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George Gazetas
State University of New York, Buffalo, NY

Panos Dakoulas
Rice University, Houston, Texas

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George Gazetas
Professor of Civil Engineering, State University of New York, Buffalo

Panos Dakoulas
Assistant Professor of Civil Engineering, Rice University, Houston, Texas

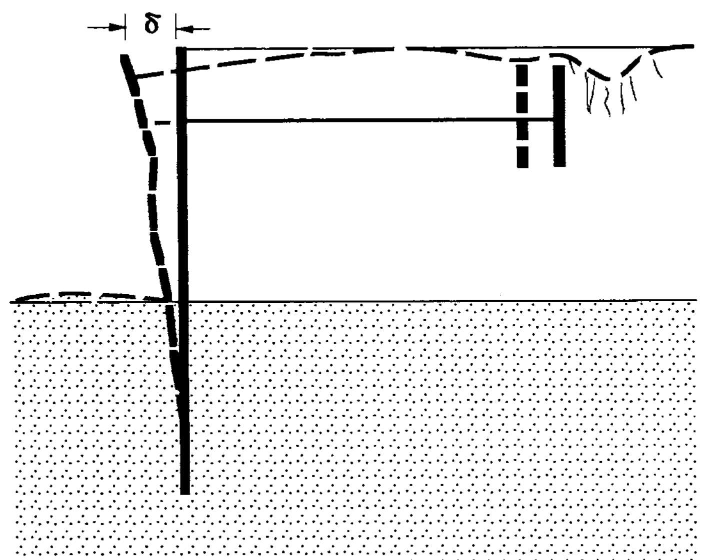
SYNOPSIS: Evaluation of numerous case histories reveals that the seismic performance of anchored sheetpile quaywalls depends primarily on the anchoring system. Current pseudo-static procedures often lead to deficient anchoring, whose excessive displacements or failure trigger excessive permanent seaward displacement at the top of the bulkhead, accompanied by cracking and settlement behind the anchor. The results of the case histories lead to a Seismic Design Chart to be used in conjunction with the pseudostatic procedure. The Chart delineates between acceptable and unacceptable degrees of damage, depending on the values of two dimensionless parameters that are functions of the material and geometric characteristics of the bulkhead, and the intensity of seismic shaking. Soil softening/degradation due to development of porewater-pressures is indirectly accounted for in the proposed method; however, the engineer must ensure that no liquefaction-flow failure of cohesionless soils will occur in the backfill or the foundation.

PAST SEISMIC PERFORMANCE OF ANCHORED BULKHEADS

Anchored bulkheads, also called anchored (steel) sheetpile walls, are quite vulnerable to strong earthquake shaking. Failures of such facilities have often resulted in major disruptions of post-earthquake emergency operations and have had serious economic consequences for the stricken regions.

Earthquake performance accounts of over a hundred anchored quaywalls in about 30 harbors in Japan (mainly), in Alaska, in West Indies, and in Chile have been published by Duke et al 1963, Hayashi et al 1970, Hung et al 1982, and Kitajima et al 1978. Detailed listings of these reported case histories may be found in the theses of Abraham (1985) and Dennehy (1985), and in a report by Agbabian Associates (1980). A study of the performance of anchored bulkheads in these harbors leads to the following main conclusions:

- Most of the observed major earthquake failures have resulted from large-scale liquefaction of loose, saturated, cohesionless soils in the backfill and/or in the supporting base (foundation). Such soils are not rare at port and harbor facility sites. Perhaps the most dramatic such failures have occurred in the Niigata, Japan, harbor during the 1964 earthquake.
- Another frequent, although not as dramatic, type of anchored bulkhead damage takes the form of excessive permanent seaward tilting of the sheet-pile wall, accompanied by excessive seaward movement of the anchor block or plate relative to the surrounding soil; such an anchor movement manifests itself in the form of settlement of the soil and cracking of the concrete apron directly behind the anchor, as sketched in Fig. 1. Apparently, and in accord with the conclusions of pertinent detailed studies, such failures are the outcome of inadequate passive soil resistance against the anchor. Development of detrimental residual excess pore-water pressures in the backfill, leading to some soil strength degradation, cannot be excluded as having contributed to this type of failures in some of the reported cases.



