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SYNOPSIS This paper deals with the dynamic analysis of block sliding on rock slope along plane surfaces, such as joints, bedding planes and faults. According to the results obtained in the shaking tests the dynamic friction along a sliding surface is dependent on the velocity of relative motion, and can be determined by using the proposed method of test. On the basis of conducted experiments a differential equation was established. The integration of the dynamics equation by numerical method provides a basis for evaluation of slope stability in terms of critical displacement and dynamic instability.

INTRODUCTION

The failure of rock slopes sometimes may be caused by the block sliding under strong vibration due to the earthquake or the blasting effect. The dynamic process of such instability has not been properly studied.

In this paper, according to the results obtained in the laboratory experiments for the rigid contact between blocks, a non-linear differential equation was established for describing the process of block instability on rock slopes. A computer program was compiled for numerically integrating the proposed equation and can be used in the dynamic analysis of rock slopes.

DYNAMIC FRICTION ALONG SLIDING SURFACE

The dynamic friction along a plane surface does not keep constant during block sliding.

The laboratory experiments show the dependance of dynamic friction on the velocity of sliding and the cumulative displacement of relative motion.

Fig. 1. schematically shows the equipment for shaking test in determining the dynamic friction of a rigid rock block on the sliding surface.

The results of one series of vibration experiments are illustrated in Fig. 2. When the base of shaking table is subjected to a vibration with the amplitude of acceleration less than the starting friction at sliding initiation of the sliding surface the rock block has the same acceleration curve as that of the base (Fig. 2a). As

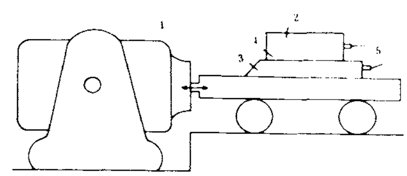

Fig. 1. Arrangement for dynamic friction test
1. Shaking table, 2. Sliding block, 3. Base block, 4. Sliding surface, 5. Accelerometers

the amplitude of the base vibration rises to a level slightly higher than starting friction, the acceleration curve of rock block begins to be shifted (Fig.2b). With the further increase of the base acceleration, the acceleration curve of rock block shows a quadratic wave form, with a concave part at the top and a phase shift (Fig. 2c,d).

The analysis of obtained curves is illustrated in Fig. 3. The acceleration of relative motion of rock block on the base can be obtained by the reduction of the absolute acceleration of the block from that of the base.

According to the curves shown in Fig. 3 the relationship between the dynamic friction in terms of the absolute acceleration of the rock block and the velocity of relative motion can be

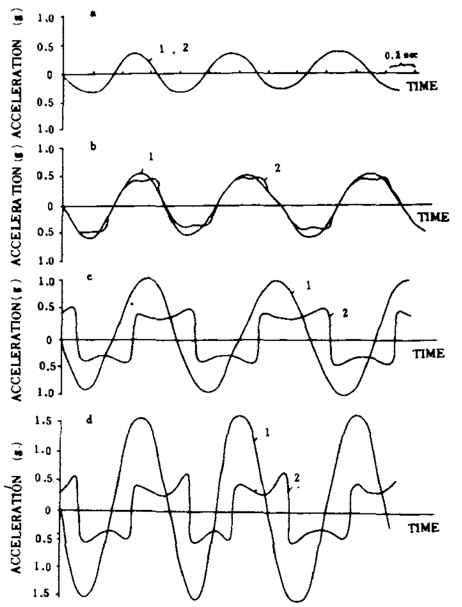

Fig. 2. Block sliding of granite along smooth surface under vibration
1. Base acceleration, 2. Block acceleration

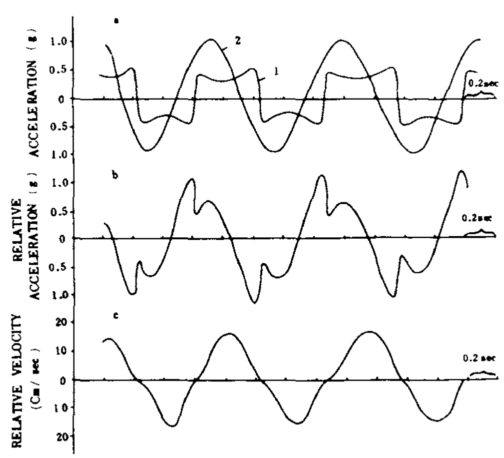

Fig. 3. Curves of dynamic friction along sliding surface
a. Acceleration of block (1) and of base (2), b. Acceleration of relative block motion, c. Velocity of relative block motion.

established(Fig. 4). And we have

$$f_d = f_d(\dot{X}) = f_{so} \cdot f_u(\dot{X}) \qquad (1)$$

Where f_{so} – the starting friction at sliding initiation.

