13 Mar 1991, 1:30 pm - 3:30 pm

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Proceedings: Second International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, March 11-15, 1991, St. Louis, Missouri, Paper No. 5.71

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SYNOPSIS : This paper deals with stiffnesses of pile groups based on single pile stiffness as worked out by Novak and Sharnouby (1983) using dynamic analysis. The interaction factors between two neighbouring piles are used as those derived by Poulos (1971) statically but assumed to hold under dynamic situation at low frequencies such as involved in earthquakes. Pile groups of 4 to 32 piles in single circular ring and upto 60 piles in multiple rings as used under raft foundations of circular chimneys and overhead water tanks are accordingly analysed. It is found that the stiffness of the group is much less than the vector sum of individual pile stiffness, the reduction factor being quite different for vertical and horizontal translational stiffnesses and the rotational stiffness of the group.

INTRODUCTION

For seismic analysis of structure supported on pile foundations, consideration of soil-pile-structure interaction is essential since it will considerably affect the natural periods of vibration as well as damping of the system, hence the seismic response too. In such analyses, the pile foundation is frequently substituted by a set of rotational and translational springs and dampers at the base of the superstructure. The problem has mainly been approached in two steps, first to determine the dynamic stiffness and damping of a single pile and, second, to combine them for the group of piles by taking suitably their interaction into account. In this paper, the approaches used so far have been briefly reviewed and a method of superposition is derived to obtain the group properties using those of one pile.

DYNAMIC ANALYSIS OF PILES

Three techniques have so far been evolved for predicting the dynamic pile behaviour. The first approach represents soil as an elastic continuum (Novak, 1974; Nogami & Novak, 1976; Novak and Nogami, 1977), the second represents the soil-pile system by a set of discrete masses, springs and dashpots (Penzien, 1970; O'Rourke and Dobry, 1979), the third one uses the finite element method (Blaney et al, 1976; Wolf and Von Arx, 1978). There are, however, certain problems difficult to be tackled by any method. These are (i) the time variable separation of the pile from soil that can appear at large displacements occurring near top of pile where the confining pressure is low, (ii) nonlinearity, (iii) variation of soil modulus with local state of stress and (iv) wave scattering among piles in a group. These aspects are difficult to incorporate even in the finite element method and some of them would call for a truly three dimensional nonlinear approach. Despite these difficulties, comparison of experimental results with theoretical prediction indicates (Novak and Sharnouby, 1983) that the theory can yield a reasonable estimate of pile stiffness, damping and dynamic response if the variation of soil properties with depth and the possible lack of fixity of the pile tip are accounted for at least approximately. Their results for stiffness and damping of single piles are used here to obtain the properties of pile groups.

STIFFNESS AND DAMPING MATRICES OF SINGLE PILE

For a single pile having area A, radius r, moment of interia I, modulus of elasticity E embedded in a soil with shear wave velocity Vs, (Fig. 1), the stiffness and damping coefficients can be written as the pile stiffness matrix $[K]^S$ and the damping matrix $[C]^S$ as follows :

$$[K]^S = \begin{bmatrix} k_{zz} & 0 & 0 \\ 0 & k_{xx} & k_{x\Psi} \\ 0 & K_{\Psi x} & k_{\Psi\Psi} \end{bmatrix}, \quad [C]^S = \begin{bmatrix} c_{zz} & 0 & 0 \\ 0 & c_{xx} & c_{x\Psi} \\ 0 & c_{\Psi x} & c_{\Psi\Psi} \end{bmatrix} \qquad ...(1)$$

where

$$k_{zz} = f_{v1}E\ A/r,\ k_{xx} = f_{x1}E\ I/r^3,\ k_{\Psi\Psi} = f_{\Psi 1}E\ I/r,$$

$$k_{x\Psi} = k_{\Psi x} = f_{c1}E\ I/r^2, \quad \text{and}$$

$$c_{zz} = f_{v2}E\ A/v_s,\ c_{xx} = f_{x2}E\ I/(r^2 v_s),\ c_{\Psi\Psi} = k_{\Psi 2}E\ I/v_s,$$

$$c_{x\Psi} = c_{\Psi x} = f_{c2}E\ I/(r\ V_s),$$

In equations (1), the functions 'f' were introduced initially by Novak (1974) and successively modified, to account for differences in the half space theory, by Novak and Aboul-Ella (1978). These depend on the system parameters the most important ones being

