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Proposal on Non-Linear Response Analysis Method in Frequency

Susumu Nakamura
Nihon University, Japan

Nozomu Yoshida
SATOKOGYO Co. Ltd., Japan

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Susumu Nakamura
College of Enginnering,Nihon University
Tamuramachi Nakagawara 1, Koriyama
Fukushima Pref. Japan 963-8642

Nozomu Yoshida
SATOKOGYO Co.Ltd.
4-12-20 Nihonbashi Honchyo
Chyuuoku Tokyo Japan 103-8639

ABSTRACT

The objective of this report is to propose a non-linear response analysis method in frequency domain based on the stress and strain relationship of soil in frequency domain. First of all, a new model for the strain and frequency dependent characteristics between shear modulus and damping constant with respect to the complex shear modulus is proposed, considering the following two characteristics; One is the similarity of stress-strain relationship between time domain and frequency domain. The other one is a non-stationary characteristics in frequency domain caused by non-linearity of soil. A new non-linear dynamic response analysis method in frequency domain is proposed by incorporating the new model of the stress-strain relationship in frequency domain into the commonly used non-linear response analysis method in frequency domain. The accuracy of this method is evaluated by the comparison of the analytical results obtained by two kind of non-linear dynamic response analysis codes. Hence, "YUSAYUSA2" and "SHAKE" is used as the co de in time domain, in frequency domain respectively. It is found that this proposed method is useful to take into account of non-stationary behavior caused by non-linearity of soil.

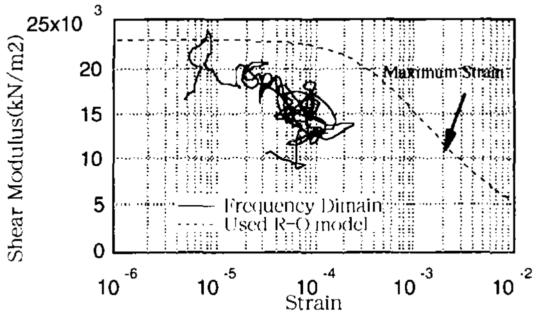
INTRODUCTION

The seismic design codes for most of civil engineering structures have been revised based on the serious damages due to the 1995 HYOGOKEN NANBU earthquake. Evaluation of dynamic behavior of ground has been required for seismic design of civil engineering structures against strong earthquake motion so called level 2 design earthquake. Not only seismic response analysis methods in time domain but also those in frequency domain have been used to evaluate such behavior. The influence of strong non-linearity on the soil has to be taken into account for those methods. Although the latter method has been used for a seismic design so far in comparison with the former method, it has been pointed out that the method is not available to evaluate the behavior under the generating strain level which is more than about 1.0%.. Because the method in frequency domain could not evaluate the following two processes caused by a strong non-stationary behavior of ground. One is the process changing the response characteristics of ground momentarily with the shear modulus changing ever moment. The other one is the process changing residual deformation characteristics. Hence, it is difficult to take into account of the latter process for the method in frequency domain. On the other hand, if it is possible to evaluate the frequency characteristics of the stress-strain relationship equivalent to that in time domain, the former process could be considered into the method in frequency domain. Hence, it has been made clear that a characteristics of stress-strain relationship in frequency domain was strongly associated with that in time domain by use of the method to transform stress-strain hysteresis in time domain to that in frequency domain (Nakamura, S., 1999).

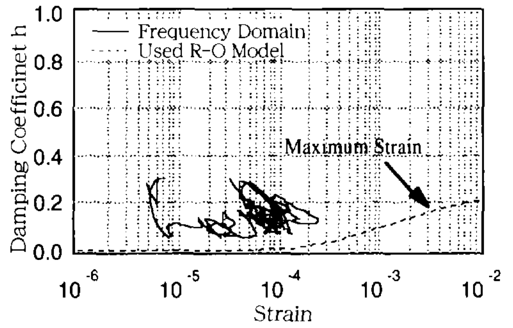
The objective of this report is to propose a method of non-linear response analysis in frequency domain considering a model of stress and strain relationship of soil in frequency domain. First of all, a new model for the strain and frequency dependent characteristics between shear modulus and damping constant with respect to the complex shear modulus is proposed, considering the following two characteristics; One is the similarity of stress-strain relationship between time domain and frequency domain. The other one is a non-stationary characteristics in frequency domain caused by non-linearity of soil. The proposed method is applied to evaluate the dynamic response at the ground of Chiba Experimental station of Institute of Industrial Science, University of Tokyo. The comparison of the analytical results with the other two kind of non-linear dynamic response analysis code is carried out to make sure the accuracy of this method. Hence, "YUSAYUSA2" (Yoshida, N. and Towhata, I., 1991)" and "SHAKE" are used as the code in time and frequency domain respectively. Furthermore, analytical results obtained by "YUSAYUSA2" are considered as the target values, in other words the virtual real behavior.

