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## Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics and Symposium in Honor of Professor I.M. Idriss

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### AN EMPIRICAL PREDICTIVE RELATIONSHIP FOR ASSESSING THE SEISMIC STABILITY OF SLOPES

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#### ABSTRACT

The objective of the study presented herein is to develop an empirical predictive relationship for permanent relative displacements for use in assessing the seismic stability of slopes, dams, and/or embankments subjected to active shallow crustal earthquake motions. A total of 330 horizontal motions, recorded at rock sites during 29 earthquakes in active shallow crustal regions (e.g., western North America: WNA), were used in this study. For each motion, the permanent relative displacements were computed using the Newmark sliding block procedure for a suite of yield-accelerations: 0.01, 0.05, 0.10, and 0.20 g. The predictive relationship proposed herein was derived by performing separate regression analyses for each yield-acceleration. This allows the relationship to be simply formulated in terms of ground motion characteristic parameters, independent of yield-acceleration ( $k_y$ ), and results in lower standard deviations than those for relations developed by regressing all the data in a single analysis. The non-linear mixed-effects technique was used to regress the relative displacement data as functions of maximum ground accelerations and velocities ( $A_{\max}$  and  $V_{\max}$ , respectively). The median permanent relative displacements predicted for WNA rock motions decreases with increasing  $k_y/A_{\max}$  but increases with increasing  $V_{\max}$ . Also, the rate of decrease in displacement with respect to  $k_y/A_{\max}$  varies as a function of  $k_y$ .

#### INTRODUCTION

The objective of the study presented herein is to develop an empirical predictive relationship for permanent relative displacements for use in assessing the seismic stability of slopes, dams, and/or embankments subjected to active shallow crustal earthquake motions at rock sites. The Newmark sliding block method was used to compute the permanent relative displacements. This method was proposed by Newmark (1965) for evaluating the seismic stability of slopes, wherein the sliding mass is modeled as a block on an inclined plane/ground. Displacement of the block relative to the plane initiates when the yield-acceleration ( $k_y$ ) is exceeded and continues until the velocities of the block and ground coincide. The permanent relative displacement is defined as the cumulative relative displacement at the end of ground shaking, as illustrated in Figure 1.

As may be surmised from Figure 1, the permanent relative displacements may vary with the orientation (or sign) of the ground motion. Accordingly, permanent relative

displacements were computed for both directions (i.e., +/-) of a ground motion. These displacements were treated as an individual data points in the regression analyses. The permanent relative displacements were computed for a suite of yield-accelerations: 0.01, 0.05, 0.10, and 0.20 g. The displacement data were then correlated to the maximum ground acceleration ( $A_{\max}$ ) and velocity ( $V_{\max}$ ).

Numerous empirical relationships for estimating permanent relative displacements have been developed over the past 30 years (e.g., Ambraseys and Srbulov, 1994; Gokhan and Rathje, 2008; Richards and Elms, 1979). Most of these previous studies used fixed-effects regression techniques (e.g., least squares method) and correlated permanent relative displacement to the ratio of  $k_y$  and the maximum peak ground acceleration ( $A_{\max}$ ), in addition to other ground motion parameters. This study differs from the previous studies in that the relationships proposed herein were developed by performing separate regressions for each  $k_y$  using the non-

linear mixed-effects (NLME) technique. Performing separate regressions for each  $k_y$  allowed the relationships to have relatively low standard deviations and to have simple functional forms that are independent of  $k_y$ . Using NLME regression analyses results in unbiased fits of the data, irrespective of the varying amount of data from different earthquakes.

Regarding the organization of this paper, first the strong ground motion dataset used in this study is described. Then, basic concepts of the NLME regression method are reviewed; the proposed functional form of the predictive model is discussed; and the results of the regression analyses are presented. It should be noted that this study did not consider the effects of vertical ground motions on the permanent relative displacements. It is also noted that the acronym "WNA" in this paper is used in a general sense to refer to "active shallow crustal" regions, not just to the western North America.

