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# PREDICTION OF VOLUMETRIC STRAIN FOR SAND UNDER CYCLIC LOADING

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## ABSTRACT

In order to investigate volume change of sand due to cyclic loading, different tests were performed on hollow cylindrical specimens under drained cyclic simple shear condition. Correlation of accumulated plastic shear energy with volumetric strain and effects of different parameters such as initial confining pressure, initial anisotropic consolidation and strain amplitude on this correlation were studied. Test results showed that volumetric strain of sand under different initial confining pressures was almost same. Volume change of specimens with different initial anisotropic consolidations was also identical, except for initial stage of shearing. It was found that the correlation of volumetric strain and normalized energy was similar for different initial confining pressures and anisotropic consolidation.

Test results showed that shear strain amplitude affected the volumetric strain-energy correlation. This effect was not significant when volume change was smaller. An average correlation of volumetric strain and energy can be used to predict volume change of sand under different strain amplitudes.

## INTRODUCTION

Study on volume change of sand under cyclic loading in laboratory tests provides very useful information for estimating subsidence of soil structure during or after dynamic loads such as earthquake shaking. Pore water pressure build-up during undrained cyclic loading is also one of the most important issues that affect the soil behavior and is being used in modeling of strength and deformation. Volume contraction during cyclic loading may be considered equivalent to pore-water pressure build-up in the course of undrained loading.

Some researchers (Nemat-Naser, S. and Shokooh, A., 1979) have expressed volume change or excess pore water pressure as a function of accumulated shear energy. Based on the results of several cyclic undrained tests, Towhata and Ishihara (1985) concluded that there is a characteristic relationship between shear energy and pore water pressure buildup at each state of shearing. This correlation was used for modeling of sand behavior under undrained cyclic loading.

In this research, a series of drained cyclic torsion shear tests were performed to investigate the different aspects of sand volume change under cyclic loading. Correlation of volume change with accumulated plastic shear energy was also studied. Different tests were performed to examine the effects of different parameters on this correlation such as initial confining pressure, anisotropic consolidation and strain amplitude.

If a unique correlation between volumetric strain and

accumulated energy could be found for sand under different conditions then this correlation can be used for prediction of volumetric strain at different stages of shearing.

## APPARATUS, SPECIMENS AND SHEARING METHOD

A hollow cylindrical torsion shear apparatus was used in this study. In this apparatus vertical load, inner and outer cell pressures and torsion shear were dynamically applied to the specimen with 19.5cm in height, 10cm in outer and 6cm in inner diameters. To ensure accurate measurement of the specimen volume change, an electronic balance was used in place of a differential pressure transducer (Pradhan et al 1986). Toyoura sand was used which mainly consists of quartz (around 90%) and chert (around 4%). Its physical properties were  $G_s = 2.65$ ,  $D_{50} = 0.16$  mm,  $U_c = 1.46$ ,  $e_{max} = 0.977$  and  $e_{min} = 0.597$ . Specimens were prepared by pluviating air-dried sand particles through the air. By changing the drop-height and rate of pluviation, specimens with different relative densities were prepared (22% to 76%). Carbon dioxide (CO<sub>2</sub>) and subsequently de-aired water were percolated through the specimen to achieve a high degree of saturation. After saturation a back-pressure of 100kPa was applied to the specimen to achieve saturation with a B-value exceeding 0.98. The specimen was then consolidated under different confining pressures and different anisotropic consolidation ratios

( $K=\sigma'_r/\sigma'_z$ ) which are listed in Table 1. Specimens were sheared after consolidation under drained cyclic torsion simple shear mode. The simple shear mode of deformation is defined as the one in which the increment of radial strain ( $d\varepsilon_r$ ) and the circumferential strain ( $d\varepsilon_c$ ) are kept zero. In these tests vertical stress was also kept constant. Stresses in hollow specimens under simple shear condition are shown in Fig.1. This mode of shearing can be generated by independent control of the axial force, the outer and the inner cell pressures in hollow torsion shear apparatus. In each test a large number of cycles were applied to the specimen to cause a large volumetric strain.

Table. 1. List of drained cyclic torsion shear tests.

Test	e <sup>(1)</sup>	Dr <sup>(2)</sup>	p' <sup>(3)</sup>	K <sup>(4)</sup>	$\gamma_{zt}$
no161	0.756	58	98	1	± 3.0%
no162	0.832	38	98	1	± 3.0%
no164	0.892	22	98	1	± 3.0%
no165	0.885	24	98	1.6	± 3.0%
no166	0.694	75	98	1	± 3.0%
no167	0.76	57	98	0.3	± 3.0%
no168	0.754	59	184	1	± 3.0%
no169	0.76	57	98	2.5	± 3.0%
no171	0.762	57	53	1	± 3.0%
no172	0.888	23	98	0.6	± 3.0%
no173	0.884	24	98	1.7	± 3.0%
no174	0.887	24	67	1	± 3.0%
no174-2	0.695	74	98	1	± 1.0%
no175	0.889	23	135	1	± 3.0%
no176	0.888	23	98	1	± 1.0%
no177	0.763	56	98	1	± 1.0%
no179	0.834	38	98	1	± 0.5%
no188	0.833	38	98	1	± 1.0%
no189	0.84	36	98	1	± 6.0%
no190	0.839	36	98	1	3(±3.0%)+100(±1.0%)+n(±3.0%)

- 1) Void ratio after consolidation and before shearing
- 2) Relative density (%)
- 3) Mean effective consolidation pressure (kPa)
- 4) Anisotropic consolidation ratio ( $K=\sigma'_r/\sigma'_z$ )