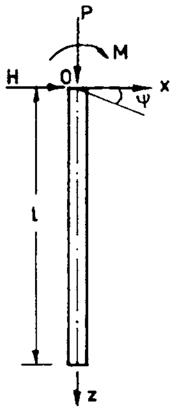

Fig. 1 - Single Pile

pile-soil stiffness ratio and the soil profile and variation of soil properties with depth. Pile tip condition is important mainly for vertical motion. The dynamic pile stiffness decreases with increasing frequency only if the soil to pile stiffness ratio is very small (Nogami and Novak, 1976). Most piles are fairly slender and in average soils, dynamic stiffness can be considered to be <u>almost frequency independent</u>. Pile damping increases linearly with frequency and thus is well modelled by a constant of equivalent viscous damping c which is frequency independent. They are insensitive to Poisson's ratio ν except for it approaching 0.5 and for high frequencies (as in case of machine foundations). For earthquake excitations, they may be taken independent of ν . Slenderness ratio (l/r) is important for short piles and for vertical vibration where pile stiffness is high. For horizontal vibration where the piles are much more flexible, the stiffness and damping parameters are independent of pile slenderness ratio and tip condition for (l/r) greater than 25 for uniform homogeneous soil conditions and 30 for stiffness increasing with depth. Accordingly, usable values of functions 'f ', which could be considered frequency independent have been presented by Novak and Sharnouby (1983). For working out pile group stiffness and damping, the 'f' valuea have been taken from this paper which should be applicable to most seismic design purposes.

STIFFNESS AND DAMPING OF PILE GROUPS

Pile foundations, in practically all cases, consist of piles in group. It is well known that the behaviour of a pile in a compact group is quite different from that of a single pile because of pile-soil-pile interaction. These interaction effects depend mainly on pile spacing. When the distance between the pile is large (greater than about 20 diameters) the piles do not affect each other (Howell, 1984) and the group stiffness and damping are simply the vector sums of the individual pile stiffness and damping. This is not often the case as the pile spacing used in groups can be as small as 3 times the diameter and generally the pile-soil-pile interaction exerts considerable influence on the stiffness and damping of the group.

(a) Pile Interaction Neglected

The individual pile stiffness, which is about its own axis, has to be transferred to the global axis which is the vertical axis passing through the centre of the cap (Fig. 2). Neglecting the interaction between the various piles, it is easily seen that the global 3x3 stiffness matrix of the entire group $[K]^g$

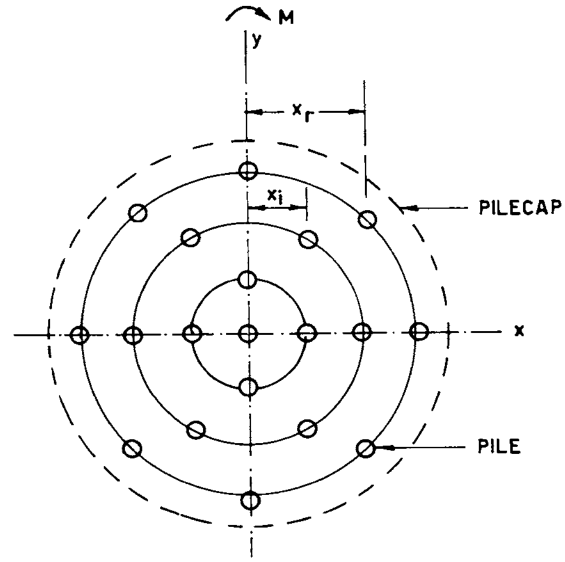

Fig. 2 - Global Axes for Pile Group

will be as follows :-

$$[K]^g = \begin{bmatrix} K_{zz} & 0 & 0 \\ 0 & K_{xx} & K_{x\psi} \\ 0 & K_{\psi x} & K_{\psi\psi} \end{bmatrix} \qquad (2)$$

Similarly the global 3x3 damping matrix of the entire group will be $[C]^g$ with letter K replaced by C. The elements of these matrices will have the following values for n piles in the group where Σ represents the summation for all n piles. For vertical displacement

$$K_{zz} = \Sigma k_{zz}, \text{ and } \quad C_{zz} = \Sigma c_{zz} \qquad (3)$$

Horizontal translation :

$$K_{xx} = \Sigma k_{xx}, \text{ and } \quad C_{xx} = \Sigma c_{xx} \qquad (4)$$

Rotation of the cap in the vertical plane, referred to pile cap base :

$$K_{\psi\psi} = \sum_{r=1}^{n} (k_{\psi\psi} + k_{zz} x_r^2), \quad \text{and}$$