δ (cm)	ASSIGNED DEGREE OF DAMAGE
≤ 2	0
2 - 10	1
10 - 30	2
30 - 60	3
> 60	4

Figure 1. Sketch of usual seismic deformation of anchored bulkheads: permanent tilting of wall due to excessive relative motion of anchor. (Degrees of damage assigned by Kitajima and Uwabe, 1978.)

SEISMIC ANALYSIS/DESIGN PROCEDURES

To the authors' knowledge, no comprehensive method of realistic dynamic analysis of anchored bulkhead systems subjected to strong shaking is well enough developed and validated to be used in practice. It is fair, however, to state that dynamic codes developed for site response or soil-structure interaction analyses have been utilized, albeit to study specific aspects (only) of the response of the system (Hung & Werner, 1982). Simplified dynamic models specific for anchored bulkheads have also been developed, including those by Karkanins (1983), Abraham (1985), and Dennehy (1985). The "beam-on-Winkler-Foundation" model developed in these studies is illustrated in Fig. 2.

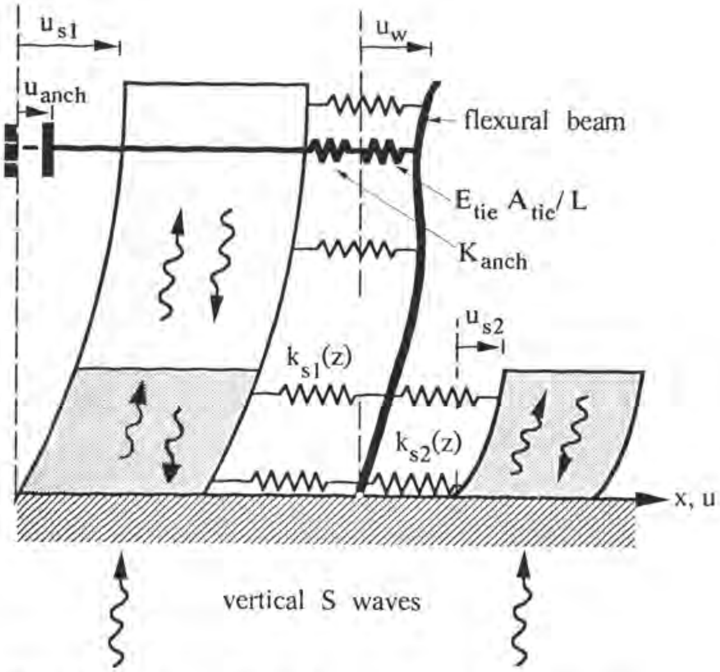


Figure 2. "Beam-on-Winkler-Foundation" model with complex "springs" (distributed and concentrated) used for obtaining a qualitative picture of the seismic response of the anchored bulkhead and for estimating the effective "point" of rotation, shown in Fig. 5.

The difficulty of providing a comprehensive rigorous method arises from several factors, which include: the complicated wave diffraction pattern due to "ground-step" geometry; the presence of two different but interconnected structural elements in contact with the soil; the inevitably nonlinear hysteretic behavior of soil in strong shaking, including pore-pressure buildup and degradation, both in front and behind the sheetpile; the no-tension behavior of the soil-sheetpile interface; the presence of radiation damping effects due to stress waves propagating away from the wall in the backfill and in the foundation; and the hydrodynamic effects on both sides of the sheetpile wall. Until codes which can properly handle all these phenomena are developed, improving the pseudo-static procedures currently used in practice so that they can lead to safe and economic design merits our effort.