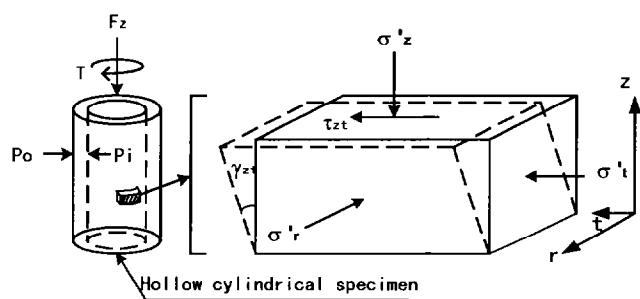


Fig. 1. Stress components in hollow torsional shear test

### CALCULATION OF VOLUMETRIC STRAIN INCREMENT AND PLASTIC TORSION SHEAR ENERGY

Volumetric strain increment  $d\varepsilon_v$  consists of a component  $d\varepsilon_v^d$  induced by plastic shear strain increment and the other component  $d\varepsilon_v^c$  induced by the change in mean effective confining pressure,  $p'$ . For any change of  $p'$  the value of  $d\varepsilon_v^c$  was calculated by using equation (1). In this equation  $m_v$  is the compression or swelling coefficient for each effective confining pressure.

$$m_v = d\varepsilon_v^c / d\log_{10} p' \quad (1)$$

Values of  $m_v$  in different confining pressure levels were calculated by using the results of some other isotropic compression tests, which were performed on hollow cylindrical specimens by authors. Measured volume change was also corrected for membrane penetration error based on the results of these tests.

Plastic torsional shear energy increment was calculated by equation (2):

$$dW^p = \tau_{zt} d\gamma_{zt}^p \quad (2)$$

in which  $d\gamma_{zt}^p$  is plastic shear strain increment and was calculated by equation (3):

$$d\gamma_{zt}^p = d\gamma_{zt} - d\gamma_{zt}^c \quad (3)$$

In this equation,  $\gamma_{zt}^c$  is the elastic component of shear strain and was evaluated by using the value of maximum tangent shear modulus ( $G_{max}$ ). For calculating maximum tangent shear modulus a mathematical equation was fitted to the stress-strain curves of different cycles. Derivative of this equation at the start of loading or loading reversal was considered as  $G_{max}$ .

### CORRELATION OF VOLUMETRIC STRAIN AND ACCUMULATED SHEAR ENERGY

In this section, results of one test, with isotropic consolidation is discussed in detail as a control case. Results of other tests will be discussed in the following sections where the effects of confining pressure, anisotropic consolidation and strain amplitude on volumetric strain, are studied.

Fig. 2a shows the stress-strain graph of specimen no162 with initial density of 38 percent under 30 cycles of three-percent shear strain amplitude. In this figure  $e_{start}$  and  $e_{end}$  are the void ratio of sand before shearing and at the end of test. Fig.2b reveals the volume change of this specimen. It can be seen in this figure that volume change accumulates by the number of cycles. One of the direct consequences of larger volumetric strain is the increase of the shear stress amplitude, which can be seen in Fig. 2a. Increase of shear stress amplitude under constant shear strain means the stiffening of sand under drained cyclic loading. Fig. 2b indicates that soil contraction in the initial cycles was more significant than in the following cycles. For example in this test, almost fifty percent of ultimate volumetric strain in thirty cycles, was generated in the first five cycles. Fig. 2c shows relation between volumetric strain and accumulated shear energy. Rate of volumetric strain in initial cycles was higher but it decreased with the number of cycles.

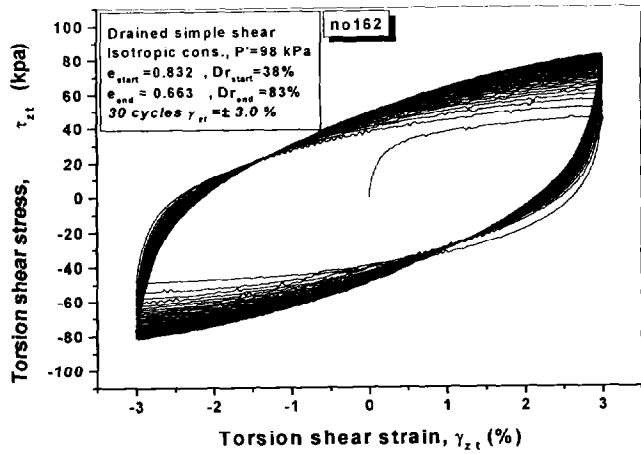


Fig. 2a. Stress-strain curves of initially medium loose sand under cyclic loading