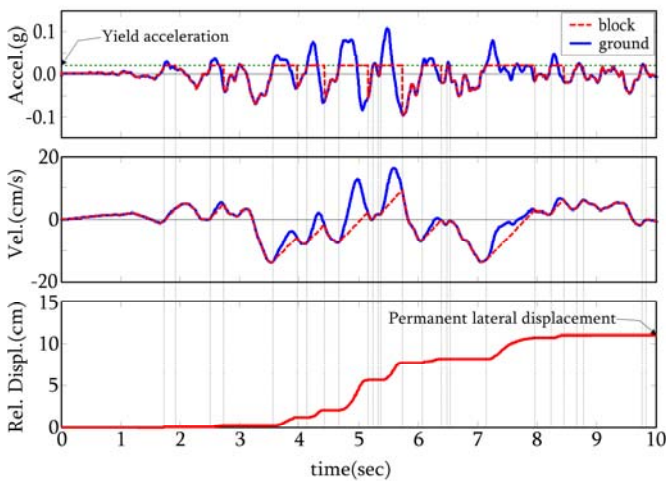


Fig. 1. Example of Newmark sliding block analyses for  $k_y = 0.03$  g and a ground acceleration time history (BES090: M6.9; R49.9km) from the 1989 Loma Prieta earthquake.

### STRONG GROUND MOTION DATASET

A total of 330 horizontal earthquake motions recorded at rock sites in active shallow crustal regions (e.g., WNA) were used to develop the empirical predictive relationships for permanent relative displacement. The ground motions were from a dataset assembled by McGuire et al (2001) to provide a library of strong ground motion time histories suitable for engineering analyses. The rock motion data for WNA were from 29 earthquakes, with the 1999 Chi-Chi earthquake being the most recent event. The moment magnitudes of these events range from 5.0 to 7.6, and the site-to-source distances range from 0.1 km to 199.1 km, where site-to-source distance is defined as the closest distance to the fault rupture plane. Figure 2 shows the earthquake magnitude and site-to-source distance distribution of the ground motion dataset.

The motions used in this study were classified by McGuire et al. as "rock" motions based on the site conditions at the

respective seismograph stations. The site classification scheme used by McGuire et al. is based on the third letter of the Geomatrix 3-letter site classification system shown in Table 1. Site categories A and B are considered to represent rock sites. This categorization is similar to that of the United States Geological Survey (USGS) shown in Table 2 in which rock sites encompass site classes A and B.

Table 1. Third letter: Geotechnical subsurface characteristics of Geomatrix 3-letter site classification.

Third letter	Site description	Comments
A	Rock	Instrument on rock ( $V_s > 600$ m/s) or < 5 m of soil over rock.
B	Shallow (stiff) soil	Instrument on/in soil profile up to 20 m thick overlying rock.
C	Deep narrow soil	Instrument on/in soil profile at least 20 m thick overlying rock, in a narrow canyon or valley no more than several km wide.
D	Deep broad soil	Instrument on/in soil profile at least 20 m thick overlying rock, in a broad valley.
E	Soft deep soil	Instrument on/in deep soil profile with average $V_s < 150$ m/s.

Table 2. USGS site classification.

Site class	Average shear wave velocity to a depth of 30 m: $V_{S30}$
A	$V_{S30} \geq 750$ m/s
B	$V_{S30} = 360 - 750$ m/s
C	$V_{S30} = 180 - 360$ m/s
D	$V_{S30} \leq 180$ m/s

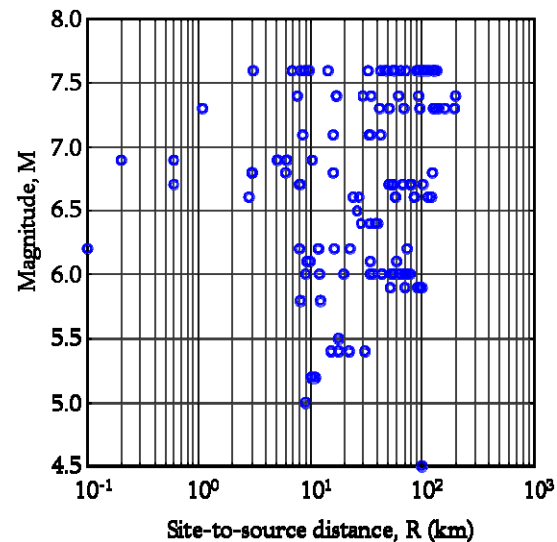


Fig. 2. Earthquake magnitude and site-to-source distance distribution.

Non-linear Mixed-effects Modeling (NLME)

As stated previously, the non-linear mixed-effects (NLME) regression technique was used to develop the empirical relationships in this study. NLME modeling is a maximum likelihood method based on normal (Gaussian) distribution and is primarily used for analyzing grouped data (i.e., datasets comprised of subsets). The NLME regression method allows regression models to account for both random-effects that vary from subset to subset and fixed-effects that do not. In this study, a subset consists of motions recorded during a given earthquake event. In comparison to applying a fixed-effects regression technique (e.g., the least squares method) to the entire dataset, a mixed-effects regression method allows both inter- and intra-earthquake uncertainty to be quantified. The inter-earthquake error is designated by  $\eta_i$  where the subscript,  $i$  represents the  $i^{\text{th}}$  earthquake (i.e., group) and has mean of zero and variance of  $\tau^2$ . The intra-earthquake error is designated by  $\varepsilon_{ij}$  where the subscript,  $ij$  indicates the  $j^{\text{th}}$  record of the  $i^{\text{th}}$  earthquake and has a mean of zero and variance of  $\sigma^2$ . The standard deviation of the total error can be determined by the following equation:

$$\sigma_{total} = \sqrt{\tau^2 + \sigma^2} \quad (1)$$

where,  $\sigma_{total}$  is the standard deviation of total error.