MODELING OF DYNAMIC DEFORMATION CHARACTEISTICS OF SOIL IN FREQUENCY DOMAIN

Based on the comarison of the stress-strain relationship in frequency domain to that in time domain (Nakamura,S. 1999), it has been pointed out that the deformation characteristics of soil in frequency was similar with those in time domain as shown in figure.1. However, according to the frequency



a)Strain dependency of Shear modulus



b)Strain dependency of Damping constrant

Fig.1 Comparison of Strain Dependency of Shear Modulus and Damping Constant in time and frequency domain

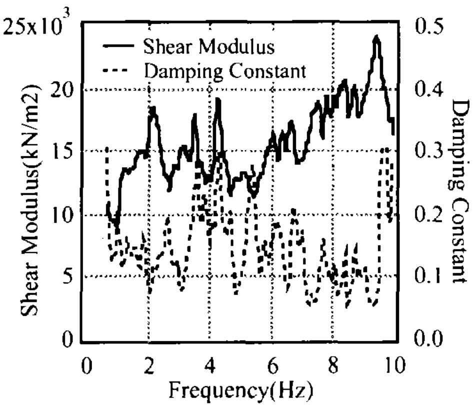


Fig.2 Frequency Characteristics of Shear Modulus and Damping Constant

characteristics of shear modulus as shown in Figure.2, the large reduction of shear modulus was observed not only at the frequency 2.9 Hz generated the maximum strain but also at the other frequencies(2.9-5.5Hz). Hence the other frequency is correspond to the higher mode natural frequency. These frequency characteristics were obtained by the analysis for the ground of Chiba Experimental station of Institute of Industrial Science, University of Tokyo. Based on these characteristics, strain dependency of shear modulus and damping constant in frequency domain are modeled as follows.

Model of Strain Dependency of Shear Modulus in Frequency Domain

First of all, it is paied attention to the similarity of the stress-strain relationship in both domain as shown in figure.3. Hence it is assumed that the strain and stress ratio in both domain is the same with each other. Then, the similarity of stress-strain relationship could be specified by the similarity of strain. Under this assumption, shear strain at an arbitrary frequency $\gamma_f(\omega)$ is possible to be expressed by multiplying coefficient Dft to a strain $\gamma_t(\omega)$ in time domain as shown equation(1). Hence, a strain $\gamma_t(\omega)$ represents a corrected strain equivalent to a strain on the stress-strain relationship specified in time domain. Herein a strain $\gamma_t(\omega)$ is named as a time equivalent strain. Then a time equivalent strain is obtained from a strain at an arbitrary frequency as equation (2).

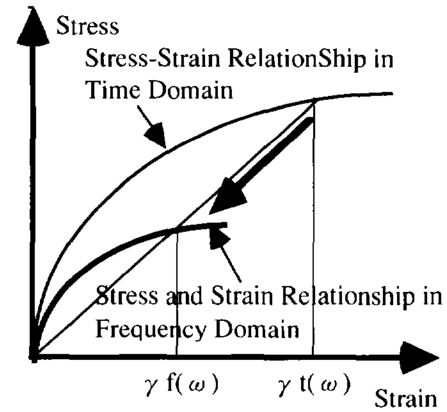


Fig.3 Image of the similarity of stress-strain relationship Between time domain and frequency domain

$$\gamma_f(\omega) = D_{ft}\cdot\gamma_t(\omega) \qquad (1)$$

$$\gamma_t(\omega) = \frac{1}{D_{ft}}\gamma_f(\omega) = C_{tf}\cdot\gamma_f(\omega) \qquad (2)$$