Pseudo-static procedures are of an empirical nature and determine dynamic lateral earth pressures with the well-known Mononobe-Okabe seismic coefficient analysis. Differences arise primarily with respect to the assumed point of application of the resultant active and passive forces P_{AE} and P_{PE} (on the two sides of the sheetpile wall), and the partial factors of safety introduced in the design.

The procedure developed and extensively used in Japan (JSCE, 1980) is perhaps the most elaborate and complete pseudo-static design procedure. As illustrated in the sketch of Fig. 3, it combines the use of the Mononobe-Okabe method with conventional static design procedures of anchored bulkheads. The vertical component of the ground acceleration is ignored, while the horizontal seismic coefficent, k, is chosen for a particular site as a product of three factors (according to the Japanese Code): a regional seismicity factor (0.10 ± 0.05), a factor reflecting the subsoil conditions (1 ± 0.2), and a factor reflecting the importance of the structure. (1 ± 0.5). To account for the presence of water in the design procedure, and "apparent" seismic coefficient k' is used for soils below the water table:

$$k' = [\gamma_s/(\gamma_s - \gamma_w)]\ k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which γ_s = the saturated unit weight of the soil, and γ_w = the unit weight of water.

Subsequently, the design proceeds as follows:

1. Estimation of the necessary length of the sheetpile embedment(D): This is computed by the free-earth support method. The safety factors usually required against the failure of embedment are 1.5 and 1.2 for static and seismic conditions respectively in sandy stratum. In cohesive soil strata, the usually required safety factor is 1.2 for both the static and seismic conditions.

2. Design of the tie rod: In the case of a sheet pile bulkhead constructed in sandy ground, tie rod tension is computed on the assumption that the bulkhead is a simple beam supported at the dredge line and the point of tie rod connection, and which carries the lateral earth pressure and the residual water pressure. In case of cohesive soil, tie rod tension is computed by the fixed-earth support method. Allowable stress of tie rods: 40% and 60% of the yield strength of steel for static and seismic conditions, respectively. These relatively low values of allowable stress are intended to account for bending moment in the tie rod due to surcharge, and for concentration of lateral earth pressure at the point of tie- rod connection.

3. Design of the sheetpile cross-section: In sandy ground, the maximum bending moment is computed for the aforementioned simple beam. This maximum moment, which is about 40-50% of that computed by the free-earth support method, corresponds to the value computed by fully taking into account the moment reduction due to the flexibility of the sheetpile (Rowe, 1952). The allowable stresses of the sheetpile for static and seismic conditions are 60% and 90% of the yield strength of steel, respectively.

4. Design of the anchor plate or block: Lateral resistance of an anchor plate should be 2.5 times the tie rod tension for both static and seismic conditions. Anchor plates should be placed behind the active failure wedge starting from the dredge line