In more traditional regression techniques (e.g., least squares method), the entire dataset is regressed in a single analysis. However, because the dataset is comprised of motions from different earthquakes, with the number of recordings from each earthquake varying, the resulting regression is inherently unduly influenced by the earthquake having the largest number of motions. On the contrary, the NLME regression method produces unbiased fittings for each subset having different numbers of ground motion recordings. This is important because of the number of motions from each earthquake can vary significantly. The statistical analysis program R (version 2.5.0) was used to perform the NLME regression analyses (e.g., Lee, 2009).

Functional Form and Regression Results

The empirical predictive relationship developed in this study was derived by performing separate regression analyses for each  $k_y$ . This approach allows the predictive relationship to be formulated only in terms of ground motion characteristic parameters, independent of  $k_y$ , and allows the standard deviations to be estimated for each  $k_y$  value. Furthermore, this approach results in lower standard deviations than those for models developed by regressing all the data in a single analysis. This approach is in contrast to previous studies where permanent relative displacements were correlated to the ratio  $k_y/A_{max}$ , which results in complex functional forms and relatively large total standard deviations. This is attributable

primarily to the large variations in displacements for a given  $k_y/A_{max}$ , as shown in Figure 3.

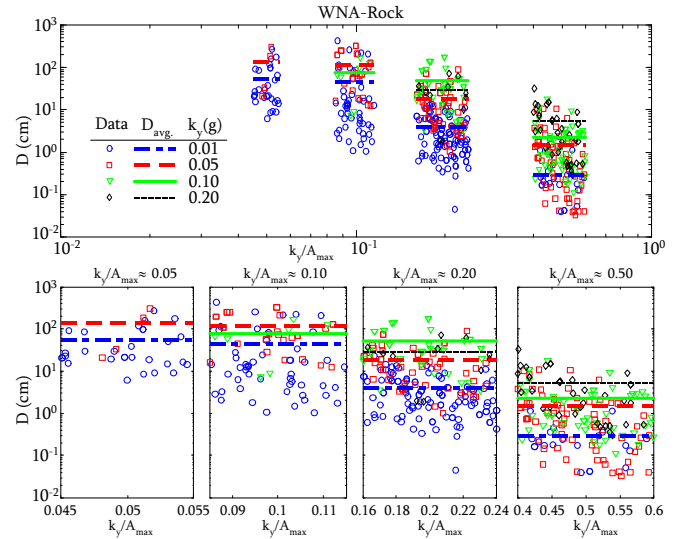


Fig. 3. Displacement data and averages for  $k_y/A_{max} = 0.05, 0.10, 0.20,$  and  $0.50$  for each  $k_y$  value considered in this study; plots for each  $k_y/A_{max}$  in semi-log scale (lower) and a plot for all the  $k_y/A_{max}$  in log-log scale (upper).

After considering numerous functional forms for the predictive relationship, the following model was found to provide the best fit of displacement data for all the  $k_y$  values:

$$\ln(D) = C_1 + C_2 \ln(A_{max}) + C_3 \ln(V_{max}) \quad (2)$$

where:  $D$  is the permanent relative displacement (cm);  $C_1, C_2, C_3$  are regression coefficients;  $A_{max}$  is the maximum ground acceleration (g); and  $V_{max}$  is the maximum ground velocity (cm/s). The regression coefficients and standard deviations determined from NLME regression analyses are listed in Table 3. Also, the  $A_{max}$  and  $V_{max}$  ranges of the displacement data used in the regression analyses for each  $k_y$  are listed in Table 4. It is accordingly recommended that the relationship (i.e., Eq. 2) be used only for  $A_{max}$  and  $V_{max}$  values that are within the ranges listed in Table 4.

Table 3. NLME Regression results: regression coefficients and standard deviations for each  $k_y$ .