Therefore, the procedure mentioned above provides a method for determination of the coefficient of dynamic friction which is an essential parameter for the dynamic analysis of block sliding.

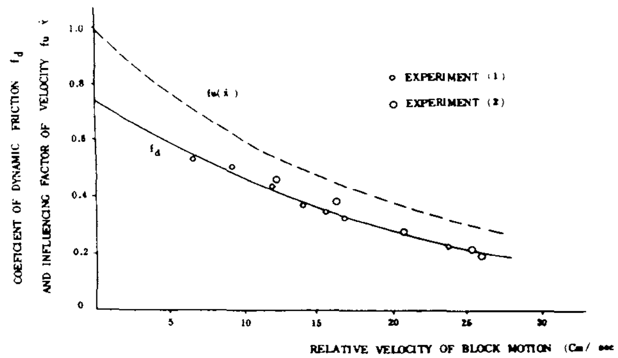

Fig. 4. Coefficient of dynamic friction f_d and the influencing factor $f_u(\dot{X})$ of sliding velocity on the dynamic friction

DYNAMIC EQUATION FOR BLOCK SLIDING ON ROCK SLOPE

The results obtained in the above section provide the basis for establishing the dynamic equation for analysis of block sliding on rock slope. Fig. 5 shows the condition of a rock slope subjected to the gravitational and vibration forces.

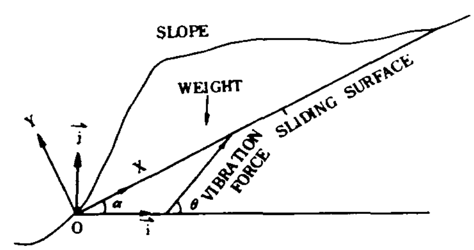

Fig. 5. Rock block on slope subjected to vibration

The expression of the vector of vibrating acceleration on the slope $\vec{a}$ is

$$\vec{a}_c = a_c \cos\theta \cdot \vec{i} + a_c \sin\theta \cdot \vec{j} \qquad (2)$$

Where a_c—amplitude of the vibrating acceleration, Then we have the sliding force F_s of a rock block on the vibrating slope in the direction X,

$$F_s = M(-\vec{a}_c - g\cdot\vec{j})\cdot\vec{T}_s \qquad (3)$$

And the resisting force developed on the sliding surface due to friction can be expressed by

$$F_r = M(-\vec{a}_c - g\cdot\vec{j})\cdot\vec{T}_n \cdot f_d \qquad (4)$$

The definition of unit vectors $\vec{i}, \vec{j}, \vec{T}_s, \vec{T}_n$ are shown in Fig.5, and we can write the relationship between them as following

$$\vec{T}_s = \vec{i}\sin\alpha + \vec{j}\cos\alpha$$
$$\vec{T}_n = \vec{i}\sin\alpha - \vec{j}\cos\alpha \qquad (5)$$

According to the principle of D'Alembert a difrential equation is derived from the resultant force along the sliding surface

$$\frac{d^2x}{dx^2}=F_s+F_r$$
$$=(-\vec{a}_c-g.\vec{j}).T_s+(-\vec{a}_c-g.\vec{j}).\vec{T}_n.\delta \qquad (6)$$

Where a_c - the vibrating acceleration, and

δ - a calculation sign

$\delta=0$, when $(-a_c \operatorname{Sin}(\theta-\alpha)+g\operatorname{Cos}\alpha)\leq 0$

$\delta=f_d$, when $\dot{X}<0$;

$\delta=-f_d$ when $\dot{X}>0$;

$\delta=\frac{a_c \operatorname{Cos}(\theta-\alpha)+g \operatorname{Sin}\alpha}{-a_c\operatorname{Sin}(\theta-\alpha)+g \operatorname{Cos}\alpha}$, when $\dot{X}=0$

and $|a_c\operatorname{Cos}(\theta-\alpha)+g \operatorname{Sin}\alpha|\leq(-a_c\operatorname{Sin}(\theta-\alpha)+g\operatorname{Cos}\alpha).f_{so}$