$$C_{\psi\psi} = \sum_{r=1}^{n} (c_{\psi\psi} + c_{zz} x_r^2) \qquad (5)$$

Coupling between horizontal translation and rotation :

$$K_{x\psi} = K_{\psi x} = \Sigma k_{x\psi}, \quad \text{and}$$

$$C_{x\psi} = C_{\psi x} = \Sigma c_{x\psi} \qquad (6)$$

The above expressions are correct for fixed head piles, the usual case. For pinned head piles $k_{\psi\psi} = k_{\psi x} = 0$ and $c_{\psi\psi} = c_{\psi x} = 0$ in these equations and k_{xx} must be evaluated for pinned head piles. For vertical motion k_{zz} and c_{zz} are independent of pile head condition.

(b) Pile Interaction Included

The pioneering work on pile-soil-pile interaction has been carried out by Poulos (1968, 1971, 1979) and Poulos and Davis (1980) under static loads. These and a few other studies show that the main effects of static pile interaction are, an increase in settlement of the group, redistribution of pile stresses and redistribution on pile loads for rigid pile caps. However, studies of dynamic pile-soil-pile interaction are very recent, relatively few and limited to linear elasticity e.g. Wolf and Von Arx (1978), Wolf et.al. (1981) using FEM approach, Waas and Hartmann (1981) using Semi-analytical approach and approximate analytical solution by Nogami (1980) and Novak and Sheta (1982). The findings of these studies are significant showing that the dynamic stiffness and damping of pile groups are more frequency dependent than the single piles. Group stiffness and damping may be increased or decreased by pile-soil-pile interaction.

Group effect is measured in terms of group efficiency ratio, GER defined as :

$$GER = K_{group} / \Sigma k_i, \qquad i = 1 \text{ to } n$$

where k_i = stiffness of the individual pile in isolation.

The dynamic behaviour of pile groups is still not well understood and whatever theoretical predictions have been made, they have not yet been verified by suitable experiments. The static interaction method (Poulos, 1971) is sufficiently accurate for dynamic analysis if the frequency range is low (Howell 1984) as is mostly the case with earthquakes and wind. Hence in this work, the approach given by Poulos has been used and extended further to facilitate computation for a large groups of piles using a computer program.

STATIC INTERACTION COEFFICIENTS FOR GROUP STIFFNESS AND DAMPING

Poulos (1971) has shown that the analysis for response of single piles can be extended to pile groups by using interaction factors between two neighbouring piles. The interaction factor α_{ij} is defined as the fractional increase in deformation of pile i due to the presence of the similarly loaded neighbouring pile 'j'. If k_i represents the stiffness of a single isolated pile, then the deformation of pile i in a group of n piles may be written as :

$$\Delta i = \frac{1}{k_i} \sum_{j=1}^{n} \alpha_{ij} P_j \qquad (8)$$

where P_j = load on pile j.

For vertical and lateral loading, interaction factors are presented in chart form in Poulos and Davis (1980). For vertical motion α_v is given for three values of slenderness, l/d. The variation of soil stiffness with depth is accounted for by a factor $\rho = G_{ave}/G_s$ (where G_{ave} = average shear modulus over the pile length, G_s = shear modulus of soil at pile base, and the pile / soil stiffness ratio, E/G_s.

For transverse (horizontal) loading, the pile group is more flexible and its behaviour depends on the length of the upper part of each pile which participates in the deformation under lateral load. This critical length l_c is developed by Randolph and Poulos (1982) as :

$$l_c = 2r \, (E/G_c)^{2/7} \qquad (9)$$

where G_c = average value of shear modulus of soil over l_c. For the solution of Eq. (9), few iterations are required. Randolph and Poulos (1982) have formulated the expression for interaction factor for horizontal translation under fixed head condition as :

$$\alpha_h = 0.6 \, \rho_c (E/G_c)^{1/7} \frac{r}{s} (1 + \cos^2 \beta) \qquad (10)$$

where α_h = horizontal interaction factor for fixed head piles, s = pile spacing.

$$\rho_c = (G_q/G_c) \text{ and } G_q = \text{Modulus at } l_c/4 \qquad ...(11)$$

β is defined as the angle between the direction of loading and the line connecting the pile centres.

If α_h exceeds 1/3, a correction is required as follows :

$$\alpha_h' = 1 - \frac{2}{\sqrt{27 \, \alpha_h}} \qquad (12)$$

For rotation in the vertical plane, the vertical and horizontal interaction factors already calculated can be used as explained subsequently.

