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# Nonlinear Time Domain Numerical Model for Pile Group Under Transient Dynamic Forces

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SYNOPSIS: A computational model of soil-pile interaction behavior in pile and pile group was developed in this paper. Particular attension was paid to making the model simple and capable of taking into account nonlinear soil behavior, such as gapping and slippage between soil and pile, and cyclic behavior of soil. The model was developed within the frame work of the Winkler model defined in plane strain conditions. In order to analyze transient dynamic response in a rigorous manner, the model was formulated in the time domain using a step-by-step method. A transfer matrix approach was also adopted in the response computation. The proposed nonlinear model was verified with rigorous solutions and the nonlinear behavior with gapping and slippage were discussed based on the computational results.

## INTRODUCTION

The behavior of pile foundations subjected to dynamic loading, such as earthquakes, machine vibration, blasting and ocean waves, is very complicated because of the complexity of soil behavior and of the interaction phenomenon between soil and pile. The complexity is increased further by grouping effects in pile groups.

Several analytical methods have been used for the dynamic response analysis of pile foundation and are discussed by Otani(1990). Those are listed as follows:

- structural approach,
- elastic continuum model,
- Winkler model, and
- finite element model

The main objectives of the paper is to present a computational model for pile foundations. In the model development, particular attension is paid in making it simple and capable of taking into account nonlinear soil behavior, and the cyclic behavior of soil within a frame work of the Winkler model defined in the plane strain conditions. The model is formulated in the time domain using step-bystep method, so that the nonlinear behavior of soils and discontinuity conditions such as gapping and slippage between soil and pile can be considered in a logical manner. A transfer matrix approach is adopted in the response computations. The proposed nonlinear model is verified with rigorous solutions and is discussed on the behavior of pile foundations.

TIME DOMAIN SOIL-PILE INTERACTION MODEL

1. Summary of the proposed model

The nonlinear soil model developed herein can account for the non-

linear soil behavior and gapping and slippage between the soil and pile. It is assumed that piles are vertical circular piles in nonlinear horizontally layered ground and the pile tip is either free(floating pile) or fixed at the bedrock(end bearing pile). Additional conditions are summarized as follows :

• The soil is divided into a number of layers as shown in Fig.1. In each layer, the soil model is divided into two parts as shown in



Fig.2 Proposed soil model in plane strain condition

Fig.2. One is an inner field model which accounts for the nonlinear characteristic, and the other is a far field model formulated in the time domain by taking the inverse Fourier tranceform of frequencey domain solutions(Nogami et al.; 1986,1988). Based on the Winkler hypothesis, the subgrade reaction in that layer is governed by the displacements of the layer only at the depth where the reaction is considered.

- Formulations of the model are in the time domain so that response can be computed by the step-by-step method in the time domain.
- Soil displacements are in either vertical or horizontal direction depending on the direction of pile shaft motion.
- The range of nondimensional frequency,  $a_0$ , is basically  $0.02 \leq$  $a_0 < 0.5$ , where  $a_0 = r_0 \omega / v_s$  with  $r_0 = a$  pile radius,  $\omega = a$ circular frequency and  $v_s = a$  shear wave velocity.
- Applied force varies with time linearly within a time step  $\Delta t$ to form a trapezoidal force time history.

#### 2. Far field model

An infinitly long rigid massless vertical circular cylinder is surrounded by an infinite elastic medium. For vertical and horizontal displacements of the cylinder, the displacements of the medium do not vary in the vertical direction. Therefore, plane strain conditions exist for those displacements. Solutions for vibration of this cylinder have already been developed by Novak et al.(1978). However, these are obtained in the frequency domain and cannot be applied to the analysis in the time domain directly. Approximate time domain explicit solutions for this were developed by Nogami and his colleagues(1986,1988 and 1991) and are used herein to describe the far field behavior.