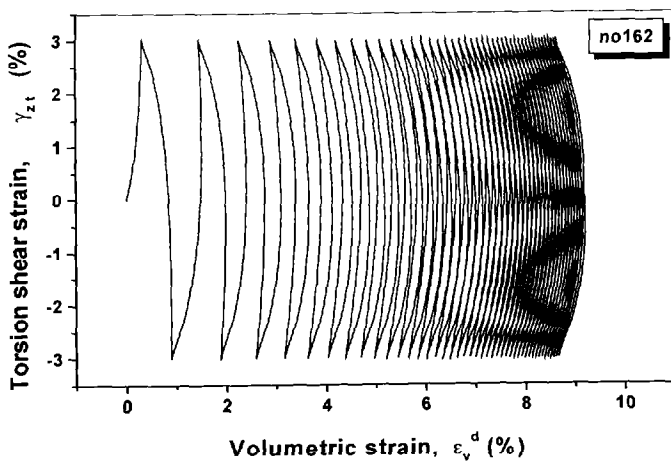


Fig. 2b. Volumetric strain of initially medium loose sand under cyclic loading

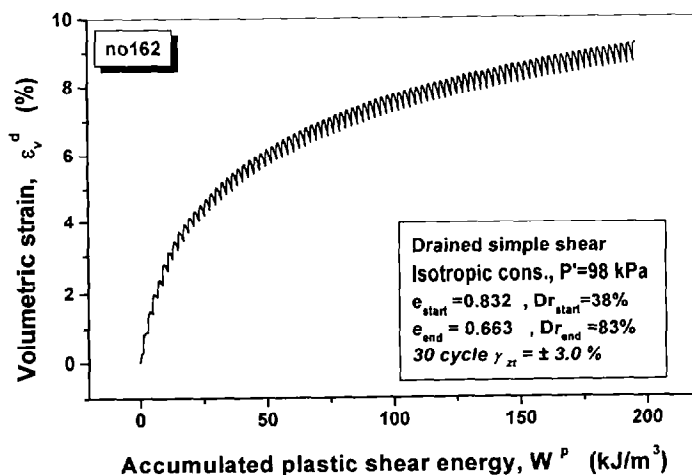


Fig. 2c. Relation between volumetric strain and energy for initially medium loose sand under cyclic loading

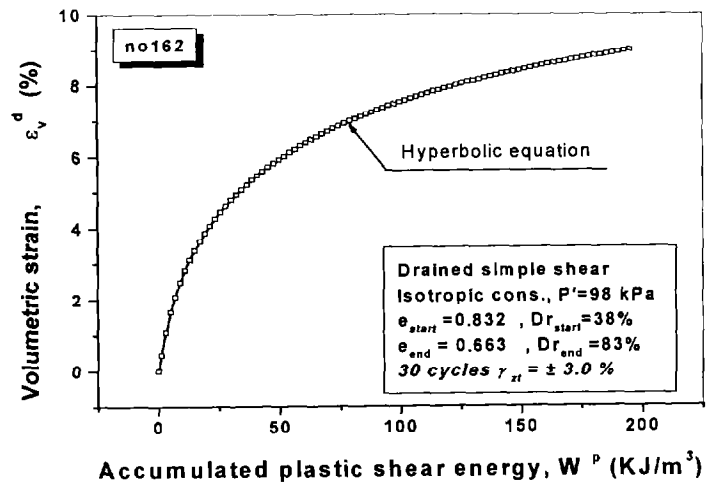


Fig. 2d. Relation of volumetric strain and energy at points where  $\tau_{zt}=0$

In Fig.2c data from all the stages of shearing were plotted. As the study of volumetric strain and its energy correlation inside the cycles is a complicated issue, study will be done only at the points where torsion shear stress is zero. Graph of volumetric strain versus energy at these points is almost a smooth curve without any remarkable fluctuation (Fig. 2d).

A hyperbolic equation may be considered for test data in Fig. 2d. It seems that for large number of cycles (higher values of energy), there is an upper bound for volumetric strain.

#### EFFECTS OF INITIAL CONFINING PRESSURE ON VOLUMETRIC STRAIN-ENERGY CORRELATION

In order to examine the effects of initial confining pressure on volumetric strain and its correlation with the accumulated energy, several tests were performed under different initial confining pressures. Simple shearing condition associated with change of the outer and the inner cell pressures to keep the cross section of the specimen constant. Due to variation of cell pressures and consequently radial and tangential stresses, ratio of  $K=\sigma'_r/\sigma'_{\theta}$  changed. Fig. 3 illustrates the history of radial stress for loose specimens with different initial isotropic confining pressures. At initial stages of shearing, radial stress decreased to some level very fast, and in the subsequent cycles fluctuated with gradual increase of its average level. In the first cycle the value of  $K$  changed from one to about 0.62 for these specimens.

Fig 4.a compares the volumetric strain of medium dense sand under different confining pressures. Volumetric strain of specimens under different initial confining pressures was almost identical after 30 cycles. This means that the initial confining pressure did not have important effects on volume change.

Although the volumetric strain of specimens under different initial confining pressure was same, its correlation with energy was different (see Fig. 4b). As it is shown in Fig. 5a, sand stiffness was higher under higher levels of confining pressure. Therefore it is clear that accumulated shear energy after the same number of cycles is different for similar strain amplitude under different confining pressure. Shahnazari (1999) showed

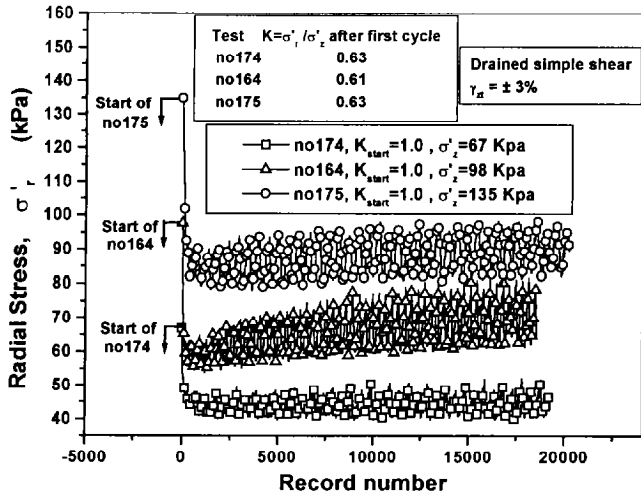


Fig. 3. Change of radial stress in drained simple shear condition with different initial isotropic confining pressures