Using these interaction factors, the following expressions have been derived for vertical, horizontal and rotational stiffness of pile group. The expressions are general and can be used for any configuration of pile group. A computer program has also been written for circular pattern which is usually the case for circular foundations.

(i) Vertical Group Stiffness

Consider a group of n piles with each pile being subjected to a vertical load P, which would have produced displacement Δ if pile was alone. Total load nP will cause uniform displacement of the group Δv.

Displacement of pile i under individual action, $\Delta = P/k_{zzi}$.

By definition, increase in displacement of pile i due to pile r = $\alpha_{vir}\Delta$ where α_{vir} = vertical interactrion factor.

Therefore, total displacement at i due to n piles

$$\Delta vi = \Delta (1 + \sum_{r \neq i}^{n} \alpha_{vir})$$

since $\alpha_{vii} = 1$, Δ_{vi} can be written as

$$\Delta vi = \Delta \sum_{r=1}^{n} \alpha_{vir}$$

Effective stiffness of pile i in the group

$$= P/ \Delta_{vi} = k_{zzi}/ \sum_{r=1}^{n} \alpha_{vir}$$

Therefore for n piles, the total group stiffness will be

$$K_{zz} = \sum_{i=1}^{n} (k_{zzi}/ \sum_{r=1}^{n} \alpha_{vir}) \tag{13}$$

(2) Horizontal Group Stiffness

On the same lines as for vertical stiffness, the horizontral group stiffness can be written as :

$$K_{xx} = \sum_{i=1}^{n} (k_{xxi}/ \sum_{r=1}^{n} \alpha_{hir}) \tag{14}$$

(3) Rotational Group Stiffness

It will have two components as evident in Eq. (5), the first represents the sum of individual rotational stiffnesses and the other giving the contribution of varying pile loads to the moment of resistance. Separate expressions are derived for both the components as follows :

(i) Due to pile vertical stiffness

Let a group of n piles be subjected to moment M (Fig. 2). Considering the pile cap to be rigid, let Ψ be the rotation of the cap.

$$\text{Then load on pile i} = \frac{M. x_i}{\sum_{i=1}^{n} k_{zzi} x_i^2} = Ax_i \text{(say)} \quad \ldots(15)$$

Similarly load on pile r $= Ax_r$ and ratio of loads of pile r to pile i= x_r/x_i.

Also, from geometry, the vertical displacement of pile i, $\Delta i = \Psi x_i$ and that of pile r, $\Delta r = \Psi x_r$. Taking interaction into account, additional deflection of pile i due to load on pile r will be Δ_i (α_{vir} x_r/x_i). Therefore

$$\text{Net displacement of pile i} = \Psi x_i [1 + \sum_{r \neq i}^{n} (\alpha_{vir} x_r/x_i)]$$

$$= \Psi x_i \sum_{r=1}^{n} (\alpha_{vir} x_r/x_i) \text{ since } \alpha_{vii} = 1. \tag{16}$$

$$= \Psi x_i \text{ F (say) where } F = \sum_{r=1}^{n} (\alpha_{vir} x_r/x_i) \tag{17}$$

$$\text{Effective stiffness of pile i} = k_{zzi}/F. \tag{18}$$

$$\text{Vertical force of pile i} = \Psi x_i\, k_{zzi}/F. \tag{19}$$

$$\text{Moment contribution of pile i} = k_{zzi}\, \Psi x_i^2/F \tag{20}$$

Hence total moment of resistance

$$= \sum_{i=1}^{n} (\Psi x_i^2 k_{zzi}/F) \tag{21}$$

Group rotational stiffness due to pile vertical stiffness

$$M/\Psi = \sum_{i=1}^{n} k_{zzi}\, x_i^2 /F \tag{22}$$

(ii) Due to pile rotational stiffness

Group rotational stiffness due to the rotational stiffness, being similar to horizontal loads,

$$= \sum_{i=1}^{n} (k_{\Psi\Psi i}/ \sum_{r=1}^{n} \alpha_{hir}) \tag{23}$$

Hence total group rotational stiffness

$$K = \sum_{i=1}^{n} \frac{k_{\Psi\Psi i}}{\sum_{r=1}^{n} \alpha_{hir}} + \sum_{i=1}^{n} \frac{k_{zzi}\, x_i^2}{\sum_{r=1}^{n} \alpha_{vir}\, x_r/x_i} \tag{24}$$