## Vertical response

The vertical displacement amplitude of the medium under plane strain conditions, as shown in Fig.3, is expressed as

$$2\pi \mathbf{G}_{s} \mathbf{w}(\omega, \bar{\mathbf{r}}) = \frac{\mathbf{K}_{0}(\mathbf{a}_{0}^{*}\bar{\mathbf{r}})}{\mathbf{a}_{0}^{*}\mathbf{K}_{1}(\mathbf{a}_{0}^{*})} \mathbf{p}_{\mathbf{v}}(\omega) \quad , \tag{1}$$

where  $p_v(a_0) = exciting$  force applied to the rigid disk,

 $w(a_0, \bar{r}) = displacement amplitude at distance \bar{r} = r/r_0$ 

from center of the disk,

 $K_0(), K_1() = modified$  Bessel functions of the first kind of

order zero and one, respectively, and

 $\mathbf{a}_0^* = (\mathbf{r}_0 \omega / \mathbf{v}_s)\mathbf{i}$  with  $\mathbf{i} = \sqrt{-1}$ .

Nogami et al. (1986) proposed the approximate solution of Eq.(1)for the case of the frequency range,  $0.02 \le a_0 \le 1.0$ , as

$$2\pi \mathbf{G}_{\mathbf{s}} \mathbf{w}(\omega, \bar{\mathbf{r}}) = \sqrt{\frac{1}{\bar{\mathbf{r}}}} \sum_{n=1}^{3} \frac{1}{\mathbf{k}_{n} + i\mathbf{c}_{n}\mathbf{a}_{0}} \mathrm{e}^{-i\mathbf{a}_{0}(\bar{\mathbf{r}}-1)} \mathbf{p}_{\mathbf{v}}(\omega)$$
(2)

where  $k_n$  and  $c_n$  are frequency independent parameters. It is realized that the medium response expressed in Eq.(2) can be reproduced by a series of three Voigt model as shown in Fig.4. Then, using an inverse Fourier transform, the impulse response function in the time domain is obtained as

$$h(t, r) = \frac{1}{2\pi G_s} \sqrt{\frac{1}{\bar{r}}} \sum_{n=1}^{3} A_n e^{-\kappa_n(t-t_s)}$$







cross section view Fig.3 Massless rigid disk in a plane strain medium



Rheological presentation of Fig.4 far field model for vertical excitation case

$$= \begin{cases} \sum_{n=1}^{3} h_{n}(t, r) & (t \leq t_{s}) \\ 0 & (t \geq t_{s}) \end{cases}$$
(3)

Loading time history is discretized at equal time interval  $\Delta t$  and is assumed to be piecewize linear. Such a history can be considered as trapezoidal loadings and are successively applied. Thus, the response to the time history can be computed by superimposing the response computed for each trapezoidal loading. The response computed in this way is written as

$$\begin{split} \mathbf{w}(\mathbf{t}_{i},\mathbf{r}) &= \sum_{j'=1}^{i'} \sum_{n=1}^{3} \{ \mathbf{H}_{n}(\mathbf{t}_{i} - \mathbf{t}_{j'-1},\mathbf{r})\mathbf{p}(\mathbf{t}_{j'-1}) + \mathbf{I}_{n}(\mathbf{t}_{i} - \mathbf{t}_{j'-1},\mathbf{r})\mathbf{p}(\mathbf{t}_{j'}) \} \\ &= \sum_{n=1}^{3} \mathbf{w}_{n}(\mathbf{t}_{i-1},\mathbf{r})\mathbf{e}^{-\kappa_{n}\Delta t} + \sum_{n=1}^{3} \mathbf{H}_{n}(\mathbf{t}_{i} - \mathbf{t}_{i'-1},\mathbf{r})\mathbf{p}(\mathbf{t}_{i'-1}) \\ &+ \sum_{n=1}^{3} \mathbf{I}_{n}(\mathbf{t}_{i-1} - \mathbf{t}_{i'-1},\mathbf{r})\mathbf{p}(\mathbf{t}_{i'}) \quad , \end{split}$$
(4)

where the disturbance applied at  $t_{i'}$  arrives at the distance r at  $t_i$ , and  $H_n$  and  $I_n$  are results of convolution integrals(Otani, 1990),  $\kappa_n$  $= k_n/c_n$  and  $w_n(t_{i-1}, r) = n$ -th displacement computed at  $t_{i-1}$ .