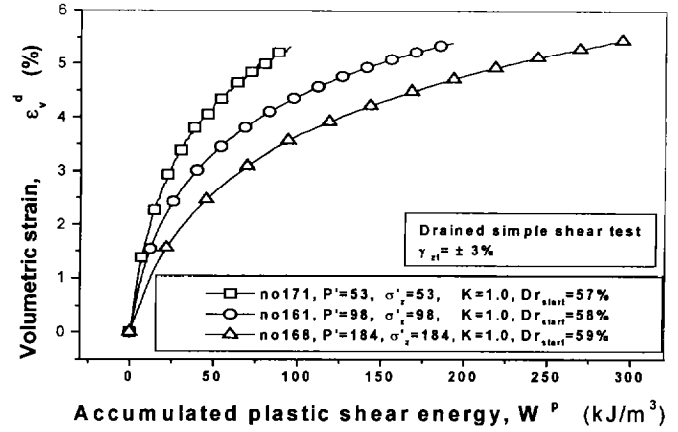


Fig. 4b. Volumetric strain and energy relations for initially medium loose sand under different initial confining pressures

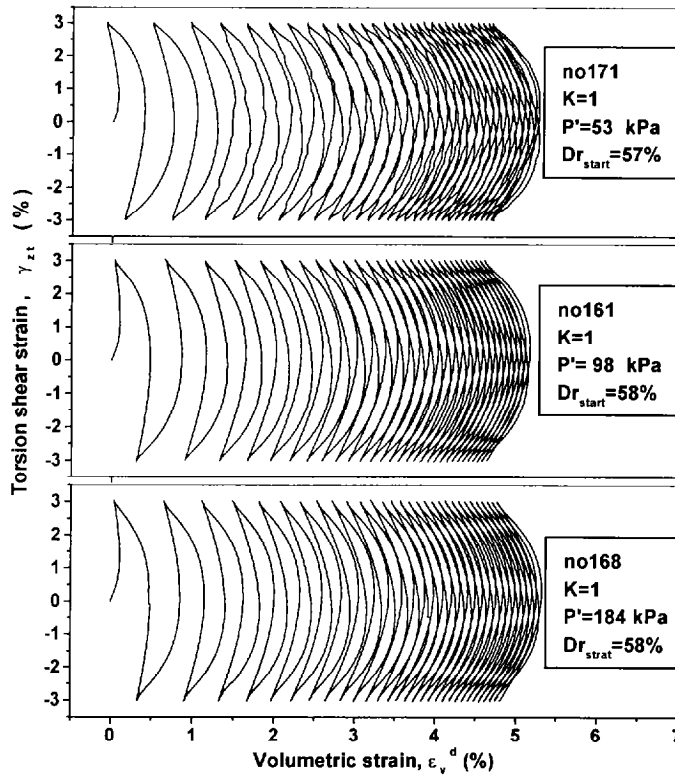


Fig. 4a. Effect of initial confining pressure on volume change of medium dense sand

that normal stress to shear plane ( $\sigma'_z$ ) is the main parameter which affects the soil stiffness in drained simple shear condition. It seems that stress-strain relationships are similar after normalizing shear stress by the value of vertical stress (Fig. 5b). It was also found that  $\epsilon_v^d - W^p$  correlation was identical when energy was normalized by a function of vertical stress. Fig. 6 shows this correlation after normalization. In this Figure  $P_{atm}$  is a reference pressure such as atmospheric pressure to

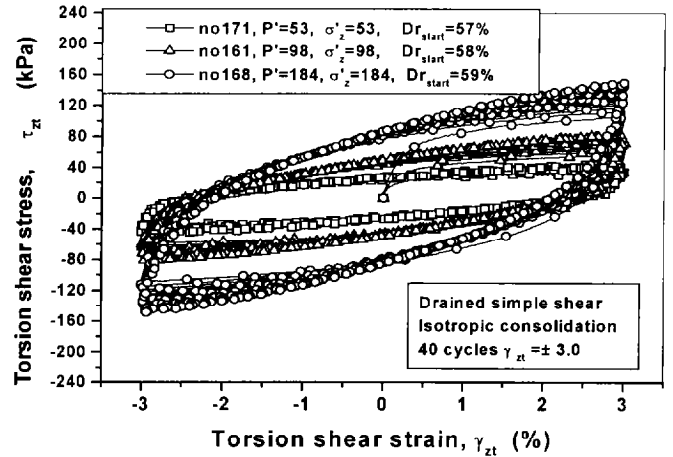


Fig. 5a. Stress-strain graph of medium dense sand under different initial confining pressure

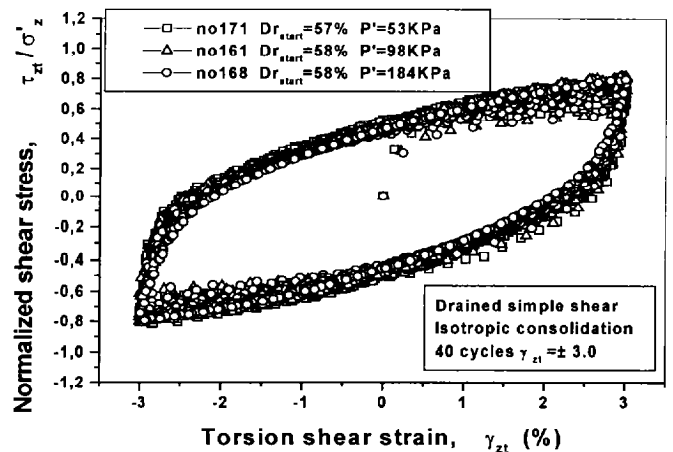


Fig. 5b. Normalized Stress-strain graph of medium dense sand under different initial confining pressure