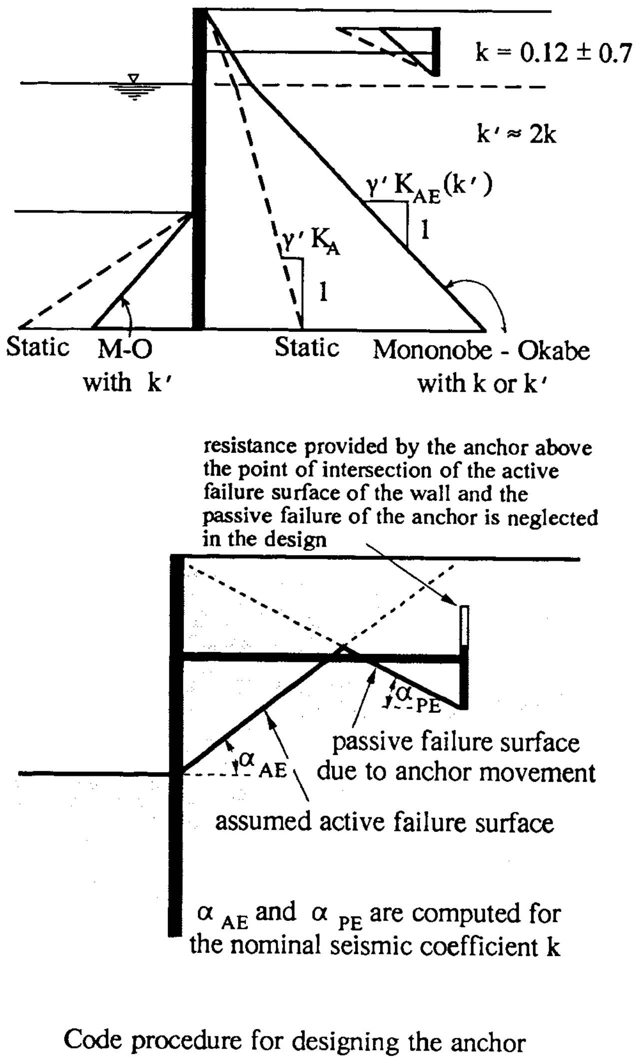


Figure 3. Illustration of the pseudo-static design procedure

(Fig. 3). When the passive wedge of the anchor plate crosses the active wedge behind the sheet pile, the passive resistance of the soil above the point of intersection should be neglected in the computation of the lateral resistance of the anchor plate.

WEAKNESS OF PSEUDO-STATIC DESIGN PROCEDURES

Kitajima & Uwabe (1978) have compiled information on the seismic performance of 110 quaywalls (mostly anchored bulkheads) in Japan. Table 1 summarizes the conclusions of their study. The conveyed message for the adequacy of the pseudo-static design methods is negative: the percentage of bulkheads that suffered some degree of seismic damage did not decline following the adoption of the previously-described design procedure... (Year of construction seems also to have had little effect on damage statistics.)

Many of the "failures" included in the statistics of the foregoing Table were clearly due to extensive

TABLE 1. STATISTICS OF SEISMIC DAMAGE TO ANCHORED BULKHEADS IN JAPAN

		Total Numbers	Number of Damaged Bulkheads	Percent of Damaged Bulkheads %
Total Numbers		110	70	64
Number of bulkheads designed according to the Japanese procedure		45	29	64
Year Constructed	Before 1950	37	22	59
	1951 - 1960	11	6	55
	1961 - 1966	40	30	75
	After 1966	22	11	55

liquefaction of the backfill and/or the supporting base stratum; these cases will not be further addressed in this paper. Carefull study (Dennehy, 1985) of the remaining "failures" leads to the following conclusions regarding the major weaknesses of the psuedo-static procedure:

First, the values of the Code-specified seismic coefficient are not representative of the actual levels of acceleration that may develop in the backfill during moderate and strong earthquake shaking. Indeed there is little justification for the selected values. As noted by Seed (1975): "it is entirely possible that such empirical values of the seismic coefficient may lead to safe designs in many cases but until some means of judging their validity is developed, their use must be considered of questionable value". Futhermore, Wood (1973) has observed that: "In general, seismic coefficients are chosen that are significantly less than the peak accelerations to be expected in a suitable design earthquake, apparently on the assumption that some permanent outward movement of the wall can be tolerated. There appears to be no rational basis for the magnitude of the reduction made."

Indeed, despite the increase of the design coefficient form k to k' ≅ 2k for soils under the water table, some of the failed bulkheads may have experienced greater "effective" peak accelerations than they were designed for. Strong ground shaking can induce accelerations in excess of 0.50g. On the other hand, moderately-strong ground shaking might be amplified by the (non-liquefiable) backfill and foundation stratum. Such an amplification could be substantial if a thick backfill-foundation profile underlain by very stiff soil or rock is excited by an earthquake motion rich in frequencies near its own natural frequency(ies). To demonstrate the possiblitiy for such an amplification, theoretical, experimental and field evidence is available.