N group piles are assumed to be equally spaced and  $\Delta$  t is taken so that the S-wave generated from one pile does not arrive at others during  $\Delta t$ . When the distance  $r_{sl}$  is defined as the distance between the s-th and l-th piles, the response of the s-th pile at time t<sub>i</sub> is formulated as follows,

$$w^{s}(t_{i}) = \sum_{n=1}^{3} I_{n}(\Delta t, 0) p^{s}(t_{i'}) + \sum_{n=1}^{3} H_{n}(\Delta t, 0) p^{s}(t_{i'-1}) + \sum_{l=1, l \neq s}^{N} \sum_{n=1}^{3} w^{s}_{n}(t_{i-1}, r_{sl}) e^{-\kappa_{n} \Delta t}$$
(5)

Eq.(5) can be rewritten as

$$\mathbf{p}_{\mathbf{v}i}^{\mathbf{s}} = \mathbf{k}_{\mathbf{v}}\mathbf{w}_{i}^{\mathbf{s}} + \mathbf{d}_{\mathbf{v}i}^{\mathbf{s}} \quad , \tag{6}$$

where  $p_v^s = p^s(t_i)$  and  $w_i^s = w^s(t_i)$ , and

$$\begin{split} k_v &= (\sum_{n=1}^{3} I_n(\Delta t, 0) p_{v(i-1)}^s)^{-1} \\ d_{vi}^j &= -k_v (\sum_{n=1}^{3} I_n(\Delta t, 0) p_{v(i-1)}^s + \sum_{l=1, l \neq s}^N \sum_{n=1}^{3} w_{i-1}^s e^{-\kappa_n \Delta t}) \end{split}$$

## Horizontal response

An explicit form of the response of a massless cylinder subjected to horizontal vibration and in plane strain conditions is obtained by Novak et al.(1978). When harmonic excitation is applied to the cylinder in x direction, the x and y direction displacements of the medium at the location(r,  $\theta$ ) are approximately expressed as

$$2\pi \mathbf{G}_{\mathbf{s}} \mathbf{u}_{\mathbf{x}}(\omega, \mathbf{r}, \theta) \approx [\{\frac{1}{\beta^2} \mathbf{f}_2(\mathbf{b}_0, \bar{\mathbf{r}}) - \mathbf{f}_2(\mathbf{a}_0, \bar{\mathbf{r}})\}(\cos^2\theta - \sin^2\theta) \\ + \frac{1}{\beta^2} \mathbf{f}_1(\mathbf{b}_0, \bar{\mathbf{r}})\cos^2\theta + \mathbf{f}_1(\mathbf{a}_0, \bar{\mathbf{r}})\sin^2\theta]\mathbf{p}(\omega)$$
(7)

$$\begin{split} &2\pi \mathbf{G}_{\mathbf{s}} \mathbf{u}_{\mathbf{y}}(\omega,\mathbf{r},\theta) \approx [\{\frac{1}{\beta^2} \mathbf{f}_2(\mathbf{b}_0,\bar{\mathbf{r}}) - \mathbf{f}_2(\mathbf{a}_0,\bar{\mathbf{r}})\} \mathrm{sin} 2\theta \\ &+ \frac{1}{\beta^2} \mathbf{f}_1(\mathbf{b}_0,\bar{\mathbf{r}}) \frac{\mathrm{sin} 2\theta}{2} - \mathbf{f}_1(\mathbf{a}_0,\bar{\mathbf{r}}) \frac{\mathrm{sin} 2\theta}{2}] \mathbf{p}(\omega) \quad , \end{split}$$