$\delta=\frac{a_c \operatorname{Cos}(\theta-\alpha)+g \operatorname{Sin}\alpha).f_{so}}{|a_c \operatorname{Cos}(\theta-\alpha)+g \operatorname{Sin}\alpha)}$, when $\dot{X}=0$

and $|a_c \operatorname{Cos}(\theta-\alpha)+g \operatorname{Sin}\alpha|>(-a.\operatorname{Sin}(\theta-\alpha)+g\operatorname{Cos}\alpha).f_{so}$

The right hand of equation (6) is a function of $t, \dot{X}, X_{ac}$, because we have

$f=f_s.f_u(\dot{X})=f_{so}.f_u(X_{ac}).f_u(\dot{X})$. In this case the equation can be solved by means of integration step by step for the given initial conditions

$$X(0)=0$$
$$\dot{X}(0)=0 \qquad (7)$$
$$X_{ac}(0)=0$$

Assume that

$$V_1(t)=-a_c \operatorname{Cos}(\theta-\alpha)-g \operatorname{Sin}\alpha \qquad (8)$$
$$V_2(t)=-a_c \operatorname{Sin}(\theta-\alpha)+g \operatorname{Sin}\alpha$$

then

$$\ddot{X}(n\Delta t)=V_1(n\Delta t)+V_2(n\Delta t).\delta \qquad (9)$$

Where t — time increment;

δ — $\delta(X_{ac.n},\dot{X}_n)$ function defined by equ.(6)

$X_{ac.n}$ — cumulative displacement of rock at $t=n\Delta t$;

$\dot{X}_n$ — Velocity of block sliding. along the surface at $t=n\Delta t$;

n — the number of integration steps;

The velocity of relative motion then can be obtained

$$\dot{X}_n=\dot{X}_{n-1}+\ddot{X}((n-1)\Delta t).\delta_{n-1} \qquad (10)$$

We assume $\dot{X}_n=0$, if $\dot{X}_n.\dot{X}_{n-1}\leq 0$, it means the change of the direction of sliding velocity.

The displacement of sliding motion X_n then can be obtained

$$X_n=X_{n-1}+\dot{X}_n\Delta t \qquad (11)$$

and the cumulative displacement at $t=n\Delta t$

$$X_{ac.n}=X_{ac.n-1}+|\dot{X}|\Delta t \qquad (12)$$

THE EVALUATION OF DYNAMIC STABILITY

Based on the quasi-dynamic principles the critical acceleration $a_{c.cr}$ be obtained

$$a_{c.cr}=\frac{-g(\operatorname{Cos}\alpha.f_{so}+\operatorname{Sin}\alpha)}{\operatorname{Cos}(\theta-\alpha)+\operatorname{Sin}(\theta-\alpha).f_{so}} \qquad (13)$$

As soon as the acceleration of slope vibration $a_c(t)$ exceeds the critical acceleration $a_{c.cr}$, the overloading condition occurs and the factor of stabitity becomes less than 1.0 quasi-instability. However, for a dynamic system this condition really means that the sliding motion can be initiated. In one case the dynamic stability may be restored after the end of the vibration, giving a certain amount of relative displacement. In other case, the dynamic instability can occur when the sliding motion is continuing after the end of the vibration.

According to the conditions mentioned above three critiria for evaluation of dynamic stability of rock slopes can be suggested.

(1) Safety factor of quasi-dynamic stability and the critical acceleration;

(2) The critical ultimate displacement of relative motion; and

(3) Dynamic instability of sliding.

COMPUTER PROGRAM AND EXAMPLE OF COMPUTATION

According to the equations (7)-(12) a computer program has been compiled for the stability evaluation of laboratory experiments.The ability of the computer program can be illustrated by an example of computation. As shown in Fig. 6, the rock slope contains a rock block intersected by a through-going joint with dipping angle $\alpha=20^{\circ}$ in case I and $\alpha=30^{\circ}$ in case II.

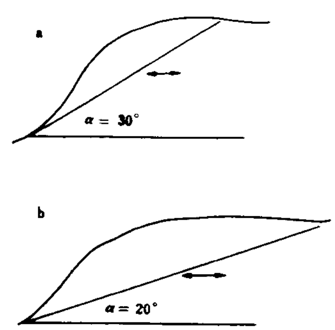

Fig. 6. Block sliding condition for computation examples

The computed results of case I are illustrated in Fig. 7.