Some examples: Nadim & Whitman (1978) have shown for rigid retaining walls that the permanent displacement computed with a finite element model incorporating a Coulomb-type sliding surface in the backfill is substantially greater than the value obtained form rigid-plastic analysis, in which soil layer response (and amplification) is ignored. Small-scale shaking-table experiments conducted by the Japanese Port and Harbor Research Institute tend to confirm this behavior for anchored bulkheads. Although both the Nadim-Whitman and the shaking table models may exaggerate such an amplification due to spurious wave reflections at the lateral boundaries, some field evidence to this effect is also available.

Furthermore, the vertical component of the ground acceleration, which is ignored by the method, increases the "effective" acceleration that controls the seismic active and passive pressures (Davies et al, 1986, Richards & Elms 1979) by a factor of $(1-k_v)^{-1}$ [see Fig.4]. On the other hand, the increase of k by a factor of about 2 for soils below the water table may only partially accomodate the detrimental effects of strength degradation due to pore-water pressure buildup. Also note that in the majority of the studied Japanese case histories the aforementioned increase in the seismic coefficient had little effect in the design of the anchor, as a significant part of the latter is located above the water table. And, finally, this increase of k was undermined by the unfortunate 20%-33% reduction in the required factors of safety, as outlined in the previous section.

In conclusion, it appears that many of the "failed" bulkheads experienced "effective" peak accelerations which were essentially 30% to 50% higher than what these walls had been designed for.

Second, the available passive soil resistance against the anchor is often seriously overestimated by the Code procedure. While there is ample indirect empirical evidence supporting the above statement (recall the most frequent modes of failure), it is important to develop an understanding of the causes of this inadequecy of the Codes.

To begin with, recall that the Japanese Code requires that the active sliding surface should start at the elevation of the dredge line. By contrast, even the static design of anchored bulkheads most often assumes that this surface originates at the point of contraflexure, or the point of zero moment in the sheetpile. Tschebotarioff's (1978) "hinge at the dredge line" concept, useful as it may be for determining maximum bending moments in the sheetpile, is un-conservative for choosing the location and size of the anchor block/plate (Tsinker 1983). In fact, it is more likely that the active failure surface originates at or near the "point of rotation" rather than at the points of contraflexure or zero-moment. The location of this point depends on the relative stiffness of the sheetpile wall and the overall rigidity of the anchoring system -- but, no doubt, is generally deeper than the points of contraflexure and zero moment.

Moreover, under seismic loading the "point of rotation" tends to move farther down, as repeatedly demonstrated in small-scale shaking-table tests (Kitajima et al 1978, Murphy 1960) and in the theoretical studies using the model of Fig. 2. The explanation is clear: when acceleration increases, the active soil pressures against the wall increase while the passive ones supporting the wall decrease (see Fig. 4). Hence, the effective "span" of the

