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SYNOPSIS The least squared, the major axis and the reduced major axis criteron are used to deduce a statistical relation between magnitude, m_{bLg}, and intensity, I, for the earthquakes in southeastern United States. Based on a catalog of 162 events during 1833 to 1987, with magnitudes between 1.1 and 6.9 and intensities between II and X, it is shown that the reduced major axis criterion produces: $m_{bLg} = (0.656 \pm 0.058)*I + (0.402 \pm 0.178)$, which is the best predictor equation of magnitude for the upper range of the observed intensities. The predictor equations based on the least squared and major axis criterion are: $m_{bLg} = (0.441 \pm 0.038)*I + (1.359 \pm 0.176)$ and $m_{bLg} = (0.544 \pm 0.047)*I + (0.898 \pm 0.424)$, respectively; the least squared equation is a better predictor for the lower range of the observations and the major axis equation yields predictions which are between the predictions from the other two equations. In mid-range of the observed data all three equations predict nearly the same results. A set of three similar equations are found between intensity, I, and magnitude m_{bLg}.

The effects of various conversion methods on values of a and b in the frequency-magnitude equation $\log N = a+b*m_{bLg}$ and values of a´ and b´ in the frequency-intensity relation $\log N = a'+b'*I$ are negligible. Three new catalogs, with 2245 events in each were formed; in the new catalogs if the intensity or the magnitude of an event was missing it was estimated based on the above equations; then, the least squared technique was used to calculate the coefficients a, b, a´, and b´; the unnormalized values of the coefficients are: $a=4.105\pm0.144$, $b=-0.591\pm0.035$, $a'=3.941\pm0.199$, and $b'=-0.400\pm0.033$, respectively.

INTRODUCTION

The majority of earthquakes in the seismic catalog of southeastern United States have occurred prior to 1900 when seismic instruments were sparse throughout the world in general, and in the North American Continent in particular; thus, the events' locations were often based on felt reports and only the Modified Mercalli intensities, MM, were reported as a measure of destructiveness of the events. For seismic risk assessment, however, it is often desirable to use magnitudes of the seismic events as well as their intensities for comparison purposes. Richter (1958), Nuttli (1974) and Sibol et al (1989) have published conversion tables for changing intensities to magnitudes for the events in California, the central Mississippi Valley, and southeastern United States, respectively. Bollinger et al (1989) have reported that the conversion of MM intensities in the historic catalog to a modern m_b magnitude is the most serious obstacle against the required synthesis for presentation of the magnitude recurrence relations. The purpose of this paper is to study the possible relations between the magnitudes and intensities of earthquakes in the southeastern region and make a limited comparison with other regions of the contiguous United States.

A catalog of 162 earthquakes which have occurred in this region between 1833 to 1987 and which have reported magnitudes and intensities is compiled. There are six events prior to 1900, including the Charleston earthquake of 1886, $m_b=6.9$, I=X and the Giles' County earthquakes of 1897, $m_b=5.8$, I=VIII; the catalog also contains 81 earthquakes during 1900 to 1976 and 75 earthquakes during 1976 to 1987.

The magnitudes of a number of historic events have been studied from felt reports by Nuttli (1974) and Sibol et al (1986). The values used in this study are a compilation from catalogs by Bollinger (1987), Bernreuter (1987), and Reagor et al (1983). If more than one value of magnitude was reported for an event, a simple arithmetic mean was adopted as the actual magnitude reported for that event. Figure 1 gives the spatial distribution of 162 earthquakes used in this study which had both magnitudes and intensities. The events form a fair representation of regional seismicity. The frequency distribution for both magnitudes and intensities are presented in Figure 2 and 3 respectively; over 70 percent of the events