where

$$f_{1}(b_{0},\bar{r}) = \frac{K_{0}(\bar{r}b_{0}^{*})}{b_{0}^{*}K_{1}(b_{0}^{*})}e^{\psi_{p}b_{0}^{*}}, \quad f_{2}(b_{0},\bar{r}) = \frac{K_{1}(\bar{r}b_{0}^{*})}{\bar{r}b_{0}^{*2}K_{1}(b_{0}^{*})}e^{\psi_{p}b_{0}^{*}} \quad ,$$
(8)

$$f_1(a_0,\bar{r}) = \frac{K_0(\bar{r}a_0^*)}{a_0^*K_1(a_0^*)} e^{\psi_s a_0^*}, \qquad f_2(a_0,\bar{r}) = \frac{K_1(\bar{r}a_0^*)}{\bar{r}a_0^{*2}K_1(a_0^*)} e^{\psi_s a_0^*}$$

and  $\theta$  = angle measured from the x- axis,  $\psi_s$  = - ( $\beta$ -1)/4 $\beta$  and  $\psi_p$  = ( $\beta$ -1)/4.

As described for the case of vertical response, Voigt type of modeling is substituted with following approximations for the functions in Eq.(7).

$$f_{1}(a_{0},\bar{r}) \approx \sum_{n=1}^{3} \frac{1}{k_{1n} + ic_{1n}a_{0}} e^{-ia_{0}(\bar{r} - \chi_{\pi 1})}$$
(9)

$$f_2(a_0,\bar{r}) \approx \{\frac{-1}{(a_0\bar{r})^2} + i\frac{1}{a_0\bar{r}} - \sum_{n=1}^3 \frac{1}{k_{2n} + ic_{2n}a_0}\}e^{-ia_0(\bar{r}-\chi_{\star 2})} \quad ,$$

where  $\chi_{s1} = 1 - \psi_s$  and  $\chi_{s2} = \frac{1}{2} - \psi_s$ . Similarly, others are

$$f_{1}(b_{0},\bar{r}) \approx \sum_{n=1}^{3} \frac{1}{k_{1n} + ic_{1n}b_{0}} e^{-ib_{0}(\bar{r}-\chi_{p1})}$$
(10)

$$f_2(b_0,\bar{r})\approx\{\frac{-1}{(b_0\bar{r})^2}+i\frac{1}{b_0\bar{r}}-\sum_{n=1}^3\frac{1}{k_{2n}+ic_{2n}b_0}\}e^{-ib_0(\bar{r}-\chi_{P^2})}\quad,$$

where  $\chi_{p1} = 1 - \psi_p$  and  $\chi_{p2} = \frac{1}{2} - \psi_p$ , and  $k_{1n}$ ,  $k_{2n}$ ,  $c_{1n}$  and  $c_{2n}$  are the parameters and are determined by the regression analysis. These approximate expressions can produce sufficient accuracy for the frequency range of  $0.02 \le a_0 \le 1.0$ . Finally, the expression of impulse response function in time domain due to a trapezoidal load are

$$2\pi G_{s}u_{x}(t_{i}, r) \approx \left\{\frac{1}{\beta^{2}}F_{2b}(t_{i}, r) - F_{2a}(t_{i}, r)\right\}(\cos^{2}\theta - \sin^{2}\theta)$$
$$+ \frac{1}{\beta^{2}}F_{1b}(t_{i}, r)\cos^{2}\theta + F_{1a}(t_{i}, r)\sin^{2}\theta$$
(11)

$$2\pi G_{s} u_{y}(t_{i}, r) \approx \left\{ \frac{1}{\beta^{2}} F_{2b}(t_{i}, r) - F_{2a}(t_{i}, r) \right\} \sin 2\theta$$
$$+ \frac{1}{\beta^{2}} F_{1b}(t_{i}, r) \frac{\sin 2\theta}{2} - F_{1a}(t_{i}, r) \frac{\sin 2\theta}{2}$$