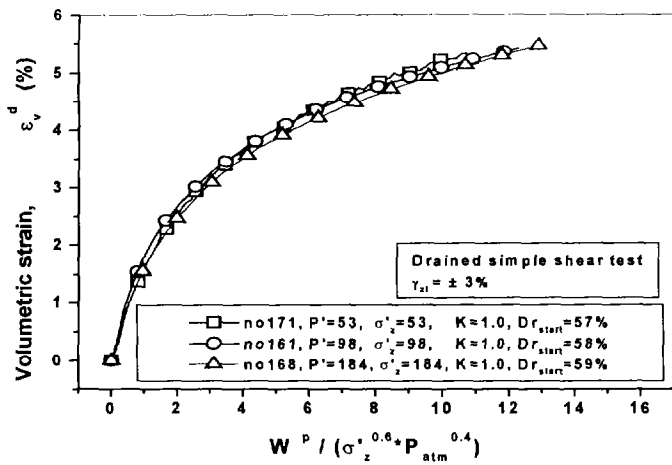


Fig. 6. Volumetric strain and normalized energy relations for initially medium loose sand under different initial confining pressures

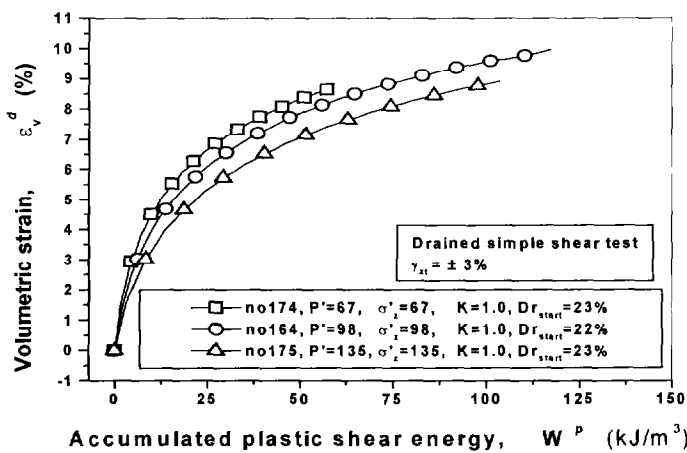


Fig. 7a. Effect of initial confining pressure on volumetric strain and energy relation for initially loose sand

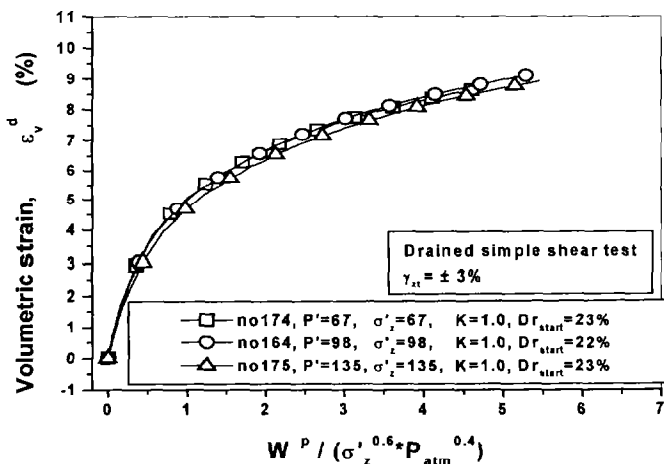


Fig. 7b. Volumetric strain-normalized energy correlation of initially loose sand under different initial confining pressures

make dimensionless normalized energy. It can be seen that by this type of normalization the effect of initial confining pressure on the  $\epsilon_v^d$ - $W^p$  correlation vanished.

Figure 7a compares  $\epsilon_v^d$ - $W^p$  relations for loose sand under different initial confining pressures. Correlation of volumetric strain and normalized energy by a function of vertical stress is shown in Fig. 7b. Also this figure suggests that the correlation of volumetric strain and normalized energy is independent of initial confining pressure.

## EFFECTS OF INITIAL ANISOTROPIC CONSOLIDATION ON VOLUMETRIC STRAIN-ENERGY CORRELATION

Different tests were performed to investigate the effects of initial anisotropic consolidation on volumetric strain and its relation with accumulated shear energy.

Figs. 8a and 8b compare volumetric strain of loose specimens under different anisotropic consolidation ratios ( $K=\sigma'_r/\sigma'_z$ ) in the first and the subsequent cycles of shearing. Volume change of sand in the first cycle of loading was different due to different initial anisotropy. But for the following cycles, almost the same volume change accumulated in specimens with different initial anisotropic consolidation ratios.

In the previous section it was explained that for isotropically consolidated specimens, radial stress changed very fast at the beginning of simple shearing and after this fast change, ratio of  $K$  decreased to about 0.62. Test results of anisotropically consolidated specimens also showed similar trend for radial stress. Fig. 9 shows the change of radial stress for specimens with different initial  $K$  values. Also in this case,  $K$  was about 0.62 at the end of the first cycle. Shahnazari (1999) showed that the vertical stress is the main parameter, which controls the behavior of anisotropically consolidated specimens under drained simple shear condition after the first cycle. This means that two specimens with the same  $\sigma'_z$  but different initial  $K$  values have identical behaviors in simple shear condition except for the first cycle. Therefore it seems that the  $\epsilon_v^d$ - $W^p$  relations will be identical for different anisotropic consolidation ratios (including  $K=1$  or isotropic consolidation) after normalizing energy by a function of vertical stress. This fact is shown in Figs. 10a and 10b for a loose specimen.