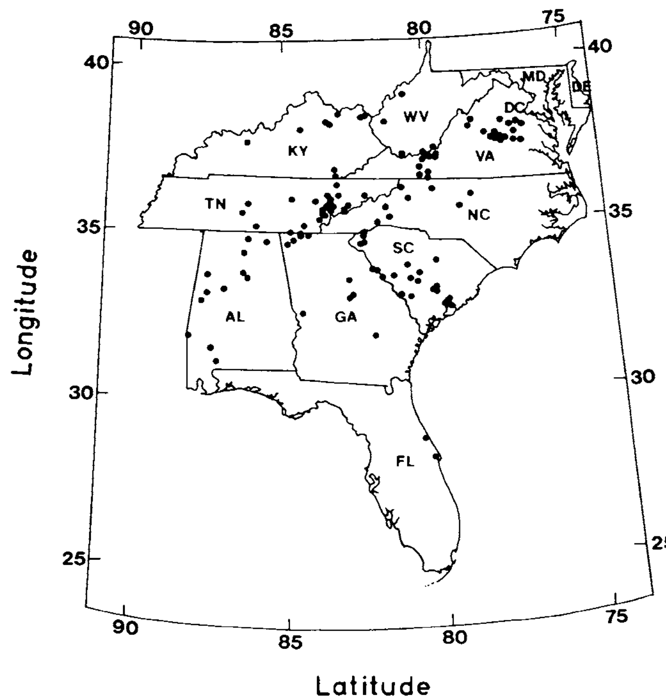

Figure 1. Location of 162 earthquakes with known intensities and magnitudes which have occurred from 1833 to 1987.

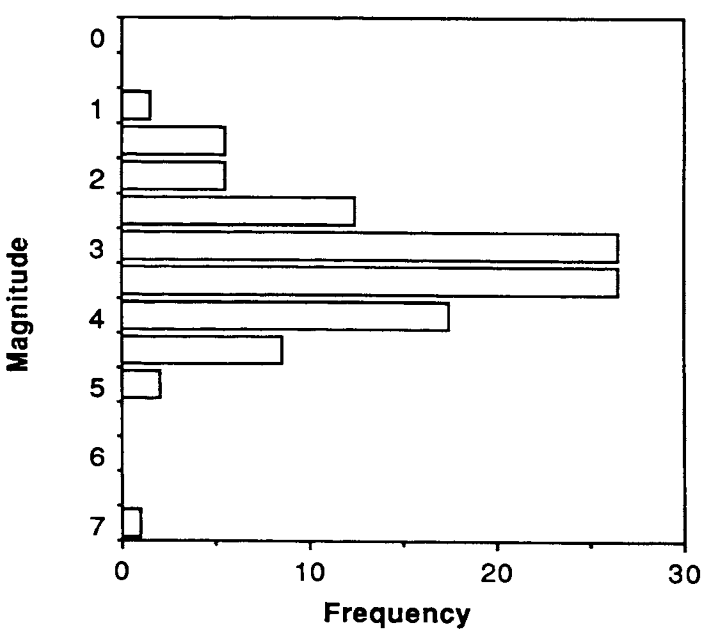

Figure 2. Frequency distribution of earthquakes' magnitudes.

have magnitudes between 2.5 and 4.5 intensities between II and VI.

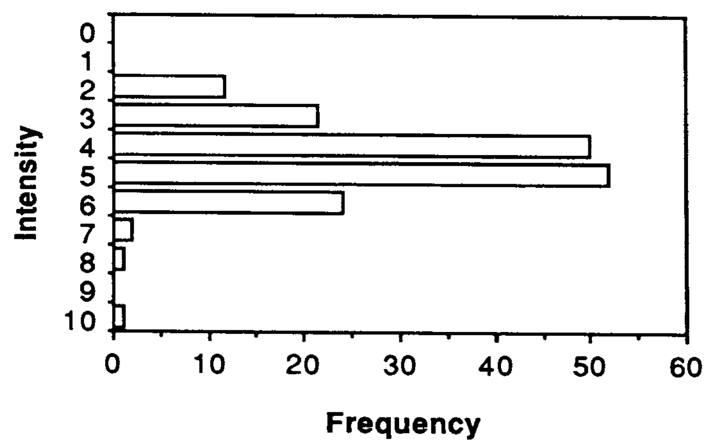

Figure 3. Frequency distribution of earthquakes' intensities.