DAMPING OF PILE GROUPS

The static procedure developed above offers no guidance as to the effect of pile-soil-pile interaction on soil damping. It is generally accepted, however, that a reasonable approach is to reduce the group damping in the same ratio as group stiffness. This is because the critical damping is proportional to stiffness and if stiffness is reduced without the damping constant also being reduced, damping ratio will increase. This increase is not observed in practice. Therefore, the expression for group damping may be written as given below :

Vertical group-damping

$$C_{zz} = \sum_{i=1}^{n} \frac{c_{zzi}}{\sum_{r=1}^{n} \alpha_{vir}} \tag{25}$$

Horizontal group-damping

$$C_{xx} = \sum_{i=1}^{n} \frac{c_{xxi}}{\sum_{r=1}^{n} \alpha_{hir}} \tag{26}$$

Rotational group-damping

$$C_{\Psi\Psi} = \sum_{i=1}^{n} \frac{c_{\Psi\Psi i}}{\sum_{r=1}^{n} \alpha_{hir}} + \sum_{r=1}^{n} \frac{c_{zzi}\, x_i^2}{\sum_{r=1}^{n} \alpha_{vir}\, x_r/x_i} \tag{27}$$

RESULTS

For working out numerical results, the following data was adopted :

Pile : $E = 10500\ N/mm^2$		Soil : $G_s = 5.0\ N/mm^2$	
r =	300 mm	G_c =	$2.2\ N/mm^2$
l =	15 m	l_c =	6.5 m
l/r =	50	ρ_c =	$G_q/G_c = 0.75$

The stiffness coefficients for single pile were taken from Novak and Sharnouby (1983) and the interaction factors between two adjoining piles were adopted from Poulos (1979) for vertical loads 'α_{vir}' and from Poulos and Randolph (1982) for horizontal loads 'α_{hir}'.

Case 1 : 4 piles in square pattern with s = 1.8 m, s/r = 6.0, gave the following values of GER: for vertical stiffness = 0.54, horizontal stiffness = 0.485, bending = 0.96.

The corresponding values given by Novak are 0.59, 0.5 and 1.25 respectively. If the rotational effect of pile top is neglected, the bending GER by the present method works out as 1.56. Thus Novak's value lies in between 0.96 and 1.56.

Case 2 : Piles in Single Circular Ring with 4 to 32 piles at varying spacing give the GER values as given in Table 1.

TABLE I. Values of GER for Piles in Single Circular Ring

No. of piles	s/r	Radius of ring mm	GER Vertical	GER Horizon.	GER Bending
	6	1150	0.448	0.470	1.574
4	12	2290	0.548	0.638	1.300
	20	3820	0.655	0.748	1.149
	6	2290	0.318	0.375	0.987
8	12	4580	0.455	0.546	0.869
	20	7640	0.673	0.668	0.795
	6	5730	0.254	0.299	0.457
20	12	11460	0.505	0.461	0.543
	20	19100	0.693	0.587	0.709
	6	9170	0.284	0.271	0.328
32	12	18330	0.508	0.426	0.522
	20	30600	0.696	0.553	0.706

Case 3 : 18 piles in two concentric rings of 6 and 12 piles at 1.7 m and 3.44 m radius respectively gave the following GER :

s/r	Vertical	Horizontal	Bending
6.0	0.178	0.228	0.630
8.0	0.214	0.222	1.079

Case 4 : 36 piles in three concentric rings of 6, 12 and 18 piles located at 1.7 m, 3.44 m and 5.16 radii respectively gave the following GER :

s/r	Vertical	Horizontal	Bending
6.0	0.119	0.155	0.365
8.0	0.157	0.196	0.620

Case 5 : 60 piles in four concentric rings of 6, 12, 18 and 24 piles located at 1.7, 3.44, 5.16 and 6.88 m radii gave the following GER :

s/r	Vertical	Horizontal	Bending
6.0	0.094	0.119	0.228
8.0	0.137	0.152	0.426

CONCLUSION

The method of pile-group stiffness calculation presented hereabove is rational and unambiguous and depends only on the correctness of the interaction factors α_{vir} and α_{hir}. The results show that the group efficiency ratio could be substatially less than 1.0 depending on the arrangement and compactness of the group, the more compact it is, lesser is the efficiency. Hence taking the stiffness of a group of piles as the sum of individual stiffnesses will very much overestimate the group stiffness, resulting in appreciable increase in the natural frequency of the total structural system, hence the seismic response too, while the seismic displacements will be much underestimated. Therefore pile group stiffness should be estimated properly to arrive at reasonable seismic response to pile supported structures.

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