Fig. 11 reveals the relations between volumetric strain and normalized accumulated energy for medium dense specimens under different initial anisotropic consolidations. In this figure specimens with the same  $\sigma'_z$  and different  $K$  had a similar correlation of volumetric strain and normalized energy.

## EFFECTS OF STRAIN AMPLITUDE ON VOLUMETRIC STRAIN- ENERGY CORRELATION

Effects of strain amplitude on volumetric strain and its correlation with energy were studied by performing several tests under different shear strain amplitudes. In this research effect of strain amplitude was studied only in simple cases. Strain amplitude was changed in different tests but in most cases it was kept constant from the start to the end of a test. No effort was made to study effects of complicated irregular shear strain amplitude.

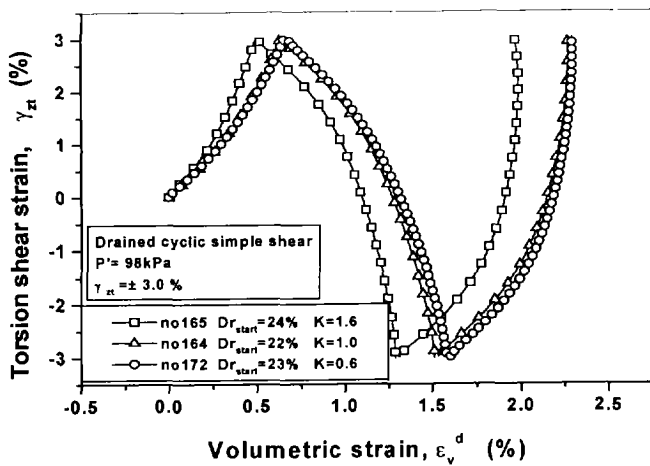


Fig. 8a. Effect of initial anisotropic consolidation on volume change of initially loose sand in first cycle

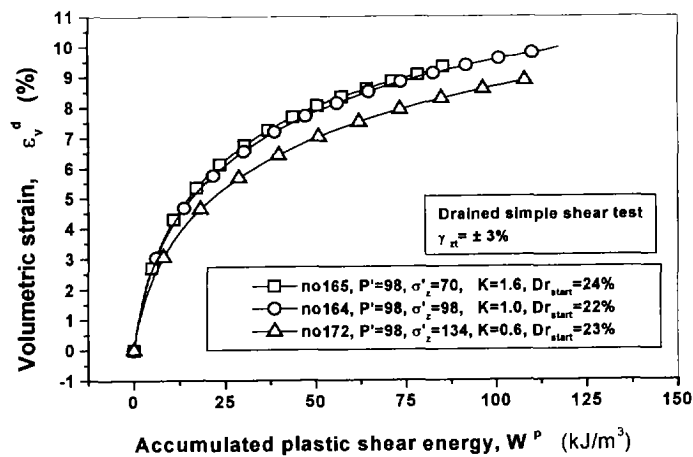


Fig. 10a. Volumetric strain and accumulated energy relations for loose sand under different anisotropic consolidations

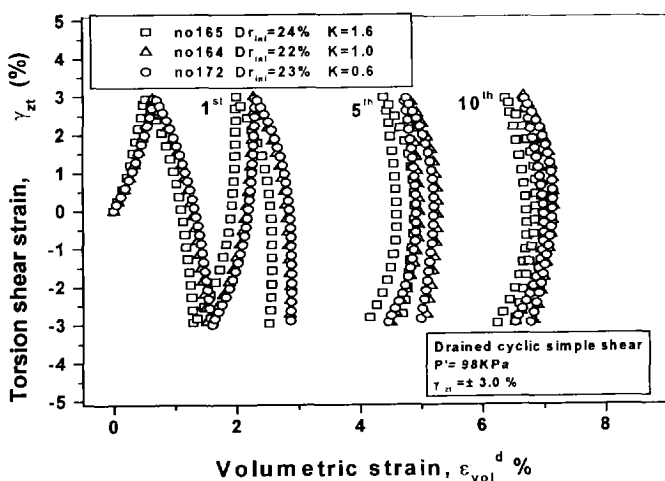


Fig. 8b. Effect of initial anisotropic consolidation on volume change of initially loose sand in different cycles

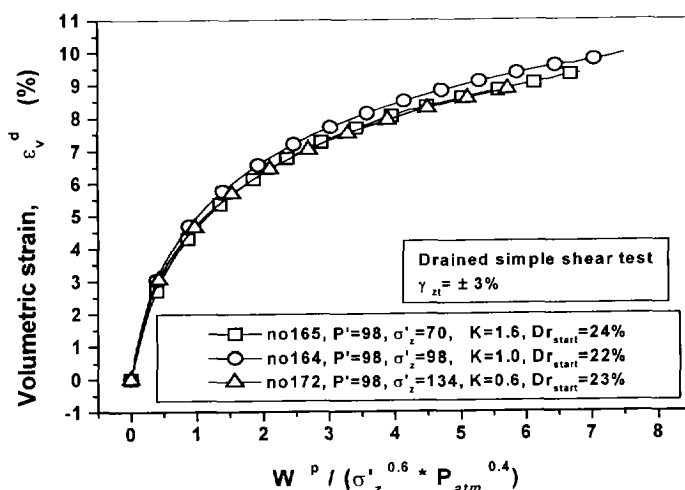


Fig. 10b. Volumetric strain and normalized energy relations for loose sand under different anisotropic consolidations