METHOD

Intensity and magnitude are two distinct properties of earthquakes. The intensity of a seismic event at a locality depends on distance from the source, energy release at the source, focal depth, variation in near surface geology, engineering style, workmanship and constructional material. The magnitude of an event depends on energy release and source radiation patterns. Both properties are representations of a multivariate distribution. Although a simple linear relation between magnitude and intensity, or conversely between intensity and magnitude may be misleading, engineering needs often require a conversion of one parameter to the other for the purposes of seismic risk analysis in an area. This is clearly seen in the work of Bernreuter et al (1987, 1989), where he suggested:

$$I = 2m_{bLg} - 3.5 \qquad (1)$$

for conversion of Lg phase magnitude, m_{bLg}, to intensity in MM scale for the earthquakes in eastern United States. However, the data base used for derivation of equation (1) or uncertainties of the coefficients are not known. The data base used for this work was presented before; in this section the method of analysis is presented.

Let y_i, be magnitude m_i and x_i be intensity I_i of an event i, where i varies from 1 to M. A majority of the magnitudes used in this work are either body wave magnitude, m_b or Lg phase magnitude, m_{bLg}; we assumed that both magnitudes are nearly the same, thus, the average is assumed to be in the m_{bLg} scale. The intensities are in MM scale. There are several ways of drawing the "best fit lines", Y=aX+b, through the scatterograph of x_i plotted versus y_i given in Figure 4, where Y is the intensity axis and X is the magnitude axis. The coefficients a and b may be estimated in several ways. Here we briefly present the least squared, major axis and reduced major axis methods. In the least squared method, a and b are determined by minimization of the value of $\sum_{i=1}^{M} d_i^2$, when d_i are the distances between each point and the best fit line parallel to the Y axis. In the major axis method the coefficients are determined by minimization of the value of $\sum_{i=1}^{M} D_i^2$, where D_i are the normal distances between each point and the best fit line. Finally, in the reduced major axis the coefficients are determined by minimization of the value $\sum_{i=1}^{M} S_i$, where S_i are the areas of the right triangles formed by the best fit line and two other line segments drawn from each point parallel to the X and the Y axes, see Figure 4.

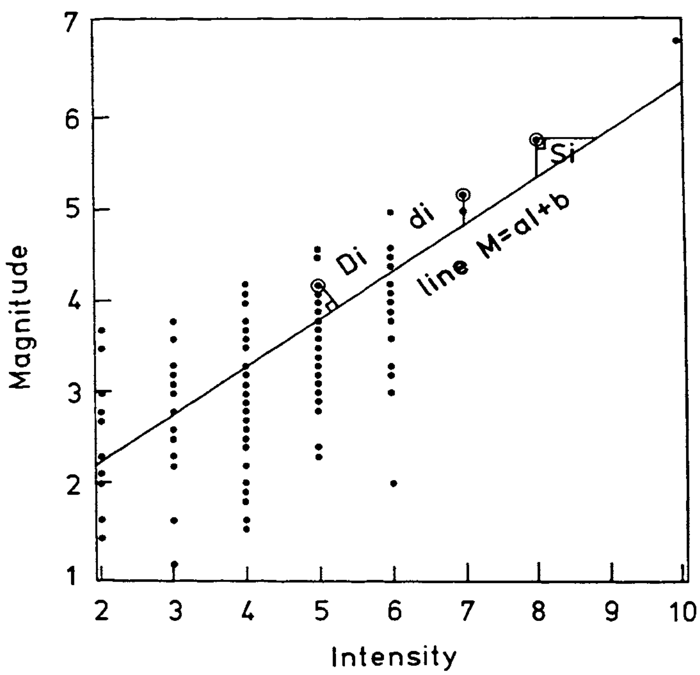

Figure 4. Plot of intensities versus magnitudes. Values of d_i D_i, and S_i are defined in the text.