where

$$\begin{split} F_{1}(t_{i},r) &= \sum_{n=1}^{3} F_{1n}(t_{i-1},r) e^{-\kappa_{n} \Delta t} + \sum_{n=1}^{3} H_{n}(t_{i} - t_{i'-1},r) p(t_{i'-1}) \\ &+ \sum_{n=1}^{3} I_{n}(t_{i} - t_{i'-1},r) p(t_{i'}), \\ F_{2}(t_{i},r) &= A \Delta t \sum_{j'=1}^{i'} p(t_{j'}) - \frac{\Delta t}{2} p(t_{i'}), \\ F_{3}(t_{i},r) &= F_{3}(t_{i-1},r) + A \Delta t^{2} \sum_{j'=1}^{i'} p(t_{j'}) \\ &- \frac{A \Delta t^{2}}{6} p(t_{i'-1}) - \frac{5A \Delta t^{2}}{6} p(t_{i'}) \quad , \quad \text{and} \end{split}$$

A is a coefficients in terms of the wave velocity, shear modulus and the distance between two piles, and the precise forms are shown in the reference(Otani, 1990).

Therefore, the response at piles due to the excitations applied at other piles in group of N piles is formulated as

$$u_{i}^{s} = \sum_{l=1, l \neq s}^{N} u(t_{i}, r_{sl}).$$
 (12)

For the response of the soil at the loaded disk(i.e.  $r = r_{ss}$ ), the stiffness of a single disk is necessary to be determined. This is because the approximation expressed above is not accurate for small value of the distance r. The force-displacement relationship of a disc subjected to horizontal excitation is expressed for a Poisson's rato ( $\nu$ ) = 0.5 as

$$\mathbf{p} = \mathbf{S}_{\mathbf{u}}(\mathbf{a}_{\mathbf{0}}^{*})\mathbf{u},$$

where

$$S_{u}(a_{0}^{*}) = 2\{2\pi G_{s} \frac{a_{0}^{*} K_{1}(a_{0}^{*})}{K_{0}(a_{0}^{*})}\} + \pi G_{s} a_{0}^{*2}$$
$$= 2S_{v}(a_{0}^{*}) - \rho \pi r_{0}^{2} \omega^{2} \quad , \qquad (13)$$

and  $\rho = mass$  density of soils.

It is noted that  $S_v(a_0^*)$  is the stiffness for the vertical vibration. Eq.(13) is modified for various values of Poisson's ratio as

$$S_{u}(\mathbf{a}_{0}^{*}) = \xi_{1}(\nu)S_{\nu}(\mathbf{a}_{0}^{*}) - \xi_{2}(\nu)\rho\pi r_{0}^{2}\omega^{2} \quad , \tag{14}$$

where the functions  $\xi_1(\nu)$  and  $\xi_2(\nu)$  are obtained by a regression analysis for various Poisson's ratios. It is realized that the soil reaction in the horizontal direction is reproduced by the system as shown in Fig.5 in which a spring, dashpot and a mass are dependent on Poisson's ratio but independent of frequency. Therefore, Eq.(14) results in the force in time domain as

$$\mathbf{p}_{\mathrm{hi}} = \mathbf{m}\ddot{\mathbf{u}}_{\mathrm{i}} + \mathbf{k}_{\mathrm{h}}\mathbf{u}_{\mathrm{i}} + \mathbf{d}_{\mathrm{hi}} \quad , \tag{15}$$

using a time marching scheme developed by Nogami et al.(1986),

$$\begin{split} k_h &= \xi_1(\nu) k_\nu, \\ d_{hi} &= -k_h \sum_{n=1}^3 (I_n(\Delta t) p_{h(i-1)} + u_{i-1} e^{-\kappa_n \Delta t}), \text{and} \\ m &= \xi_2(\nu) \rho \pi r_0^2 \end{split}$$