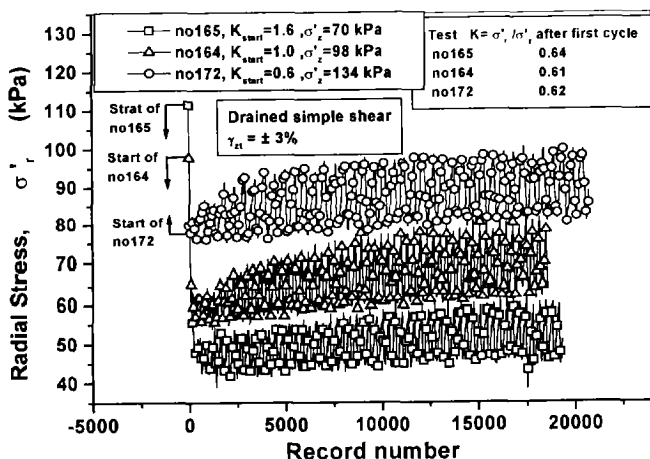


Fig. 9. Change of radial stress in drained simple shear condition under different initial anisotropic consolidation

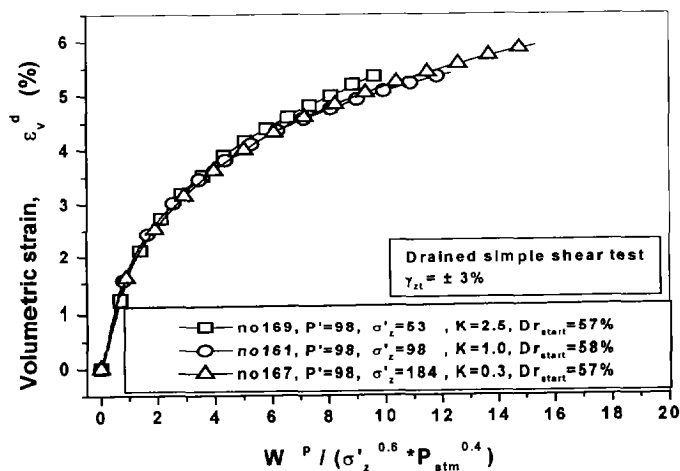


Fig. 11. Volumetric strain and normalized energy relations for medium dense sand under different initial confining pressure.

Fig.12a compares the volumetric strain of medium loose sand under different shear strain amplitudes. It is clear that volume change was larger for larger shear strain amplitudes. For example, after one cycle of six-percent shear strain amplitude two-percent volumetric strain accumulated. For generation of the same volumetric strain, 15 cycles of 0.5% shear strain amplitude were required. Relationship between volumetric strain and accumulated shear energy of these specimens are shown in Fig.12b. It can be seen that the correlation of volume change and energy was affected by shear strain amplitude. Fig. 13 reveals the volumetric strain- energy relation for medium dense specimens under different strain amplitudes. This figure also shows the effect of strain amplitude on  $\epsilon_v^d$ -  $W^p$  correlation. Figs 12b and 13 may suggest that for smaller value of volumetric strain, effect of strain amplitude on  $\epsilon_v^d$ -  $W^p$  relation is not remarkable. However in the range of larger volume change, sand had a larger volumetric strain under larger shear strain amplitude at a same value of energy.

Fig 14 illustrates  $\epsilon_v^d$ -  $W^p$  relationship for a medium loose specimen under variable shear strain amplitude. At the beginning of this test, three cycles of three-percent shear strain amplitude were applied and then the strain amplitude was reduced to one-percent (100 cycles). It was changed later to three-percent again (27 cycles). In this figure it can be seen that the slope of  $\epsilon_v^d$ -  $W^p$  graph changed by increase or decrease of strain amplitude. The graph was steeper for larger shear strain amplitude.

Test results suggest that when shear strain amplitude changes, only accumulated energy can not predict volume change of sand. It seems that energy must be normalized by or combined with another parameter to have a volumetric strain correlation independent of shear strain amplitude. Based on a limited number of experimental results, a proper type of energy normalization can not be easily founded, especially for irregular changes of strain amplitude. Therefore no further effort was made to do this in current paper.

It was mentioned before that the effect of strain amplitude is not remarkable in lower range of volumetric strain. An easy way to exclude the effect of strain amplitude on  $\epsilon_v^d$ -  $W^p$  relation can be use of an average correlation for different strain amplitudes. As it is shown in Figs.12b and 13, volume change of sand may be roughly predicted under different strain amplitudes, by using an average  $\epsilon_v^d$ -  $W^p$  correlation.

**CONCLUSIONS**

The following major conclusions are obtained from a series of drained cyclic simple shear tests:

- 1) Except for initial stage of shearing, initial confining pressure and anisotropic consolidation did not have any significant effect on volumetric strain of sand.
- 2) Initial confining pressure and anisotropic consolidation affected the relation between volumetric strain and accumulated shear energy. After normalizing energy by a function of effective vertical stress,  $W^p$ - $\epsilon_v^d$  relations were almost identical.