Detail calculations and usages of each method are discussed in the literature, Davis (1986), Imbrie (1956), Kruskal (1953), and Kermack and Haldane (1950). A brief summary is given in this section. Often based on theoretical or experimental grounds, the dependent variable Y and the independent variable X are known; thus, it is clear which variable should be regressed against the other. Under this condition the least squared method will be used often. In our case both magnitude, m_{bLg}, and intensity, I, are subject to observational errors and neither can be regarded as a function of the other based on an acceptable theory or physical model. Yet it is useful to form a cross-plot of the two variables and find their statistical linear relation. An agreeable solution perhaps is to fit the line based on simultaneous minimization of the value of the sum of deviations of the observations from the best fit line in both X and Y directions. Such a line would closely resemble the visual trend in the observations and would attribute the deviations from the fitted line to both variables.

Davis (1986) discussed two methods by which the coefficients of the best fit line could be calculated. The first method involves simultaneous minimization of the squared deviations from the line in both the X and Y directions; this is equivalent to minimization of the squared deviation D_i; where D_i are perpendicular distances from the best fitted line discussed previously. This best fitted line is the major axis line and can be found by the principal eigen vectors of the variance-covariance matrix of X and Y. The second procedure involves the minimization of the product of the deviations in the X and Y directions which is equivalent to the minimization of the sum of the areas of triangle S_i as discussed before. This line is called the reduced major axis line. Although there are several mathematical equations for the calculation of a and b for each case, here we present the compact summary results derived by Kermack and Haldane (1950).

i. For Least Squared Regression line:

$$a = r\frac{S_y}{S_x} \pm \frac{S_y}{S_x}\left(\frac{1-r^2}{M}\right)^{1/2} \quad (2)$$

and

$$b = \bar{y} - \bar{x}a \pm S_y\left[\left(\frac{1-r^2}{M}\right)\left(1 + \frac{\bar{x}^2}{S_x^2}\right)\right]^{1/2} \quad (3)$$

ii. For Major Axis line:

$$a = \mathrm{Tan}\beta\left[1 \pm \left(\frac{1-r^2}{M}\right)^{1/2}\right]; \quad \mathrm{Tan}\beta = \frac{2S_xS_yr}{S_x^2 - S_y^2} \quad (4)$$

and

$$b = \bar{y} - \bar{x}\,a \pm \left\{\frac{1}{M}\left(S_y - S_x\,\mathrm{Tan}\beta\right)^2 + (1-r)\,\mathrm{Tan}\beta \, * \left[2\,S_y\,S_x + \frac{\bar{x}\,\mathrm{Tan}\beta\,(1+r)}{r^2}\right]\right\}^{1/2} \quad (5)$$

iii. For Reduced Major Axis line:

$$a = \frac{S_y}{S_x} \left\{ 1 \pm \left[\frac{1-r^2}{M} \right]^{1/2} \right\} \qquad (6)$$

and,

$$b = \bar{y} - \bar{x}\, a \pm S_y \left\{ \frac{1-r}{M} \left[2 + \bar{x}^2 \; \frac{1+r}{S_x^2} \right] \right\}^{1/2} . \qquad (7)$$

Where variables S_x^2, S_y^2 are variances of x_i and y_i, and r is the correlation coefficient between them. $\bar{x}$, $\bar{y}$ and, M are the means of x_i and of y_i and number of data points respectively.

RESULTS

Relation between Magnitude m_{bLg} and Intensity I:

In this section we assume m_{bLg} as a dependent variable and I as an independent variable. The resulting equations based on the least squared, major axis and reduced major axis are:

$$m_{bLg} = (0.441 \pm 0.038)I + 1.359 \pm 0.176, \qquad (8)$$

$$m_{bLg} = (0.544 \pm 0.047)I + 0.898 \pm 0.424, \qquad (9)$$

and,

$$m_{bLg} = (0.656 \pm 0.058)I + 0.402 \pm 0.178 \qquad (10)$$

respectively.

