq.(10) is assumed represent the attenuation characteristics maximum ground strains, and riangleq in Eq.(10) assumed to be 30 km. Then the empiric formulae of maximum ground strains a written as follows:

where

 ε : Maximum normal strain γ : Maximum shear strain

The coefficients a, b and c are obtained multiple regression analysis for the maxim strains calculated, that is ε and γ (= γ_{xy} as shown in Table 1. From Table 1, t attenuation formulae of the maximum grour strains are obtained as follows:

```
Upper Level ground(GL-2m) at Field-A:
   ε =1.237 × 10<sup>°. 493M</sup> × ( ∠ +30)<sup>-°. 741</sup> × 10<sup>-6</sup>
   \gamma = 0.894 \times 10^{0.54BM} \times ( \ 2 + 30)^{-0.774} \times 10^{-6}
Lower Level ground(GL-46m) at Field-A:
```

 ε =1.285 \times 10^{°. 309M} \times (\angle +30) ^{°°. 370} \times 10^{°°} $\gamma = 1.549 \times 10^{\circ} \times 20^{\circ} \times (\ 2 + 30)^{-\circ} \times 10^{-\circ} \times 10^{-\circ}$

Lower Level ground(GL-53m) at Field-B:

ε =1.506 × 10^{°. 358M} × (∠ +30)^{-°. 569} × 10⁻⁶ γ =4.860 × 10^{0.312M} × (\triangle +30)^{-0.596} × 10⁻⁶ (14)

Fig. 10 shows the attenuation of the maximu ground strains ϵ and γ calculated by Eq. (E for each event, together with predicted value b

							and the second se	
Field	Strain		Coefficient			Correlation	Standard	Number
TICIU			а	b	с	Coefficient	Error	of Data
А	Upper Level	ε	1.237	0.493	-0.741	0.829	0.206	65
	Ground Strain	γ	0.894	0.548	-0.774	0.856	0.210	65
	Lower Level	ε	1.285	0.309	-0.370	0.708	0.213	74
	Ground Strain	γ	1.549	0.293	-0.319	0.699	0.212	76
В	Lower Level	ε	1.506	0.358	-0.569	0.736	0.197	66
	Ground Strain	γ	4.860	0.312	-0.596	0.690	0.190	53

Table 1 Coefficients of Attenuation Equation

Attenuation Equation : $\frac{\varepsilon}{\tau}$ } = a × 10^{bM} × (\varDelta + 30)^c × 10⁻⁶

q. (14). The followings are pointed out from ig. 10.

) The maximum normal strains at the upper level ground and the lower level ground at Field-A are distributed in the range from 5×10^{-e} to 200 \times 10⁻⁶ and from 4 \times 10⁻⁶ to 60 × 10^{-6} . respectively. The maximum shear strains are distributed in the range from 5 \times 10 $^{-6}$ to 350 \times 10 $^{-6}$ and from 4 \times 10 $^{-6}$ to 100 \times 10 $^{-6}$ for the upper level ground and the lower level ground, respectively. The maximum normal and shear strains for the lower level ground at Field-B are distributed in the range from 3×10^{-6} to 70×10^{-6} and from 6×10^{-6} to 100×10^{-6} , respectively. Thus the maximum shear strain is larger than the maximum normal strain at the same depth,

and the upper level ground strain is larger than the lower level ground strain.

2) According to the empirical attenuation equations of ground strain at Field-A, the coefficient b, which represents the effect of earthquake magnitude on the maximum ground strains, of the upper level ground is larger than that of the lower level ground. The coefficient c, which represents the effect of epicentral distance on the maximum ground strains, of the upper level ground is smaller than that of the lower level ground. Those facts indicate that the strains in the

upper level ground are more sensitive to rate earthquake magnitude, and its attenuation with epicentral distance is larger, as compared with that in the lower level ground.





Fig. 10 Attenuation Characteristics of Maximum Ground Strains (Continued)

- 3) Comparing the empirical formulae of attenuation of the strains in the lower level ground at Field-A with that at Field-B, the coefficient b is almost same, and the coefficient c for Field-A is a little lager than that for Field-B.
- 4) Compared with the coefficient c of the attenuation equations for maximum ground accelerations based on SMAC accelerograph, which is about -1.2[see Eq. (11)], the coefficient c for maximum ground strains is larger. This means that the attenuation rate of maximum ground strains with epicentral distance is smaller than that of maximum ground accelerations.

CONCLUSION

The ground strains induced during earthquakes were evaluated by a finite element method, with use of the dense instrument array data obtained at the Public Works Research Institute. The empirical formulae of attenuation equation of maximum ground strains (Eq.(14)) were presented by multiple regression analysis based on the observed data of 78 earthquakes. The result of this study might be regarded as basic information for assessing the ground strains during earthquakes. However, it should be noted that those results were derived from the data recorded by relatively small ground motions. The accumulation of strong motion

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records and further investigations should be

REFERENCES

- Arakawa, T., Kawashima, K. and Tamura, F (1985), "Finite Ground Strains Induced durir. Earthquake for Application to Seismic Desig of Underground Structures", Proc. of Fifth International Conference on Numerical Methoc in Geomechanics, Nagoya.
- Katayama, T., Iwasaki, T. and Saeki, M. (1978) "Statistical Analysis on Acceleration Respons Spectrum", Proc. of JSCE, No.275(in Japanese)
- Kawashima, K., Aizawa, K. and Takahashi, F (1984), "Attenuation of Peak Ground Motion ar Absolute Acceleration Response Spectra", Proc of Eighth World Conference on Earthquake Engi neering, Tokyo.
- Ohkubo, T., Iwasaki, T. and Kawashima, K (1981), "Dense Instrument Array Program of th Public Works Research Institute an Preliminary Analysis of the Records", Proc. c 13th Joint Meeting U.S.-Japan on Wind an Seismic Effects, UJNR, Tokyo.
- Public Works Research Institute (1977), " Proposal for Earthquake Resistant Desig Method", Technical Memorandum of PWRI, No.118 (in Japanese).
- Sasaki, Y., Tamura, K. and Aizawa, K. (1989) "Analysis on the Finite Ground Strains Induce During Earthquakes", Proc. of Third U.S.-Japa Workshop on Earthquake Disaster Prevention fo Lifeline System, UJNR, Tokyo.
- Tokida, K., Tamura, K. and Aizawa, K. (1990) "Attenuation Characteristics of Ground Strain During Earthquake Based on Dense Array", Th Eighth Japan Earthquake Engineering Symposiu 1990, Tokyo (under contribution).
- Zienkiewicz, O. C. (1971), "The Finite Elemen Method in Engineering Science", McGraw-Hill.