$k_y$ (g)	$C_1$	$C_2$	$C_3$	$\tau_{ln}$	$\sigma_{ln}$	$(\sigma_{ln})_{total}$
0.01	1.08	0.57	1.21	0.24	0.49	0.55
0.05	0.54	1.13	1.07	0.27	0.50	0.57
0.10	-0.077	1.38	1.03	0.37	0.49	0.61
0.20	-2.02	1.41	1.24	0.43	0.50	0.66

Table 4. Ranges of  $A_{max}$  and  $V_{max}$  of the displacement data used in the regression analyses for each  $k_y$ .

$k_y$ (g)	$A_{max}$ (g)		$V_{max}$ (cm/s)	
	Min.	Max.	Min.	Max.
0.01	0.019	1.58	1.32	125.1
0.05	0.091	1.58	4.66	125.1
0.10	0.139	1.58	8.22	125.1
0.20	0.290	1.58	16.38	125.1

The median permanent relative displacements predicted for WNA rock motions using Eq. 2, in conjunction with the coefficients listed in Table 3, are shown in Fig. 4. As may be observed from this figure, the permanent relative displacements decrease with increasing  $k_y/A_{max}$  (i.e., decreasing  $A_{max}$  for a given  $k_y$ ) but increase with increasing  $V_{max}$ . It is also observed that the rate of decrease in displacement with respect to  $k_y/A_{max}$  varies as a function of  $k_y$ .

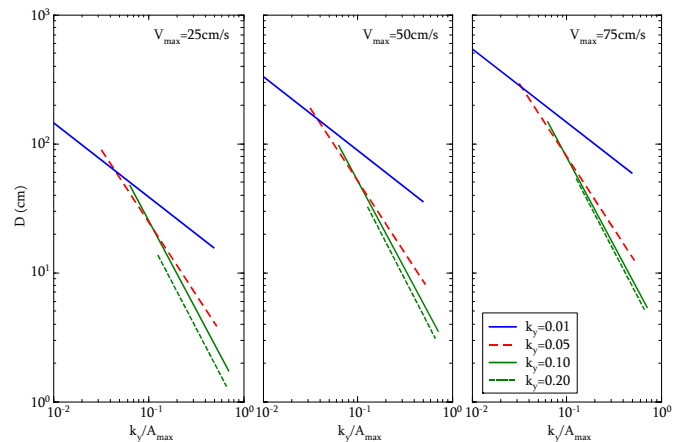


Fig. 4. Regression results for varying  $V_{max}$  and  $A_{max}$  for the  $k_y$  values considered in this study.

## CONCLUSIONS

Empirical predictive relationships for permanent relative displacement for use in assessing the seismic stability of slopes, dams, and/or embankments subjected to WNA rock motions have been developed. The predictive relationships proposed herein differs from existing relationships in that the non-linear mixed-effects (NLME) regression technique was used and separate regression analyses were performed for each  $k_y$  (i.e., 0.01, 0.05, 0.10, and 0.20). The resulting relationships have simple functional forms and correlate permanent relative displacement to  $A_{max}$  and  $V_{max}$ . The predicted median permanent relative displacements decreased with increasing  $k_y/A_{max}$  but increase with increasing  $V_{max}$ . It is also observed that a larger  $k_y$  tends to have a higher rate of decrease in displacement with respect to  $k_y/A_{max}$ .

## ACKNOWLEDGEMENTS

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## REFERENCES

- Ambraseys, N.N. and Srbulov, M. [1994]. "Attenuation of Earthquake-induced Ground Displacements." *Earthquake Engineering & Structural Dynamics*, Vol. 23, No. pp. 467-487.
- Gokhan, S. and Rathje, E.M. [2008]. "Empirical Predictive Models for Earthquake-Induced Sliding Displacements of Slopes." *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 134, No. 6, pp. 790-803.
- Lee, J. [2009]. "Engineering Characterization of Earthquake Ground Motions." Ph.D. Dissertation, Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, MI, USA, 246p.
- McGuire, R.K., Silva, W.J. and Costantino, C.J. [2001]. "Technical basis for revision of regulatory guidance on design ground motions: Hazard-and risk-consistent ground motion spectra guidelines." NUREG/CR-6728, US Nuclear Regulatory Commission, Washington, DC.

Newmark, N.M. [1965]. "Effects of earthquakes on dams and embankments." *Geotechnique*, Vol. 15, No. 2, pp. 139-160.

Program-R [version 2.5.0]. "A language and environment for statistical computing and graphics": <http://www.r-project.org/>.

Richards, R. and Elms, D.G. [1979]. "Seismic Behavior of Gravity Retaining Walls." *Journal of the Geotechnical Engineering Division, ASCE*, Vol. 105, No. GT4, pp. 449-464.