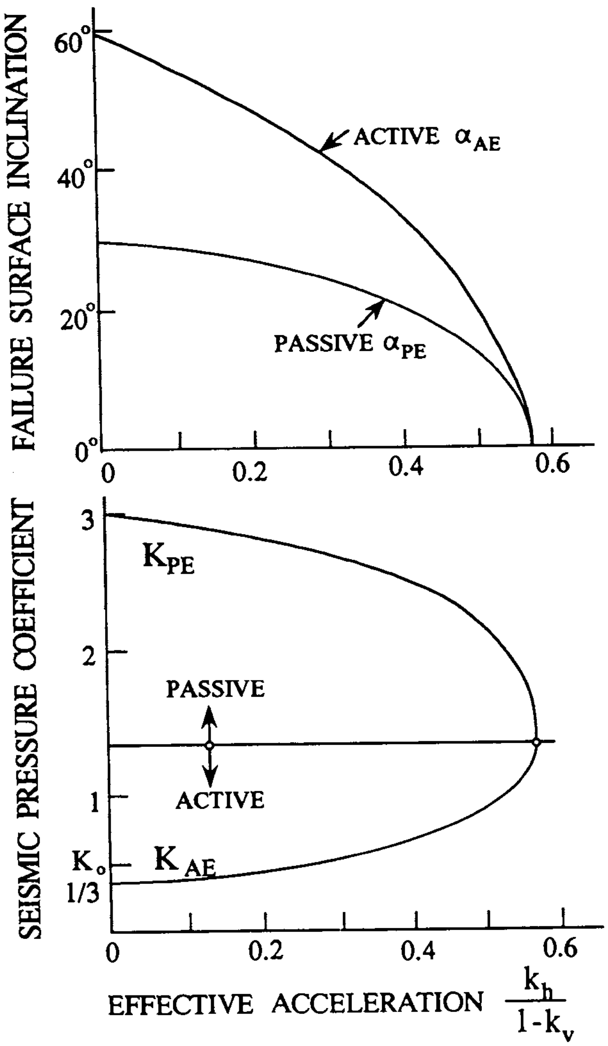


Figure 4. Effect of horizontal and vertical seismic coefficients on : (a) the angles of active and passive sliding wedges, and (b) the active and passsive earth pressure coefficients (Davies et al, 1986; Richards and Elms, 1979.)

sheetpile beam (in the terminology of the free-earth support method) tends to increase, and the origin of the active sliding surface (≅ "point of rotation") tends to be pushed downward. It appears that in several of the studied Japanese cases this "point" might have been located at a depth $f \geqq D/2$, where the depth of embedment, D, usually takes values in the range of 50% to 80% of H. It is thus evident that the Code recommendation of placing the origin at the dredge line (Fig.3) would in most cases underestimate the required tie-rod length, L.

An additional factor in the Code procedure contributing to an overestimation of the available passive anchor resistance stems from the use of the seismic coefficient k rather than the "effective" peak acceleration in the backfill, or at least of the increased coefficient k'. As illustrated in Fig. 4, increased acceleration levels imply not only reduced passive forces, but also flatter failure surfaces. And it is obvious that a smaller in reality angle α_{PE} than that assumed in the Code design would (further) reduce the capacity of the anchor.

EMPIRICAL SEISMIC DESIGN CHART

To arrive at a practical design chart (using the results of those case histories that did not involve liquefaction flow failures), two simple dimensionless indices have been selected. Their definition, significance, and methods of computation are explained below.

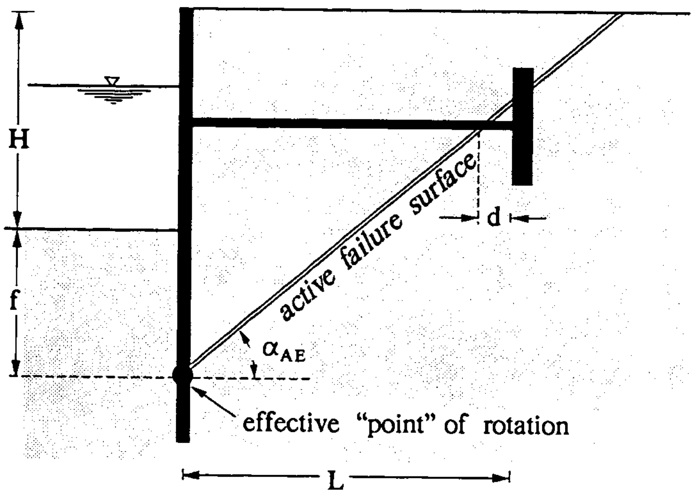


Figure 5. Definition of the Effective Anchor Index: EAI = d / H

(a) The "Effective Anchor Index" (EAI), representing the relative magnitude of the available passive anchor force: EAI is defined in Fig. 5, in terms of the horizontal distance d from the active failure surface to the tie-rod-anchor connecting point:

$$EAI = \frac{d}{H} \quad \text{.....................................(2)}$$

Note that the width of the anchor, 2B, does not appear directly in this index, despite its importance for the anchor resistance. This was a reluctant choice, out of necessity: in only a few of the analyzed case histories was this width reliably known! But, at least, 2B is indirectly reflected in Eqn 2 through the height H; indeed, according to the Code procedure, 2B depends chiefly on H and the backfill angle of shearing resistance ϕ.