Hence, Ctf represents the coefficient to evaluate a time equivalent strain. Under the assumption of the similarity with respect to the stress and strain relationship in both domain, the coefficient has to be the strain ratio at the same shear modulus in both domain. Hence, it is possible to evaluate the ratio because not only strain but also stress are obtained at an arbitrary frequency by use of a calculated stress-strain hysteresis in time domain. However, the hysteresis is not obtained before carrying out the dynamic response analysis. Then as the method to evaluate the appropriate coefficient during a execution of analysis, the ratio between the maximum strain in frequency domain and that in time domain is also considered as the coefficient. The relationship between the coefficient calculated for the above two ideas and strain in frequency domain was shown in Figure.3. Hence, the coefficient based on the strain ratio for the same shear modulus in both domains is named as model 2. It is found that the values changes with the strain. On the other hand, the coefficient based on the maximum strain ratio is 12.2 at the layer between 2.0m and 3.0m, 11.1 at the layer between 4.0m and 5.0m respectively and is named as model 1. Next, figure.4 shows the comparison of the frequency characteristics of shear modulus based on a time equivalent strain estimated by use of these two models coefficients with the target characteristics as shown in figure.2. Hence, 5.0 which is the value at maximum strain in each layer is taken as the coefficient for model 2. Although the minimum shear modulus of the target is good agreement with that calculated by the method 2 at the frequency given the maximum strain, shear modulus at the other frequency is larger than the target. On the other hand, the shear modulus calculated by model 1 at the frequency, except the frequency given the maximum strain, is good agreement with the target. However the shear modulus calculated by model 1 at the frequency given the maximum strain is smaller than the target. Hence, model 1 is taken as the method to evaluate the coefficient Ctf by paying attention to such characteristics as the mentioned above. However, the frequency characteristics of the target shear modulus at the frequency between 2.9Hz and 5.5Hz is not correspond with that calculated by model 1. In order to make clear the difference of shear modulus between the target and the estimated frequency characteristics, a non-stationary response spectrum of strain calculated by "YUSAYUSA2" at the layer between 4.0m and 5.0m is shown in figure.6. This spectrum shows the frequency characteristics of strain at a time changing momentary. It is found that the predominant frequency around 5.5Hz observed at the time of 9.0 seconds changes the lower frequency with a time. This tendency supposed to be caused by the reduction of shear modulus at the layer, in other words the effect of non-linearity of soil. Based on this evidence, the characteristics of shear modulus at the frequency range between 2.9Hz and 5.5Hz is considered as the non-stationary behavior of soil caused by the non-linearity of soil. Hence, the frequency rang is named as a non-linearity affected frequency range. Then, a time equivalent strain obtained by equation(2) has to be corrected by taking into account of such a non-stationary behavior as shown equation(3).

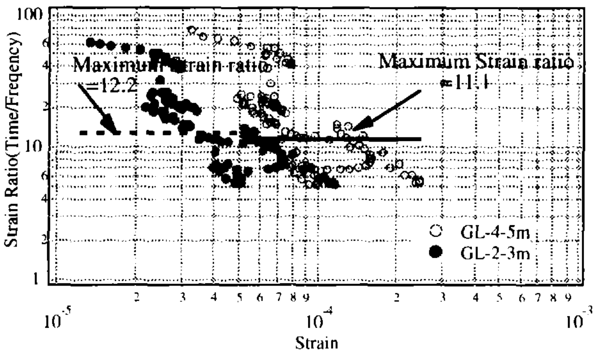


Fig.4 Comparion of the coefficient Ctf calculated by two Method

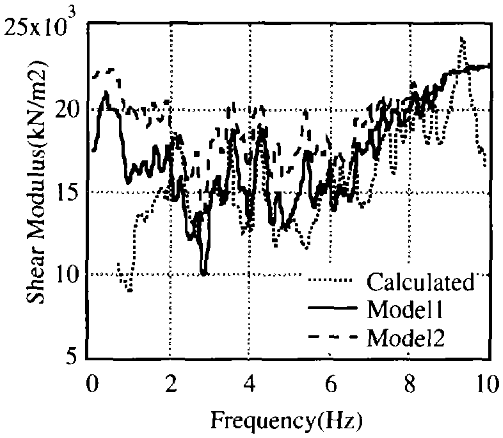


Fig.5 Comparion of the Frequency Characteristics of Shear Modulus calculated by the two Method with the target

$$\gamma_{tN}(\omega) = C_{fN}(\omega) \cdot \gamma_t(\omega) \le \gamma_t(\omega)_{\max} \qquad (3)$$

Hence, $C_{tN}(\omega)$ represents the coefficient to correct the effect of the non-stationary behavior and is named as a corrected time equivalent strain. The coefficient evaulates by use of the following equation.