The three lines are plotted in Figure 5. It is clear from this figure that for higher values of I and m_{bLg} the reduced major axis line represents the data properly but the least square line has the maximum deviation and the major axis line lies between the two other lines. All three lines pass through the means of intensities and magnitudes; thus, at the mid-ranges the lines have the least deviation from each other. For comparison purposes the magnitude intensity conversion tables for California, Richter, (1958), for the Central Mississippi region, Nuttli (1974), and for southeastern region, Sibol et al (1987), are also presented in Figure 5 which indicate that values in the conversion tables are nearly equal to the upper limit of our observational values and that the reduced major axis line closely represents the higher limit of the intensity I $\geq$ VI for California, and the central Mississippi region. In addition, the Sibol et al (1987) data are very close to this line; thus the reduced major axis given by equation 10 is recommended to be used for the calculation of a magnitude, given an intensity from the historic catalog.

Relation between Intensity, I, and Magnitude, m

Here it is assumed that intensity is the dependent variable and magnitude is the independent variable. The resulting equations can be useful when the magnitude of an earthquake is available from related seismograms but no field investigation has been conducted to estimate the intensity of the event. Again, th resulting equations based on the least squared, the major axis and the reduced major axis are:

$$I = (1.025 \pm 0.089)m_{bLg} + 1.047 \pm 0.303, \qquad (11$$

$$I = (1.837 \pm 0.159)m_{bLg} - 1.650 \pm 0.783, \qquad (12$$

and,

$$I = (1.525 \pm 0.58)\, m_{bLg} - 0.613 \pm 0.305 \qquad (13$$

respectively.

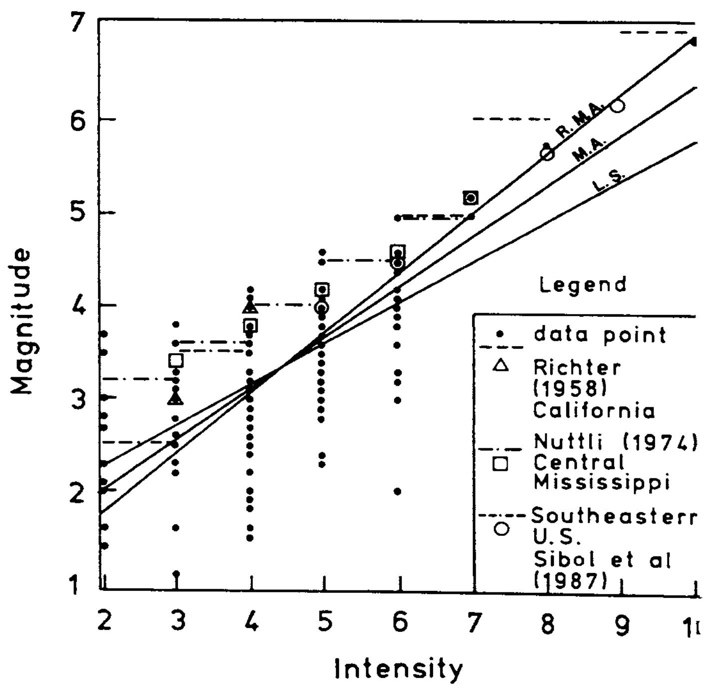

Figure 5. Plot of intensities versus magnitudes. The R.M.A., M.A., and L.S. lines are the reduced major axis, the major axis and the least squared line respectively.

These equations together with other conversion tables given by Richter (1958), Nuttli (1974) and Sibol et al (1987) are presented in Figure 6. The reduced major axis line which is now between the major axis and the least squared line has the least visual deviation for a magnitude higher than 4.5. The reduced major axis line is also in good agreement with the upper limit of magnitudes for data from California and Central Mississippi. In addition for magnitudes higher than 4.5, the data points presented by Sibol et al are very close to the reduced major axis line. Seismologists customarily express intensities in integer numbers but the preceding equations yield results in real numbers; thus, it is recommended that the resulting intensity from equations 11 to 13 be rounded to the nearest proper integer number when these equations are used.