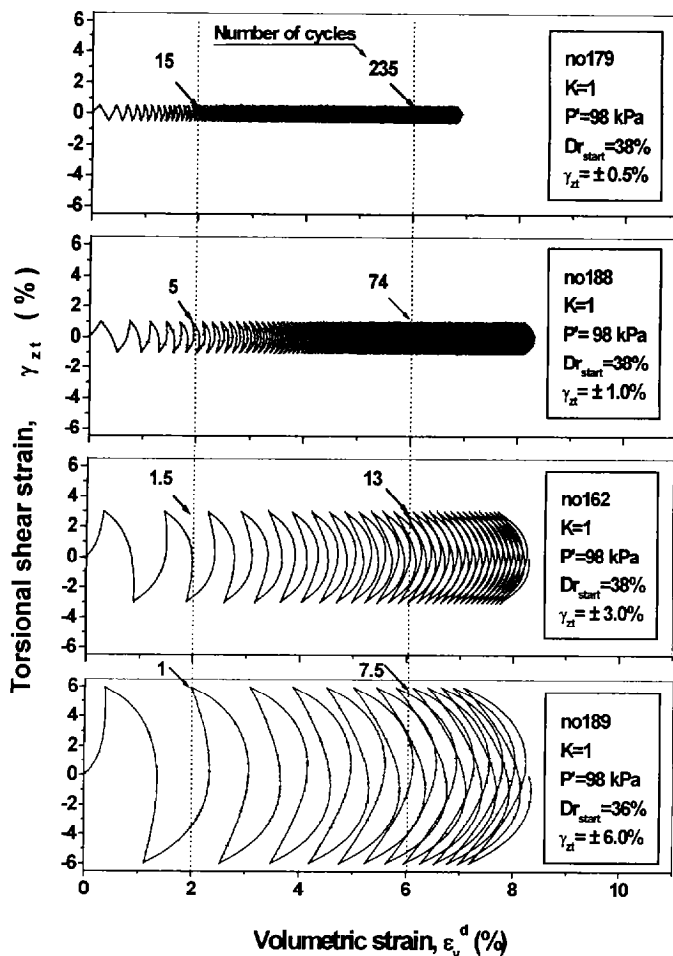


Fig. 12a. Volume change of medium loose sand under different shear strain amplitudes

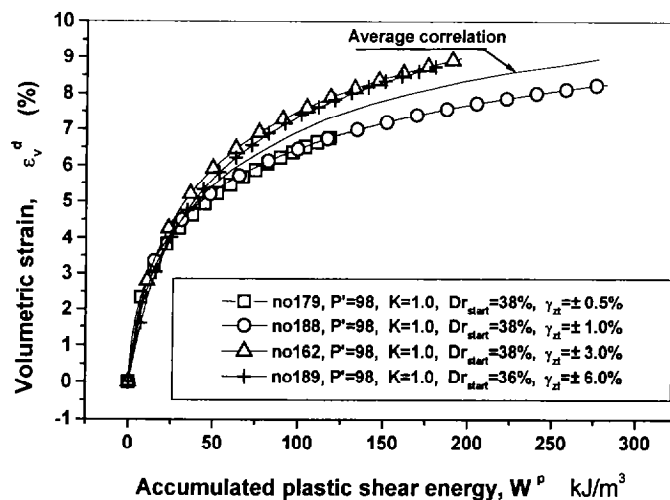


Fig. 12b. Volume change of medium loose sand under different shear strain amplitude



3) Shear strain amplitude affected the volume change and energy correlation mainly on the range of large volumetric strain. An average correlation can be used to predict the volumetric strain under different strain amplitudes.

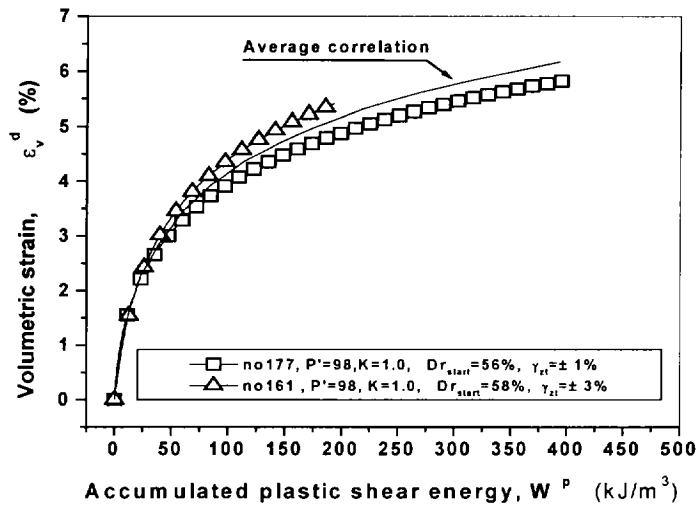


Fig.13. Energy-volumetric strain relation for medium dense sand under different strain amplitudes.

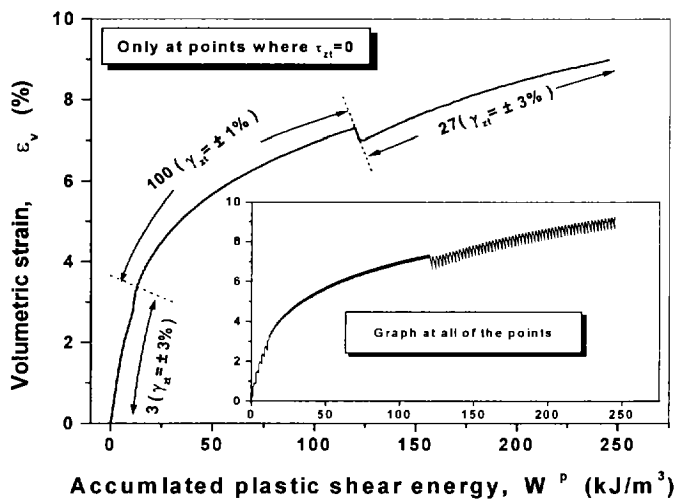


Fig. 14. Effect of strain amplitude on energy-volumetric strain correlation

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