$$C_{fN}(\omega) = 1 + (\alpha - 1) \cdot \frac{\omega - \omega_{lN}}{\omega_{hN} - \omega_{lN}}, \alpha = \frac{\gamma_t(\omega_{lN})}{\gamma_t(\omega_{hN})} \quad \omega_{lN} \le \omega \le \omega_{hN} \qquad (4)$$

$$C_{fN}(\omega) = 1 \quad \omega < \omega_{lN} \quad \omega_{hN} < \omega$$

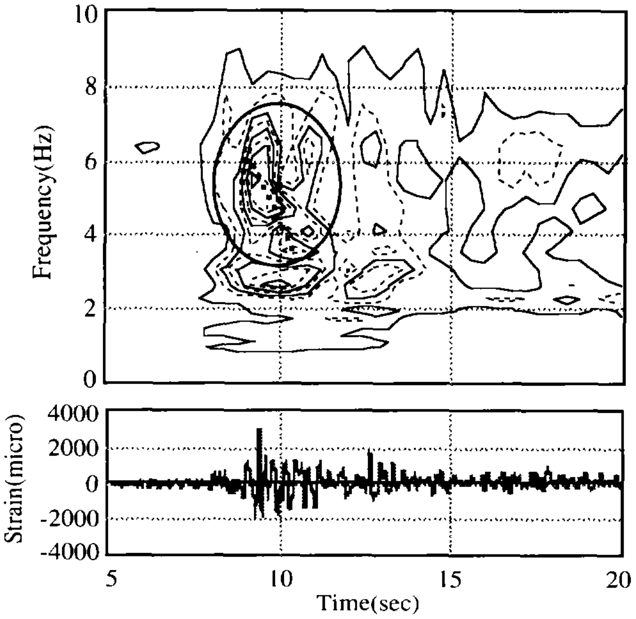


Fig.6 Non-Stationary Response Spectrum of Strain at the Layer between 4.0m and 5.0m

Based on the target frequency characteristics of the shear modulus as shown in figure.2, the coefficient is evaluated as the value to let the peak strain at the highest frequency in the non-linearity affected frequency range be the same with the maximum strain. Furthermore, the non-linearity affected frequency range frequency range is required to carry out this correction. Hence, the lower and higher frequency limit ω_{lN}, ω_{hN} in equation (4) have to be determined respectively. The lower frequency limit ω_{lN} is determined as the frequency given the maximum value of the strain in frequency domain. On the other hand, the higher frequency limit ω_{hN} supposed to be associated with the frequency for the largest change of the transfer function when the shear wave velocity at a layer changes. Then, the value is possible to be evaluated as the predominant frequency of the sensitivity function SF(ω) obtained by differentiating the transfer function with respect to shear wave velocity at the object layer.

$$SF(\omega) = \left| \frac{\partial U(\omega)}{\partial V_i} \right| \qquad (5)$$

Model of Damping Constant depend on strain in Frequency Domain

The frequency characteristics of damping constant h(ω) is modeled as the sum of the hysteresis damping in first term and the scattering damping in second term as shown in equation (6).

$$h(\omega) = h_h(\gamma_{tN}(\omega)) + h_s \cdot f^a \qquad (6)$$

This model has been used as the evaluation of the damping characteristics for a deeper ground structure (Dainity,M., 1981) and applied to evaluate the characteristics for the surface soil deposits(Nakamura,S,1994). Hence, first term represents damping characteristics due to large strain and is determined to substitute the corrected time equivalent strain to stress-strain relationship in time domain. Second term represents scattering damping characteristics during wave propagating and is used as the evaluated values (hs =0.097,a=-0.57) by author (Nakamura, S., 1994)

PROPOSE OF ONE DIMENTIONAL NON-LINEAR RESPONSE ANALYSIS METHOD IN FREQUENCY DOMAIN

The analytical flow of the proposed one dimensional non-linear dynamic response analysis in frequency domain is shown in figure.7. First of all, initial ground condition such as ground structure, elastic modulus, stress-strain relationship of each soil in time domain and input motion have to be evaluated as well as the other method (ie, SHAKE etc). Next, higher limit frequency ω_{Nh} of the non-linearity affected frequency range at each layer is calculated for initial ground condition. Furthermore, the frequency ω_{Nl} at the maximum response strain in each layer is calculated by the response analysis based on multi-reflection theory. Finally, shear modulus and damping constant which are used for the calculation on the next step are calculated for the corrected time equivalent strain based on the method as the mentioned previous chapter. These process carry out until the difference between the maximum strain in time domain at the existing step i and that at the previous step i-1 becomes less than the specified value.