Fig.5 Rheological presentation of far field model for horizontal excitation case

Since the response of the s-th pile due to the load applied at the piles other than the s-th pile (i.e. Eq.(12)) is known at  $t_i$ , the force-displacement relationship at the s-th pile in N group piles can be expressed after combining Eqs. (12) and (15) as

$$\mathbf{p}_{\mathrm{hi}}^{\mathrm{s}} = \mathrm{m}\ddot{\mathrm{u}}_{\mathrm{i}} + \mathbf{k}_{\mathrm{h}}\mathrm{u}_{\mathrm{i}}^{\mathrm{s}} + \mathrm{d}_{\mathrm{hi}}^{\mathrm{s}} \tag{16}$$

where

$$d_{hi}^{s} = -k_{h} \{ \sum_{n=1}^{3} (I_{n}(\Delta t) p_{h(i-1)}^{s} + u_{i-1}^{s} e^{-\kappa_{n} \Delta t}) + \sum_{l=1, l \neq s}^{N} u(t_{i}, r_{sl}) \}$$
(17)

Although the formulation implemented for the horizontal response is discussed only for the x-direction, the same formulations are conducted using values in the y-direction. Eqs.(7) and (18) are the soil reactions for the far field model and can be coupled with the inner field model. It is noted that the solutions described here are for the far field model so that the radius  $r_0$  in those equations corresponds to the results of the inner field model  $r_1$ .

#### 3. Inner field model

When the pile foundations are excited by vibrational forces, some areas relatively close to the pile shaft show highly nonlinear behavior. It is considered, therefore, that this plastic behavior mobilized the discontinuity conditions such as gapping and slippage. Here in the model, this plastic area is defined as an inner field model. The inner field model consists of one nonlinear spring and consistent mass as shown in Figs.6 and 7 for each vertical and horizontal excitation case. The model can also reproduce the discontinuity such as gapping and slippage.



Fig.6 Inner field model in vertical excitation case



Fig.7 Inner field model in horizontal excitation case

## Continuum condition

Parallel Jenkin's model, which is shown in Fig.8, is used to simulate the nonlinear behavior in the soil medium. This model does not include the effects of slippage and gap between pile and soil, so that the condition of soil nonlinearity only is called continuum condition herein. The formulation of continuum condition is exactly the same



Fig.8 Behavior of inner field model based on multi-linear model

between the vertical and horizontal excitations, and therefore only the horizontal excitation case is presented. The equation of motion for this condition is simultaneously formulated as

$$k_{rs}(u_{i}^{a} - u_{i}^{b}) + m_{11}\ddot{u}_{i}^{a} + m_{12}\ddot{u}_{i}^{b} = p_{i}^{a}$$
$$-k_{rs}(u_{i}^{a} - u_{i}^{b}) + m_{21}\ddot{u}_{i}^{a} + m_{22}\ddot{u}_{i}^{b} = -p_{i}^{b}$$
(18)

where  $m_{11}$ ,  $m_{12}$ ,  $m_{21}$  and  $m_{22}$  = component of consistent mass.  $k_{rs}$  = stiffness of inner field model.

Finally, coupling this inner field model behavior with the far field model behavior, the time domain soil behavior for the lateral response is derived as

$$\begin{cases} p_{i}^{a} \\ 0 \end{cases} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} + m \end{bmatrix} \begin{cases} \ddot{u}_{i}^{a} \\ \ddot{u}_{i}^{b} \end{cases} + \begin{bmatrix} k_{r} & -k_{r} \\ \\ -k_{r} & k_{r} + k_{h} \end{bmatrix} \begin{cases} u_{i}^{a} \\ u_{i}^{b} \end{cases} + \begin{cases} 0 \\ d_{hi} \end{cases}$$
(19)

Similarly, that for the vertical response is obtained as Eq.(19) with replacing  $k_h$  and  $u_i$  respectively with  $k_v$  and  $w_i$ .