The active failure surface is assumed to originate at the effective "point" of rotation, at depth f from the dredge line. In actual design f could be estimated from a numerical analysis, for example using the "Beam-on-Winlker-Foundation" model of Fig. 2. Taking f = D would lead to a slightly conservative length L.

The angle, α_{AE}, of inclination of the active sliding wedge is a decreasing function of the effective acceleration coefficient, k_e, as plotted in Fig. 4 for $\phi = 30^o$ and dry soil, where

$$k_e = \frac{k_h}{1 - k_v} \quad \text{...............................(3)}$$

The horizontal and vertical seismic coefficients, k_h and k_v, should be taken as fractions of the anticipated peak ground acceleration components during the design earthquake shaking; e.g., as suggested by Seed,

$$k_h \cong \frac{2}{3}\frac{\max a_h}{g} \quad \text{...............................(4)}$$

For cohesionless soils under the water table, to indirectly take into account both the potential strength degradation due to pore-water pressure buildup and the hydrodynamic effects, we suggest that k_e should increase to

$$k_e' \cong 1.50\, k_e \cong \frac{\max a_h}{1 - \frac{2}{3}\max} \quad \text{.....................(5)}$$

Finally, having established k_e', the angle α_{AE} can computed from the Coulomb-Mononobe-Okabe sliding-wedge analysis (Prakash 1981, Richards & Elms 1979).

(b) The "Embedment Participation Index" (EPI), provides a measure of the likely contribution of the embedment depth. If the wall were acting as a free cantilever (with no anchor), it would undergo horizontal displacement and rotation the magnitude of which would depend on the potential active and passive forces, F_{AE} and F_{PE}, and the respective moments of these forces about the 'point' of rotation. In the interest of simplicity, and being restricted by the available data of the analyzed case histories, EPI is defined as:

$$EPI = \frac{F_{PE}}{F_{AE}}\left(1 + \frac{f}{f + H}\right) \quad \text{........................(6)}$$

which for uniform backfill and foundation can be approximated as:

$$EPI \cong \frac{K_{PE}}{K_{AE}}\, r^2\, (1 + r) \quad \text{......................(7a)}$$

where

$$r = \frac{f}{f + H} \quad \text{....................................(7b)}$$

The ratio K_{PE}/K_{AE} of the passive to the active earth pressure coefficient is, in general, obtained from a Coulomb-Mononobe-Okabe analysis. K_{PE}/K_{AE} is a monotonically decaying function of the seismic coefficient and the angle of shearing resistance. Note that, for a wall-soil friction angle $\delta = 0$ and cohesionless soil,

$$1 \leq \frac{K_{PE}}{K_{AE}} \leq \tan^4(45^o + \phi/2) \quad \text{.................(8)}$$

where the upper bound is the familiar ratio of the static (k = 0) Rankine earth pressure coefficients, whereas the lower bound is reached at a critical effective acceleration (Richards & Elms, 1979).

$$\frac{k_h}{1-k_v} = \tan\phi \quad \text{.............................}(9)$$

It is noted that the flexural rigidity E_pI_p, of the sheetpile does not appear explicitly in the above definition of EPI, despite its obvious importance on the magnitude and shape of the wall deformation. Again, this was done reluctantly since the sectional moment of inertia, I_p, was only rarely reported in the studied cases. Nonetheless E_pI_p relates to the wall height H and the depth f, and hence it does affect (indirectly) EPI.