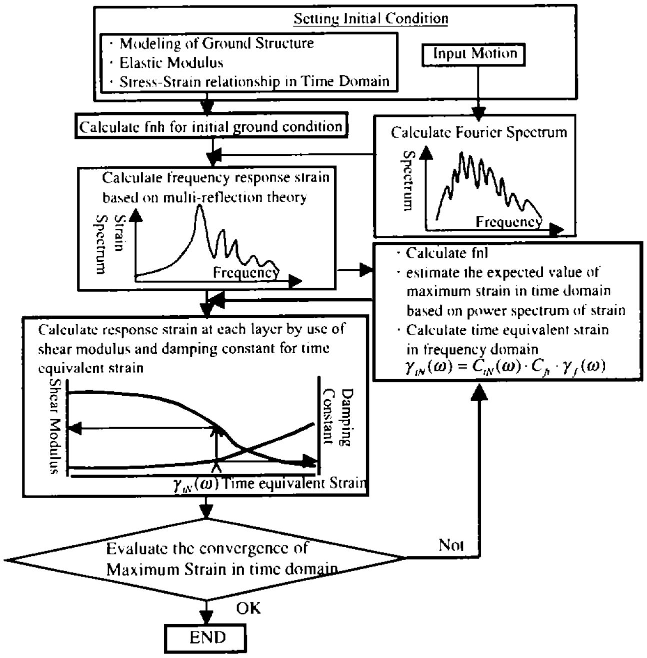


Fig.7 Analytical Flow of Proposed Method

ANALYTICAL RESULTS

The proposed method was applied to evaluate the dynamic response of ground at Chiba Experimental station of Institute of Industrial Science, University of Tokyo. The analytical results obtained by this method was compared with those obtained by the analytical code "SHAKE". Hence, the value 0.65 was used as the coefficient to evaluate a effective strain for SHAKE. Except the value, the input data is the same with each method. Furthermore, the accuracy of this method was evaluated by the comparison with the analytical results obtained by use of dynamic response analysis code "YUSAYUSA2" (Yoshida,N. and Tohata, I., 1991)" in time domain. Hence, all of the analytical results obtained by "YUSAYUSA2" is named as the target.

First of all, maximum response acceleration, displacement, stress and strain with depth obtained by the three methods are shown in figure.8. Hence, the maximum stress obtained by this method is almost the same with the target. As for the maximum acceleration, although the values around ground surface obtained by this method is the same with the values obtained by SHAKE, the values is larger than the target value. According to maximum displacement, the value at ground surface obtained by this method is smaller than the target. However, the value at ground surface is close to the target value in comparison with the value obtained by SHAKE. Next, acceleration time histories obtained by the proposed method and SHAKE are compared with the target as shown in figure.9. At the time between 9 and 10 seconds, maximum accelerations are generated and the wave form obtained by the proposed method and SHAKE is good agreement with the target. Furthermore, the phase of wave form obtained by the proposed method is correspond with the target in comparison with that by SHAKE. Next, stress-strain hysteresis obtained at the layer between 4 and 5 m below the ground surface by the two methods are shown in figure.10 with the skeleton curve of R-O model which was used for the stress-strain relationship in "YUSAYUSA2". In comparison of the hysteresis obtained by SHAKE, the hysteresis obtained by the proposed method has similar characteristics with the skeleton curve of the target stress-strain relationship. Next, converged shear modulus and damping constant for the proposed method and SHAKE are shown in figure.11. As for the frequency characteristics of shear modulus, the difference between the values obtained by the proposed method and those by SHAKE are less than 10% at the non-linearity affected frequency range. Therefore, the wave forms obtained by the both method are good agreement with the target wave form each other around the time between 9 and 10 seconds. On the other hand, damping constant obtained by the proposed method at the non-linearity affected frequency range are larger than that by SHAKE because the frequency dependent term for SHAKE has not been considered. However, the accuracy of the scattering damping term in equation (6) with respect to damping characteristics has to be improved in future.