DISCUSSION AND CONCLUSION

The majority of earthquakes in the historic catalog of southeastern United States have only reported intensities but no magnitudes. However, in many engineering studies, it is desirable to

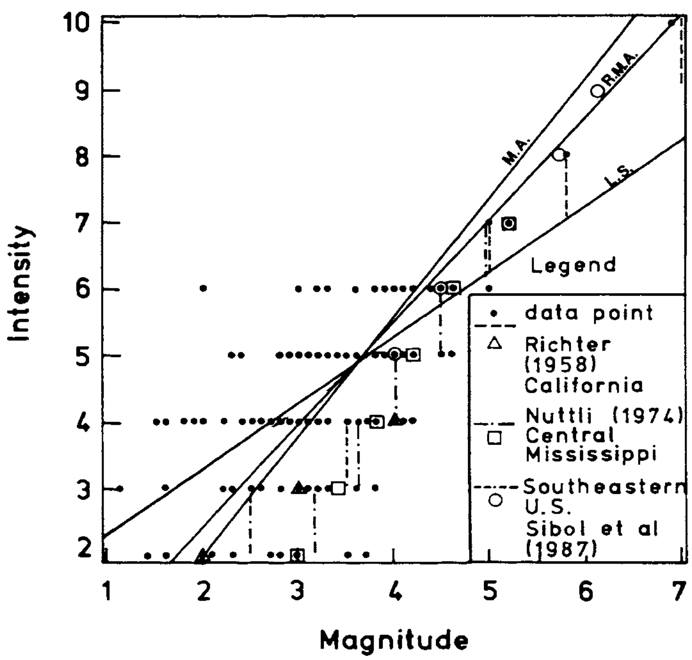

Figure 6. Plot of magnitudes versus intensities. For notation refer to Figure 5.

to have the corresponding magnitudes for the synthesis of the seismic risk analysis; here, we used 162 reported events with both magnitudes and intensities and three methods to derive a relation between magnitude and intensity or vice-versa. Tables 1 and 2 present the results of the conversion of intensity to magnitude and magnitude to intensity using equation 8 to 13 and the observed ranges for each parameter. The closest estimation is derived by the reduced major axis method for both magnitude and intensity.

Equations 8-13 were used and three new earthquake catalogs of southeastern United States have been produced. In the new catalogs, the events which had only a magnitude or intensity previously now have both magnitude and intensity. The frequency-magnitude and frequency-intensity plots of the new catalogs are shown in figures 7 and 8 respectively. The numerical values of the unnormalized coefficients a, b, a´, and b´ of the frequency-magnitude relation, $\log N = a + b * m_{bLg}$ and frequency-intensity relation $\log N = a' + b' * I$ are presented in Table 3 and 4. The values of the coefficients in the tables are estimated by the least squared method. They are very close. The maximum deviation for a, a´, b and b´ are 0.22, 0.91, 1.0, and 1.2 percent respectively. Thus, the effects of conversion on the shape of frequency-magnitude or frequency-intensity curve are negligible. For the entire area of southeastern United States and a time exposure of about 285 years, various coefficient sets are used to predict number of earthquakes with a given magnitude, Table 5, or intensity, Table 6. For the highest level of either magnitude or intensity the predictions are very close to the observations. For example, all three sets of coefficients predict that the number of earthquake with magnitude 7 or intensity 10 is about one; this is the same as what the observed seismic catalog indicates since only Charleston earthquake of 1987 has magnitude of 6.9 and intensity of X. Two events with intensity IX are also predicted, but none are reported; in addition there is a prediction of three or four earthquakes with magnitude 6,

TABLE 1 Comparison of Calculated Intensities based on Equation 13,12 , 11 and 1, for several Magnitudes.