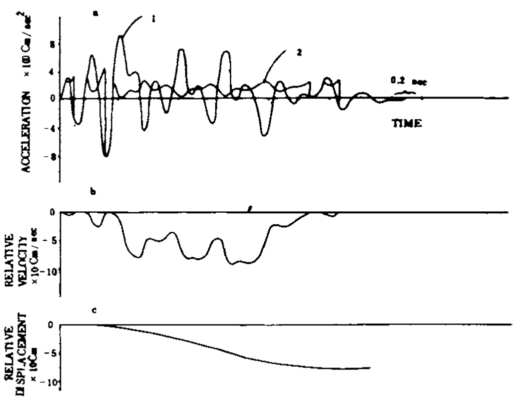

Fig. 7. Sliding velocity and displacement of rock block on a joint surface dipping at $\alpha=20^{0}$

1. Acceleration of rock slope in the direction of sliding surface
2. Acceleration of rock block in the direction of sliding surface

The curve 1 in Fig. 7a is the acceleration of slope vibration in the direction along joint surface according to the field seismic record. The resuts of computation are shown in Fig. 7b.c. In this case the quasi-dynamic analysis gives a safety factor less than 1.0. The dynamic analysis shows the block will slide along the joint surface. However, as shown in Fig. 7b, the velocity of block sliding becomes smaller and smaller before the end of vibration, so that the the motion eventually stops and the stability of block is restored again. The ultimate sliding displacement of the block along joint surface reaches 73 cm. According to the velocity and displacement curves within the time interval of 0.4 sec the block sliding is of low velocity and small displacement. After then the velocity of block sliding increases gradually. The maximum velocity reaches 85 cm/sec, and the displacement is linearly increasing during this time interval. At 0.9 sec before the end of slope vibration the velocity of block sliding decreases down to 25cm/sec and since then the displacement practically stoped. The situation for $\alpha=30^{0}$ in case II differs from that in the abovementioned case. As shown in Fig. 8 the velocity of block sliding continuously increases and reaches 320cm/sec by the end of slope vibration. The displacement reaches 300 cm. After the end of slope vibration the velocity of block sliding is continuing to increase, so the stability can't be restored again and the displacement progressively increases with time. At 0.7 sec after the end of vibration the velocity of block sliding reaches 350cm/sec and displacement over 690cm. The sliding is still accelerated.This situation indicates the occurrence of dynamic instability.

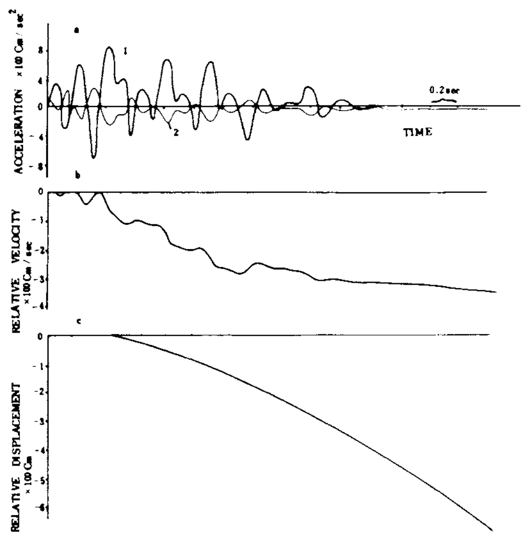

Fig. 8. Sliding velocity and displacement of rock block on a joint surface dipping at $\alpha=30^{0}$

CONCLUSIONS

1. The labratory experiments indicated the variation of friction along a joint surface during sliding. An obvious decrease of dynamic friction is observed with increase of the velocity of sliding motion.
2. The coefficient of dynamic friction can be determined using a shaking table and acceleremeters
3. A non-linear dynamic equation was established. The integration of the equation using step method provides the basis for stability evaluation.
4. In addition to the conventional quasi-dynamic factor of safety, this paper suggested the criteria of safety in terms of critical displacement and dynamic instability.

ACKONWLEDGEMENT

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REFERENCE

Wang Sijing, (1977) Preliminary Notes on the dynamic stability of Rock Slopes, Scientia Geologica Sinica, No.4(In Chinese).