CONCLUDING REMARKS

A new non-linear response analysis method in frequency domain was proposed based on the stress and strain relationship of soil in frequency domain. First of all, in order to evaluate the strain and frequency dependent characteristics with respect to shear modulus and damping constant of the complex shear modulus, a corrected time equivalent strain as shown the following equation was proposed.

$$\gamma_{tN}(\omega) = C_{tN}(\omega)\cdot C_{tf}\cdot\gamma_f(\omega) \qquad (7)$$

The strain and frequency dependent characteristics with respect to shear modulus and damping constant is possible to evaluate by substituting the strain into the stress – strain relationship in time domain. Hence, C_{tf} is the coefficent to consider the similarity of stress-strain relationship between time domain and frequency domain. $C_{tN}(\omega)$ is the coefficent to consider the non-stationary characteristics in frequency domain caused by non-linearity of soil. A new non-linear dynamic response analysis method in frequency domain is proposed by incorporating the new model of the stress-strain relationship in frequency domain into the commonly used non-linear response analysis method in frequency domain. The accuracy of this method was evaluated by the comparison of the analytical results obtained by non-linear dynamic response analysis codes in time domain. Furthermore, the comparison with the analytical results obtained by the code "SHAKE" which has been used for a seismic design was carried out. As a result of these, it is found that the proposed method is useful to evaluate a non-linear dynamic response of ground in comparison with SHAKE. It will be expected for this method to apply the problem generating high strain in the ground.

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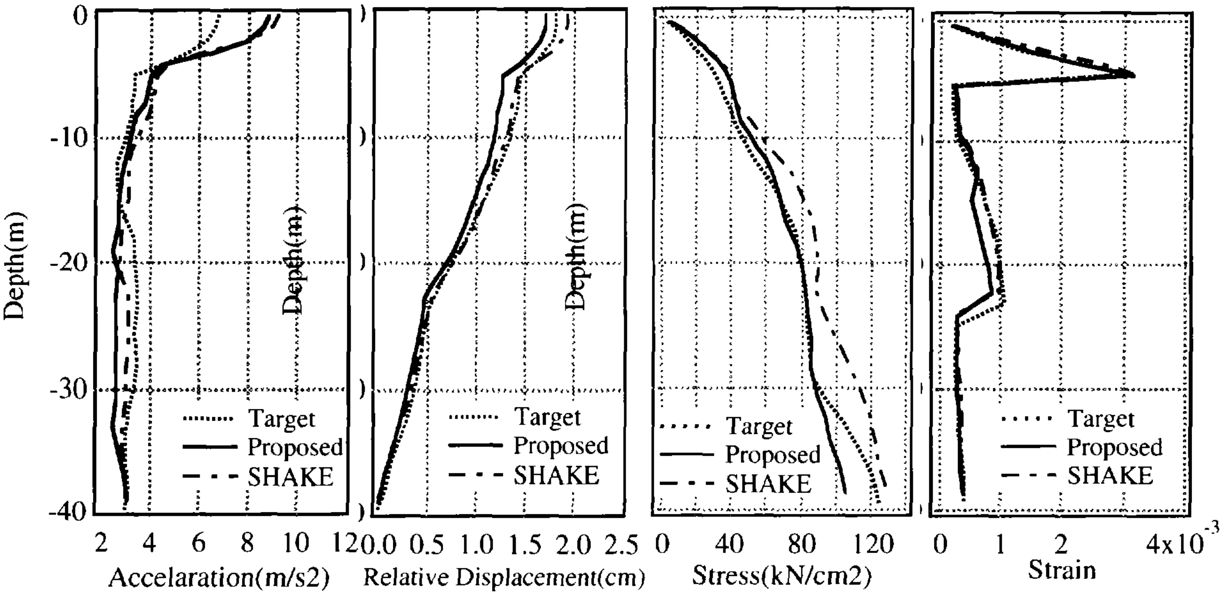
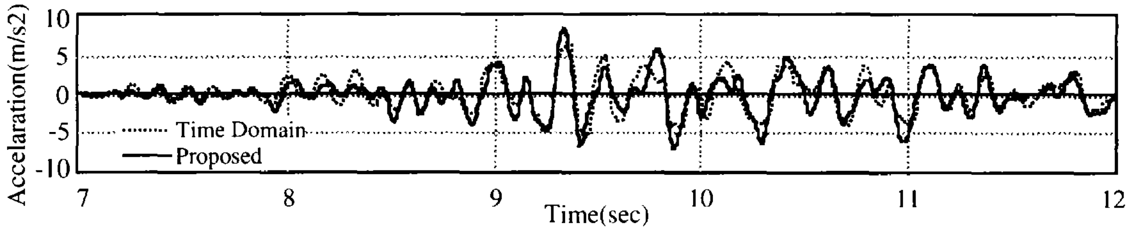
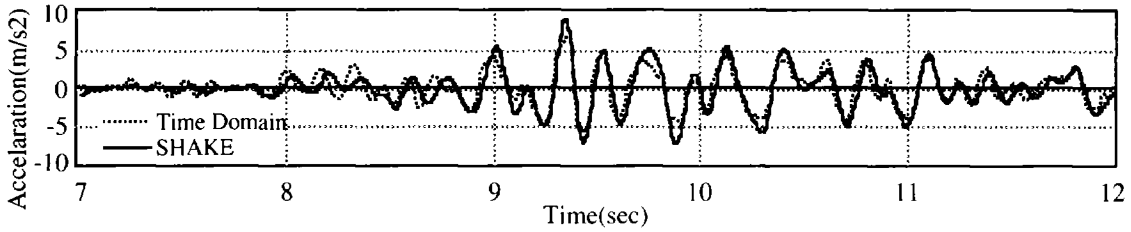