m_{bLg}	Obs	R.M.A.	M.A.	L.S.	Ber.
6.9	10	9.91 ± 0.71	11.03 ± 1.88	8.12 ± 0.92	10.3
5.8	8	8.23 ± 0.64	9.01 ± 1.70	6.99 ± 0.83	8.1
5.2	7	7.32 ± 0.61	7.90 ± 1.61	6.38 ± 0.76	6.9
4.2	4 - 6	5.79 ± 0.55	6.07 ± 1.45	5.35 ± 0.68	4.9
3.2	3 - 6	4.27 ± 0.49	4.23 ± 1.29	4.33 ± 0.59	2.9
2.2	1 - 5	2.74 ± 0.13	2.35 ± 1.13	3.30 ± 0.50	0.5

Where m_{bLg} = Lg phase magnitude, obs - observed intensity, R.M.A., M.A., L.S. and Ber. are calculated intensities based on the reduced major axis, major axis, least squared and Bernreuter et al (1987) equations. Uncertainties are the standard errors of estimation. Note for M ≥ 5.2, the Bernreuter et al equation and the reduced major equation give exactly similar values if intensities are rounded to the nearest integers, however for M ≤ 4.2, their equation underestimates the intensities.

again, only the Giles county event of 1897 has a magnitude of 5.8 and intensity of VIII. It is very probable that a few events with a magnitude of about 6 and intensity of about VIII or IX have occurred but so far researchers have not deciphered them. For the mid-range of either magnitude or intensity predictions are reasonable as well; however, the number of small earthquakes are over estimated, probably because smaller events have been neither observed nor reported.

TABLE 2 Comparison of Calculated Mmagnitudes based on Equation 10 ,9 and 8 for Several Intensities.

I	Obs	R.M.A.	M.A.	L.S.
10	6.9	6.96 ± 0.76	6.34 ± 0.90	5.77 ± 0.56
8	5.8	5.65 ± 0.64	5.25 ± 0.80	4.88 ± 0.48
7	5.0 - 5.2	4.99 ± 0.58	4.71 ± 0.75	4.44 ± 0.44
6	2.0 - 5.0	4.34 ± 0.53	4.16 ± 0.71	4.00 ± 0.41
5	2.3 - 4.6	3.68 ± 0.47	3.62 ± 0.66	3.56 ± 0.37
4	1.5 - 4.2	3.02 ± 0.41	3.08 ± 0.61	3.12 ± 0.33
3	1.1 - 3.8	2.37 ± 0.35	2.53 ± 0.57	2.68 ± 0.29
2	1.4 - 3.7	1.71 ± 0.29	1.99 ± 0.52	2.24 ± 0.25

Refer to Table 1 for notations.

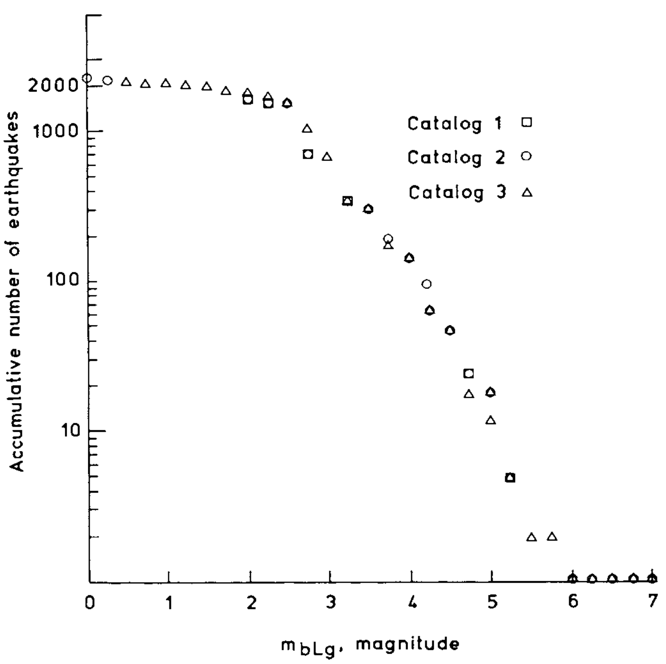

Figure 7. Magnitudes plotted versus accumulative number of earthquakes for three new catalogs.