The two indices, EAI and EPI, computed for each one of the studied 75 anchored bulkheads, produce a point on the diagram of Fig. 6. The degree of damage of the particular bulkhead is reflected on the size and shading of the circle. Thus five different degrees of damage (0 - 4) are distinguished, as explained in the Table of Fig. 1, according to Kitajima & Uwake (1978). With justified reservation, in view of the rather crude way of characterizing the adequacy of the anchoring system and the effectiveness of embedment, and of the uncertainties regarding soil strength parameters and estimated ground acceleration, a clear picture emerges in Fig. 6. Two fairly distinct zones can be identified: Zone A, comprising mostly anchored bulkheads that suffered acceptable damage (degrees of damage 0, 1, or 2); and Zone B, within which bulkheads suffered unacceptable deformation or even failure (degrees of damage 3 and 4).

The shape of the line delineating the two zones does indeed suggest that the degree of damage suffered by an anchored bulkhead is dependent on both the adequacy of the anchoring system and the relative depth of embedment. Notice, however, that the flat shape of these lines implies that the importance of the Effective Anchor Index (EAI) is far greater than that of the Embedment Participation Index (EPI), as one might have anticipated from the earlier discussion on the types of observed failures.

Fig. 6 can serve as a Seismic Design Chart to be used in conjunction with (and to rectify the inadequacies of) the aforementioned pseudo-static design procedures. For instance, one can first follow the design steps 1,2,and 3 that were outlined in the second section of this article, but then determine the required length of the tie-rod form the following geometric expression:

$$L \geq (h + f).\cot\alpha_{AE} + (EAI)_c.H \quad \text{...........}(10)$$

where the critical value of the Effective Anchor Index, $(EAI)_c$, is read from the delineating line of the Chart for the specific value of the Embedment Participation Index (EPI), as illustrated in Fig. 6.

CONCLUSION

An empirical chart has been developed (Fig. 6) for guiding the design of anchored steel sheetpile bulkheads against strong earthquake shaking. Use of this Chart, along with the Mononobe-Okabe-based pseudostatic design procedure would lead to safer anchored bulkheads.

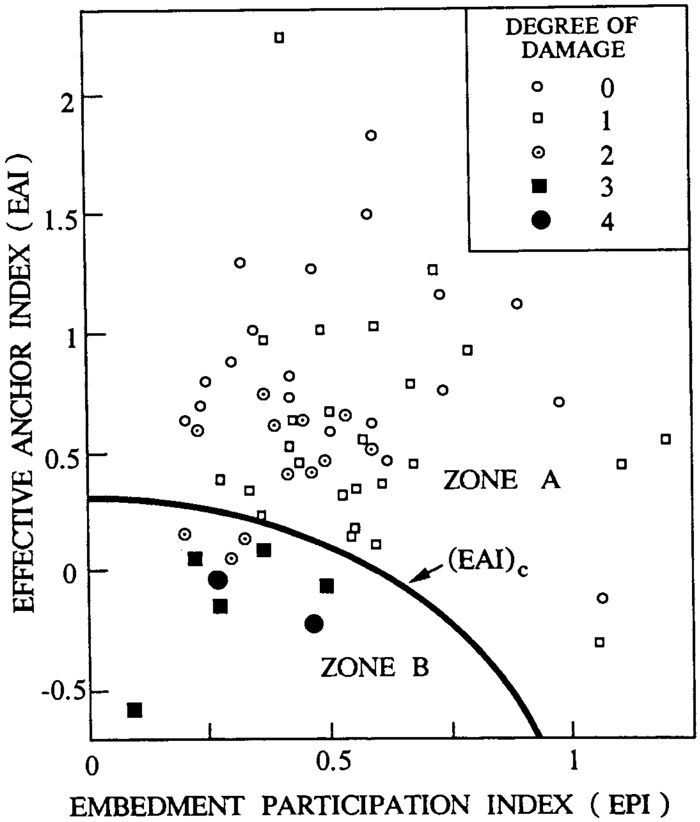


Figure 6. The developed Seismic Design Chart

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