a)Acceleralation *b)Relative Displacement* *c)Stress* *d)Strain*

Fig.8 Comparison of Maximum Response with Depth for Each Method

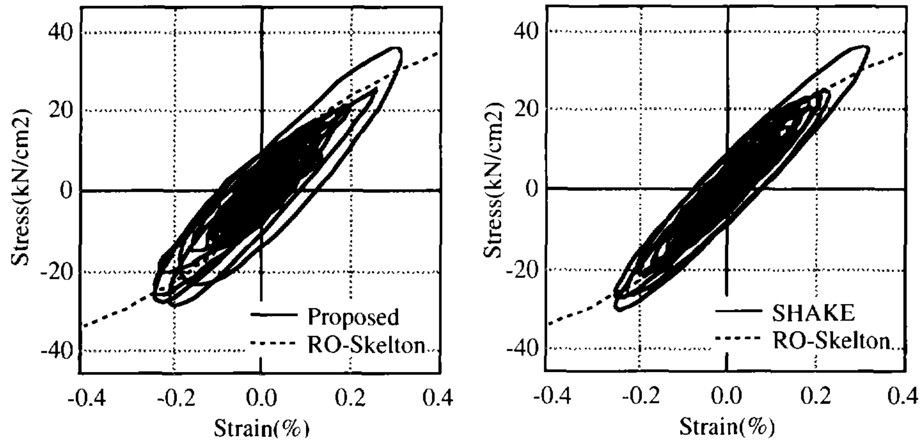


a)Proposed Method



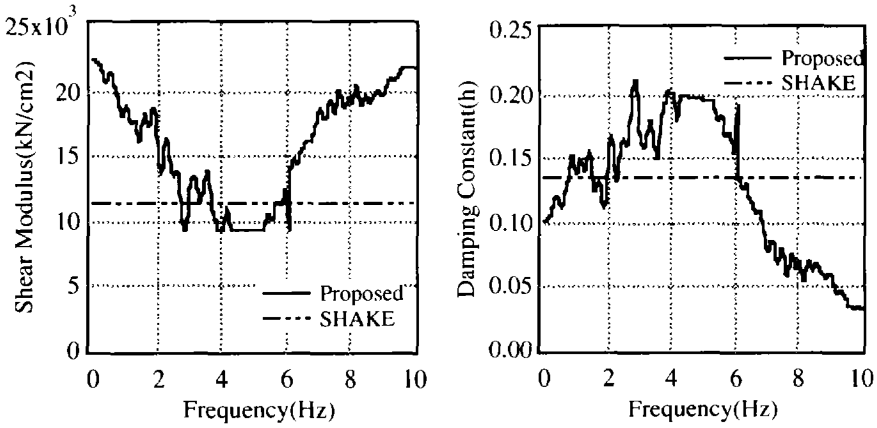
b)SHAKE

Fig.9 Comparison of Acceleration time histories obtained by Proposed Method and SHAKE to the Target



a)Proposed Method *b)SHAKE*

Fig.10. Comparison of Hysteresisi obtained by Proposed Method and SHAKE



a)Converged Shear Modulus *b)Converged Damping Constant*

Fig.11 Comparison of Converged Shaer Modulus and Damping Constant Obtained By Proposed Method and SHAKE