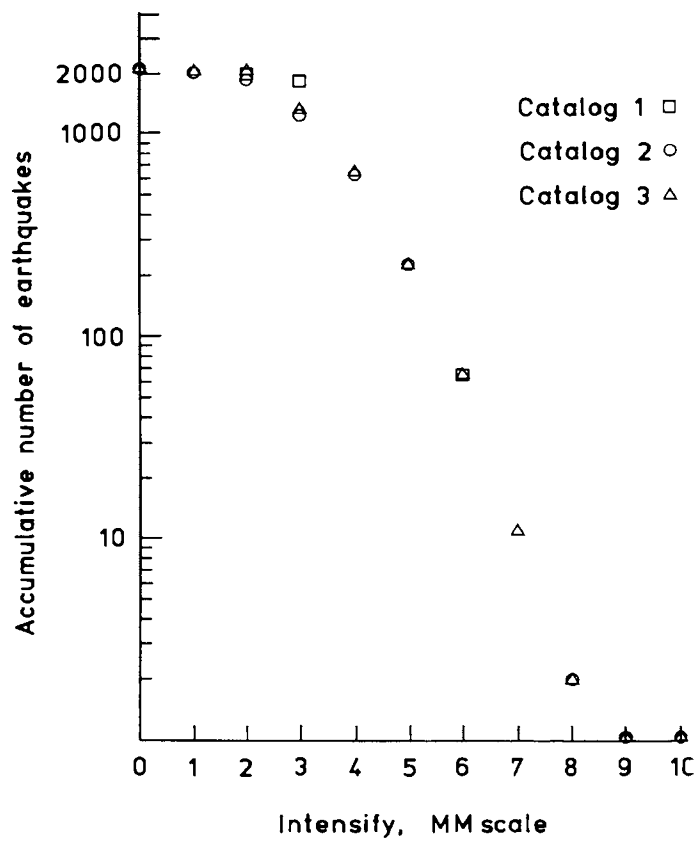

Figure 8. Intensities plotted versus accumulative number of earthquakes for three new catalogs.

TABLE 3 Value of a and b in Log N = a +b*m_{bLg}

data	a	b
catalog 1	4.105 ± 0.144	-0.591 ± 0.035
catalog 2	4.104 ± 0.141	-0.594 ± 0.034
catalog 3	4.114 ± 0.142	-0.598 ± 0.035

Magnitudes and intensities in catalog 1,2 and 3 are based on the reduced major axis ,major axis, and least squared equations ,8-10 The coefficients a b, a',and b' in this table and table 4 are based on the least square method.

TABLE 4 Value of a',and b' in Log N = a' + b'*I

data	a'	b'
catalog 1	3.941 ± 0.199	-0.400 ± 0.033
catalog 2	3.930 ± 0.199	-0.399 ± 0.033
catalog 3	3.966 ± 0.202	-0.403 ± 0.034
Original data	3.878 ± 0.197	-0.392 ± 0.033

Refer to table 3 for notations.

TABLE 5. Predicted Number of Earthquakes with a given Magnitude.

m_{bLg}	N1	N2	N3
2	837	824	828
3	214	210	209
4	55	53	53
5	14	14	13
6	4	3	3
7	≅1	≅1	≅1

N1,N2,and N3 are based on coefficient set for catalog 1,2,and 3 respectively.

TABLE 6. Predicted Number of Earthquakes with a given Intensity.

I	N1	N2	N3	N4	O.N.
3	551	541	572	503	727
4	219	216	226	204	429
5	87	86	89	83	160
6	35	39	35	34	56
7	14	14	14	14	9
8	6	5	6	6	1
9	2	2	2	2	0
10	≅1	≅1	≅1	≅1	1

Where I is intensity ; N1 ,N2 ,N3 ,and N4 are calculated from coefficients obtained for catalog 1,2,3,and original data respectively. O.N. is the observed number of earthquakes.

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