## Discontinuity conditions

The discontinuity condition for vertical response is caused by a Coulomb damper which starts to slip when the shear forces in the slider reaches the maximum friction force allowed at the soil-pile interface as shown in Fig.6. When the gap is not formed during the lateral pile displacement, the normal stress acting on the pile shaft increases at the front side of the pile but decreases at the back side. This leaves the summation of the normal stress acting on the pile shaft unchanged, and thus the maximum friction forces are also considered to be unchanged. On the other hand, when gap is formed, no vertical friction forces is induced at the back side, and thus the maximum vertical friction force is considered to be linearly proportional to the normal stress acting on the front side of the pile. During the slippage between the soil and pile, the soil model behaves independently of the pile response. The stick conditon resumes when the soil response velocity relative to the pile response velocity becomes zero.

The discontinuity condition for horizontal response is caused by a rigid frame with an expansion joint as shown in Fig.7. The expansion joint controls the opening space of the frame and thus creates the gap. It is assumed that the two vertical walls of the frame move in the identical direction in the following manner:

$$u^{e} + u^{p} (= u_{a})$$
 front side ,  
 $u^{e} + \alpha_{\sigma} u^{p}$  back side .  
(20)

where  $u^e$  and  $u^p$  = elastic and plastic components of the lateral displacement, respectively;  $\alpha_g = \text{gap}$  factor in the range of  $0 \le \alpha_g \le 1$ . Thus, the expansion joint produces the gap when the plastic deformation develops and the size of the gap is

$$\mathbf{u}_{\mathbf{g}} = \mathbf{u}^{\mathbf{p}} (1 - \alpha_{\mathbf{g}}) \tag{21}$$

It is noted that the gap cannot be formed when  $\alpha_{\mathbf{g}} = 1$ .

## Cyclic conditions

Here in the model, Idriss et al.(1978) type of degradation factor is substituted for the resistance and the stiffness in each half cycle, which is defined as

$$\delta_d = N^{-t} \quad , \tag{22}$$

where N is the number of cycles and t is a parameter related to the cyclic displacement level.

# FORMULATION OF DYNAMIC RESPONSE OF PILE HEADS

# 1. Governing equation for a soil-pile system

A pile in a subsoil is divided into a number of horizontal layers containing pile segments. The governing equations of motion of those segments are described for both vertical and horizontal responses.

## Vertical response

The response of a pile segment in each layer at time  $t_i$  due to vertical excitation is governed by the following differential equation,

$$-E^{\mathbf{p}}A\frac{\mathrm{d}^{2}W_{i}}{\mathrm{d}z^{2}} + m^{\mathbf{p}}\ddot{W}_{i} = -p_{\mathbf{v}i}$$

$$\tag{23}$$

where  $W_i$  and  $\ddot{W}_i$  are vertical displacement and acceleration of the pile segment at time  $t_i$ .  $E^p$ , A and  $m^p$  are Young's modulus, cross-sectional area and mass density of piles, respectively.  $p_{vi}$  is a soil-pile interaction force.

## Horizontal response

The motion of the piles is assumed as a transverse vibration on the Winkler type of ground. Then, the horizontal response of each pile segment at time  $t_i$  is calculated from the equation,

$$E^{p}I\frac{d^{4}U_{i}}{dz^{4}} + m^{p}\ddot{U}_{i} = -p_{hi} \quad ,$$
 (24)

where  $U_i$  and  $\ddot{U}_i$  are the horizontal displacement and acceleration of piles at time  $t_i$ ,  $E^{pI}$  and  $m^{p}$  are the bending stiffness and mass density of piles, respectively.  $p_{hi}$  is the horizontal interaction force in the horizontal direction.

# 2. Force-displacement relationship at pile heads

A transfer matrix approach is used to obtained the force-displacement relationships at the pile heads. The basic idea of the transfer matrix approach is to correlate the force and displacement at one end of the beam with those at the other end by knowing the relationships for the small segments and successive matrix productions using those relationships. Detailed formulations have been described by Otani(1990) and Nogami et al.(1986, 1988).

# VERIFICATIONS OF THE PROPOSED MODEL

A plot of complex stiffness vs. nondimensional frequency  $a_0$  up to the value of 0.5 is presented. Fig.9 shows the vertical stiffness of single piles under the linear condition. Three different cases are considered as shown in the figure. Those computed by a more rigorous approach developed by Nogami(1980) are also shown in the figure. The size

of inner field model is fixed at  $r_1/r_0 = 2.0$ . The figure shows that the model can simulate the linear elastic behavior well. For vertical vibration of two-pile group, Fig.10 shows the results computed by the present model together with that computed by a more rigorous solution. It can be seen that satisfactory agreement between the two are obtained for a linear elastic case.



Fig.9 Model behavior of complex stiffness variation for a single pile(vertical case)



Fig.10 Model behavior of complex stiffness for an end bearing two-pile group(vertical case)



Fig.11 Model behavior of complex stiffness for a single pile (horizontal case)



Fig.12 Model behavior of complex stiffness for two-pile group (horizontal case  $\theta = 0$ )



Fig.13 Model behavior of complex stiffness for two-pile group (horizontal case  $\theta = 90$ )

Similar comparison studies are described for horizontal vibrations. Fig.11 shows complex stiffness(multiplied by  $L^3/E^{PI}$ ) for a single pile foundation. Figs.12 and 13 show those for two-pile groups subjected to horizontal loading in the  $\theta = 0$  and 90 directions, respectively. Agreement between the results computed by the two different methods are generally good. Thus, the present model can reproduce the linear elastic behavior well for both single and pile groups.

# MODEL BEHAVIOR WITH GAP AND SLIPPAGE

The behavior of the developed model is studied. The input nonlinear characteristics for the inner field model are defined by finite element results computed by Otani(1990). A harmonic displacement, which each amplitude of applied displacement is 0.02inch for vertical case and 0.03inch for horizontal case, is applied at the pile shaft and the force to cause the displacement is computed in each excitation. The frequency of applied displacement is  $a_0 = 0.2$ .

#### 1. Vertical response

Fig.14 shows the vertical response of the model for a single pile. Three different cases are shown in the figure, including the elastic condition (case 1), the inelastic condition allowing slippage (case 2) and inelastic condition allowing degradation and slippage (case 3). The degradation factor  $\delta_d$  is defined by the number of cycle  $N^{-4}$  and t = 0.3 is used. The effects of the slippage can be  $\epsilon$  in the response for case 2 and case 3 in the figure. The hy loops of reaction-displacement relationships during those re are presented in Fig.15. The hysteresis loops for the elastic c. the inelastic case with slippage become steady state by the cycle. The hysteresis loop for the elastic case entirely results fraction damping and thus the enclosed area increases as ir ing frequency. This trend is clearly observed in the hysteresis although Fig.15 shows the hysteresis loop only at one frequency.

### 2. Horizontal response

In the analysis of the horizontal response, the gap factor is sele to have a value of  $\alpha_{g} = 0.7$ . The results are shown in Fig.16 Fig.17, in which three cases are the inelastic case (case 1), the elastic case allowing gapping (case 2), and the inealstic case allow both gapping and degradation effects (case 3). As is seen in th figures, the effect of gapping appears clearly in both response t histories and hysteresis loops. The gap becomes large until it reac a certin value as the number of cycles increases in case 2. The a plitude of the cyclic force response decreases with the number of t cycles when the gap is formed. According to the test results by M. lock(1970), similar trends have been observed in the hysteresis loobtained for displacement control pile load tests in soft clay.



Fig.14 Time response for vertical excitation



Fig.15 Hysteresis loop for vertical excitation



Fig.16 Time response for horizontal excitation



Fig.17 Hysteresis loop for horizontal cexcitation

#### CONCLUSIONS

A simplified computational model is presented in the paper. The model is capable of handling not only the linear but also nonlinear behavior of pile-soil system, including the nonlinear soil behavior, gapping and slippage between pile and soil. Despite significant simplification involved in modeling the complex behavior, the proposed nonlinear pile group model can reproduce the behavior reasonably well compared with the behavior computed by more rigorous numerical methods. Therefore, the model may be conveniently used as a tool for the design of pile foundations.

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